Hydrodynamical Trigger Mechanism for Pulsar Glitches

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We describe a new instability that may trigger the global unpinning of vortices in a spinning neutron star, leading to the transfer of angular momentum from the superfluid component to the star's crust. The instability, which is associated with the inertial r modes of a superfluid neutron star, sets in once the rotational lag in the system reaches a critical level. We demonstrate that our simple model agrees well with the observed glitch data. This new idea should stimulate work on more detailed neutron star models, which would account for the crustal shear stresses and magnetic field effects we have ignored.

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Introduction.—Even though pulsars are generally very stable rotators, in some cases with an accuracy that rivals the best terrestrial atomic clocks, many systems exhibit a variety of timing "noise." The most enigmatic features are associated with the so-called glitches, sudden spin-up events followed by a relaxation towards steady long-term spin-down. Several hundred glitches, with magnitude in the range $\Delta\Omega_c/\Omega_c \approx 10^{-9} - 10^{-6}$ where Ω_c is the observed rotation frequency, have now been observed in over 100 pulsars [1]. The archetypal glitching pulsar is Vela, which exhibits regular large glitches. Glitches have also been reported in several magnetars [2] as well as one, very slowly rotating, accreting neutron star [3].

Despite the relative wealth of observational data our theoretical understanding of glitches has not advanced considerably in recent years. The standard "model" for large pulsar glitches envisages a sudden transfer of angular momentum from a superfluid component to the rest of the star [4], which includes the crust (to which the pulsar mechanism is assumed rigidly attached) and the charged matter in the core. A superfluid rotates by forming a dense array of vortices, and the vortex configuration determines the global rotation. The key idea for explaining glitches is that, if the superfluid vortices are pinned to the other component, a rotational lag builds up as the crust spins down due to electromagnetic braking. Once the rotational lag reaches some critical level, the pinning breaks. This allows the vortices to move, which leads to a transfer of angular momentum between the two components and the observed spin-up of the crust.

Most theoretical work has focused on either the strength of the vortex pinning [5,6] or the post-glitch evolution [7]. There have not been many suggestions for the mechanism that triggers the glitch in the first place. It is generally expected that this role will be played by some kind of instability, but there are few truly quantitative models. The results presented in this Letter change the situation dramatically. We present evidence for a new instability, acting on the inertial modes of a rotating superfluid star, that sets in beyond a critical rotational lag. The predictions of this model agree well with the observational data making it plausible that this mechanism provides a missing piece in the pulsar glitch puzzle.

Inertial mode analysis.—We want to improve our understanding of the hydrodynamics associated with a pulsar glitch. Even though this should be a key issue, it has not been discussed in detail previously. In principle, one should be able to express the dynamics in terms of global oscillation modes of the system. In this Letter we present the first ever results for inertial modes of neutron star models with the two main features required for the standard glitch models, a superfluid neutron component that rotates at a rate different from that of the crust and "pinned" neutron vortices.

We use the standard two-fluid model for superfluid neutron stars (see for example [8]), identifying the two components with the neutron superfluid and a conglomerate of all charged particles (the "protons"). In the following, the index $x = \{n, p\}$ identifies the distinct fluids. Our aim is to model small amplitude oscillations with respect to a background configuration where both fluids rotate rigidly with (parallel) angular velocities $v_x^i = \epsilon^{ijk}\Omega_j^x x_k$ and where the magnitudes are different, $\Omega_n \neq \Omega_p$. The linear perturbations of this system (assuming a time dependence $\sim \exp(i\sigma t)$) are, in the inertial frame, described by the two coupled Euler equations,

$$(i\sigma + v_n^j\nabla_j)\delta v_n^i + \delta v_n^j\nabla_j v_n^i + \nabla^i\delta\psi_n = \delta f_{\rm mf}^i, \quad (1)$$

$$(i\sigma + v_p^j\nabla_j)\delta v_p^i + \delta v_p^j\nabla_j v_p^i + \nabla^i\delta\psi_p = -\delta f_{\rm mf}^i/x_p. \quad (2)$$

Here $\delta\psi_x = \delta\tilde{\mu}_x + \delta\Phi$ represents the sum of the perturbed specific chemical potential and gravitational potential. We have also introduced $x_p = \rho_p/\rho_n$. This ratio, which is roughly equal to the proton fraction, is assumed constant throughout the star. For simplicity, we assume that the two fluids are incompressible, which means that $\nabla_i\delta v_x^i = 0$. In general, the two fluids are coupled (i) chemically, (ii) gravitationally, (iii) via the entrainment effect, and (iv) by the vortex mediated mutual friction $f_{\rm mf}^i$.

For clarity, we will ignore the entrainment in the present analysis. A detailed discussion of how the entrainment affects our results will be provided elsewhere.

For the inertial modes, the main coupling mechanism is provided by the mutual friction force. The general expression for this force is [9],

$$f_i^{\mathrm{mf}} = \mathcal{B}\epsilon_{ijk}\epsilon^{kml}\hat{\omega}_n^j\omega_m^n w_l^{np} + \mathcal{B}'\epsilon_{ijk}\omega_n^j w_{np}^k, \qquad (3)$$

where $w_{np}^i = v_n^i - v_p^i$ and $\omega_n^i = \epsilon^{ijk}\nabla_j v_k^n$. A "hat" denotes a unit vector. When the two fluids are not corotating, the perturbed force δf_{mf}^i is quite complex [10]. The form (3) for f_{mf}^i results from balancing the Magnus force that acts on the neutron vortices and a resistive "drag" force between the vortices and the charged fluid. Representing the drag force by a dimensionless coefficient $\mathcal{R}$, one finds that $\mathcal{B}' = \mathcal{R}\mathcal{B} = \mathcal{R}^2/(1+\mathcal{R}^2)$. The range of values that $\mathcal{R}$ takes in a neutron star is not well known. The standard assumption has been that the drag is weak [11], which means that $\mathcal{B}' \ll \mathcal{B} \ll 1$. Then the second term in (3) has no effect on the dynamics. However, it may well be the opposite limit that applies. The vortices in a neutron star core may experience a strong drag force if their interaction with the magnetic fluxtubes is efficient [12–14]. The drag on the superfluid in the crust may also be strong due to vortex "pinning" by the lattice nuclei. Even though the current evidence [5,6] favors weak crustal pinning, the existence of strong pinning regions has not been completely ruled out. In these cases one must consider the strong coupling limit $\mathcal{R} \gg 1$, i.e., $\mathcal{B} \approx 0$ and $\mathcal{B}' \approx 1$.

The hydrodynamical equations (1) and (2) allow for a rich set of oscillation modes. Here we focus on a subset of the inertial modes, the purely axial r modes. The r modes have attracted attention since they may suffer a gravitational-driven instability [15]. They have been studied in superfluid neutron stars previously [16–18], but these studies have not accounted for both mutual friction and a rotational lag. Expressed in terms of spherical harmonics, the r mode velocity fields take the form

$$\delta v_x^i = \left(-\frac{imU_l^x Y_l^m}{r^2\sin^2\theta}\hat{e}_\theta^i + \frac{U_l^x\partial_\theta Y_l^m}{r^2\sin\theta}\hat{e}_\varphi^i\right)e^{i\sigma t}. \qquad (4)$$

In the corresponding single fluid problem such purely axial solutions exist, corresponding to a single $l = m$ multipole. The solution (4) satisfies the continuity equations automatically, which means that we only have to consider the two Euler equations. Inserting (4) in (1) and (2) and eliminating $\delta\psi_x$ one arrives at two equations for U_l^x. As in the single fluid inertial mode problem, these relations feature couplings between different l multipoles. In the present problem these couplings are further complicated by the presence of δf_{mf}^i. As a result, one would expect the problem to be difficult to solve. Remarkably, this is not the case. It turns out that, even when $\Omega_n \neq \Omega_p$, there exists a simple $l = m$ solution of the form $U_m^x = A_x r^{m+1}$. This solution is exact at leading order in rotation and for all values of $\mathcal{R}$. The amplitudes A_x follow from the solution of a 2×2 algebraic system, the determinant of which provides the dispersion relation for the mode frequency.

A new superfluid instability.—The strong coupling limit, $\mathcal{B} = 0$ and $\mathcal{B}' = 1$, provides a good illustration of the main new result. In this case we have, in terms of the dimensionless parameters $\kappa = (\sigma + m\Omega_p)/\Omega_p$ and $\Delta = (\Omega_n - \Omega_p)/\Omega_p$, the two frequency solutions

$$\kappa_{1,2} = -(1 - x_p + \Delta \pm \mathcal{D}^{1/2})/(m+1)x_p, \qquad (5)$$

where

$$\mathcal{D} = (1+x_p)^2 + 2\Delta[1 + x_p\{3 - m(m+1)\}] + O(\Delta^2). \qquad (6)$$

For the amplitudes we find $A_n/A_p = 2(1+\Delta)/(m+1)\kappa$.

Let us focus on the short length scale modes. Taking $m \gg 1$ and recalling that x_p and Δ are both small (generally $\Delta \ll x_p$), we find that one of the r mode solutions becomes unstable ($\mathrm{Im}[\kappa] < 0$) for $m > m_c$, where

$$m_c \approx (2x_p\Delta)^{-1/2} \approx 320 \qquad (7)$$

for the typical value $\Delta = 10^{-4}$. Here, and in the following, we use $x_p = 0.05$. For $m \gg m_c$ the instability growth time scale $\tau_{\mathrm{grow}} = 1/(\Omega_p\mathrm{Im}[\kappa])$ is well approximated by

$$\tau_{\mathrm{grow}} \approx (P/2\pi)(x_p/2\Delta)^{1/2} \approx 0.25(\Delta/10^{-4})^{-1/2}\ \mathrm{s}, \qquad (8)$$

where we have taken $P = 2\pi/\Omega_p = 0.1$ s as a typical observed spin period (we associate the charged component with the crust, $\Omega_p = \Omega_c$). We see that the instability can grow rapidly, on a time scale comparable to the rotation period of the star.

Although we cannot yet claim to understand the detailed nature of this new r mode instability, we have some useful clues. We find that the unstable modes are such that $|A_n/A_p| \sim x_p$. Thus, the fluid motion is predominantly in the proton fluid. There should also be a close connection with the short wavelength instability that we recently discovered for precessing superfluid stars [19]. In that case, the result followed from a local plane-wave analysis of the inertial modes. An attempt to link these two results would be useful. Finally, since the present system has two distinct rotation rates one might expect the instability to belong to the general two-stream class [18]. Such instabilities are generic in multifluid systems. The intuitive condition for such an instability dictates that the mode's pattern speed $-\mathrm{Re}(\sigma)/m$ should lie between Ω_n and Ω_p [18]. This translates into $m > \sqrt{2}m_c$, which is, indeed, satisfied for most of the instability regime.

In order for the new instability to affect the dynamics of realistic neutron stars it must (at least) overcome viscous damping. For young and mature neutron stars, dissipation is dominated by shear viscosity due to electron-electron collisions [20]. For a uniform density star with $M = 1.4M_\odot$ and $R = 10$ km (the values used in the following) the corresponding viscosity coefficient is $\eta_{\mathrm{ee}} \approx 2.7\times 10^{20}T_8^{-2}$ g/cm s, where $T_8 = T/10^8$ K represents the core temperature [21]. We can use the standard energy-integral approach to estimate the viscous damping time scale. In fact, since $|A_n| \ll |A_p|$ we can use existing results for r modes of uniform density stars [22], remembering that shear viscosity only acts on the proton fluid. Thus, a simple calculation leads to $\tau_{\rm sv} \approx 6 \times 10^4 T_8^2/m^2$ s for our canonical parameters. The unstable modes will grow fast enough to overcome viscous damping if $\tau_{\rm grow} < \tau_{\rm sv}$. This leads to

$$m < 500(\Delta/10^{-4})^{1/4}(P/0.01\ {\rm s})^{-1/2}T_8, \qquad (9)$$

which gives the range of unstable m modes. Since the instability sets in when $m > m_c$ we conclude that the system becomes unstable once it reaches the critical lag

$$\Delta_c \approx 6 \times 10^{-5}(P/0.1\ {\rm s})^{2/3}T_8^{-4/3}. \qquad (10)$$

Making contact with observations.—Within a two-component model, it is straightforward to estimate the critical lag required to explain the observations. Assuming that angular momentum is conserved in the process, one must have $I_c\Delta\Omega_c \approx -I_s\Delta\Omega_s$, where I_s and I_c are the two moments of inertia, while $\Delta\Omega_s$ and $\Delta\Omega_c$ represent the changes in the spin frequencies. The glitch data suggest that about 2% of the total spin-down is reversed in the glitches [23], indicating that $I_s/I_c \approx 0.02$. In order to permit Vela-sized glitches with $\Delta\Omega_c/\Omega_c \sim 10^{-6}$ we need (assuming $\Omega_c = \Omega_s$ after the event)

$$\Delta_g \approx (I_c/I_s)(\Delta\Omega_c/\Omega_c) \approx 5 \times 10^{-4}. \qquad (11)$$

The observational estimate of the lag Δ_g at which large glitches occur is similar to our estimate (10) for the onset of the superfluid r mode instability. We do not think this is a coincidence. Even though it is difficult to compare the parameters of our two-fluid neutron star model to the global quantities used in the phenomenological discussion directly, it is clear that our new instability has the features expected of a glitch trigger mechanism. It operates in the strong drag limit, where vortices are effectively pinned to the charged component. As long as the system is stable, a rotational lag should build up as the crust spins down. Once the system evolves beyond the critical level (10) a range of unstable r modes grow on a time scale of a few rotation periods. We cannot say what happens when these modes reach large amplitudes, but it seems inevitable that the fluid motion associated with the instability will break the vortex pinning, allowing a glitch to proceed.

Let us compare the "predictions" of our model to the data for pulsars exhibiting large glitches. To do this, we estimate the maximum glitch size allowed if $\Delta_g = \Delta_c$, assuming a completely relaxed system and $I_s/I_c = 0.02$. Since we do not have temperature data for most glitching pulsars, we estimate T by combining the heat blanket model from [24] with a simple modified URCA cooling law. Calibrating this model to the Vela pulsar, for which $T \approx 6.9 \times 10^7$ K [25], we find $T_8 \approx 3.3(t_c/1\ {\rm yr})^{-1/6}$ K. Here $t_c = P/2\dot{P}$ is the characteristic pulsar age. The results are shown in Fig. 1. This Figure shows that our model does well in predicting the maximum glitches one should expect. The data are consistent with the idea that a system needs to evolve into the instability region before a large glitch happens. It should be noted that, even though the instability first appears at $m = m_c$, the growth time is much longer than the estimated $\tau_{\rm grow}$ until $m > 1.2m_c$ or so. It is also interesting to note that two of the systems with actual temperature data [25], Vela and PSR B1706-44, both sit on the $m \approx 1.6m_c$ curve in Fig. 1. Moreover, it is worth noting that $\tau_{\rm grow}$ is shorter than the currently best resolved glitch event even for $\Delta \sim 10^{-7}$ (e.g., for the Crab). Finally, it is worth keeping in mind that since we are assuming that the system is completely relaxed in each individual event, our estimates provide an upper limit on the observed glitches. In reality the rotational lag may only be partially relaxed, which would explain why some glitches are smaller.

Discussion.—We have described a new instability that may operate in rotating superfluid neutron stars. We have demonstrated that this instability sets in at parameter values that compare well with those inferred from pulsar glitches. This suggests that this kind of instability may be the mechanism that triggers large pulsar glitches. This model is consistent with a number of observed properties of glitching pulsars: (i) Adolescent pulsars, like Crab and PSR J0537-69, should only exhibit small amplitude glitches. For fast spin and a relatively high temperature the instability sets in at smaller values of Δ. (ii) More

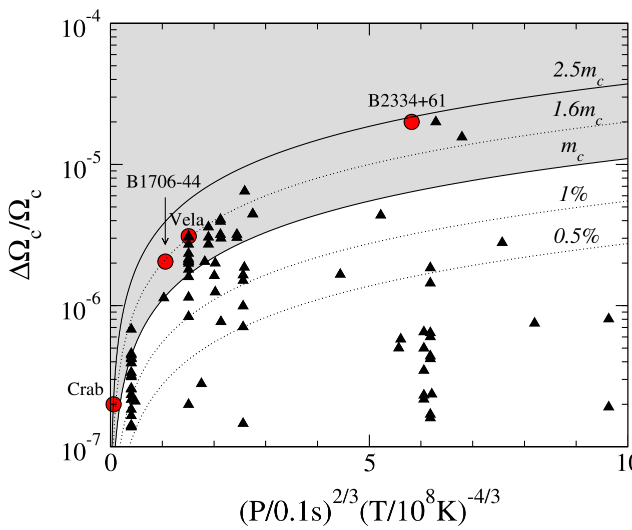

FIG. 1 (color online). The maximum glitches predicted from (10) are compared to observations. The m_c curve represents the onset of the superfluid instability (the instability region is gray, and for $2.5m_c$ there is a wide range of unstable inertial modes) and a glitch involving the relaxation of 2% of the total moment of inertia (corresponding curves for 1% and 0.5% are also shown). The circles are systems with temperature data (taken from [25]). The triangles represent systems for which T is estimated. The glitch data is taken from [1,27].

mature and slower spinning neutron stars have colder cores which means they can produce larger glitches, provided the required Δ can build up (cf. PSR J1806-21 with $\Delta\Omega_c/\Omega_c \approx 1.6 \times 10^{-5}$ [26]). Since Δ_c increases as the star ages one would expect neutron stars to cease to glitch eventually. (iii) For any glitch mechanism that relies on a critical spin lag between a superfluid component and the rest of the star, it is easy to estimate the time interval t_g between successive glitches. Assuming that each glitch relaxes the system completely (see comment earlier), we estimate $t_g \approx 2\Delta_g t_c \gtrsim 2\Delta_c t_c$. For Vela we then find $t_g \gtrsim$ 750 d, which compares well with the observed averaged time of about 1000 d. The most regular known glitcher, PSR J0537-69, has an average interglitch time of about 120 d [27]. In the absence of temperature data we use the simple cooling law for this object and find $t_g \gtrsim 90$ d. Again, the agreement with the observations is good, and consistent with the notion that the system evolves into the unstable regime before a glitch occurs. One should also remember that we have assumed perfect vortex pinning in between the glitch events. A more detailed analysis would account for vortex creep [7]. This will tend to increase the time it takes to build a given rotational lag. This may bring the estimates closer to the observed interglitch times. (iv) There is no reason why the instability should not operate in all spinning neutron stars in which a rotational lag builds up. In particular, one may expect accreting neutron stars to "glitch" occasionally. So far, there has only been one suggested event, in the slowly rotating transient KS 1947 + 300 [3]. For this system our model suggests (combining $P = 18.7$ s with $T \approx 10^8$ K, a typical temperature for an accreting star) a maximum glitch of $\Delta\Omega_c/\Omega_c \approx 4 \times 10^{-5}$. This is close to the suggested observed glitch with $\Delta\Omega_c/\Omega_c \approx 3.7 \times 10^{-5}$ [3]. Such events should, of course, be extremely rare.

Since we have considered the nonmagnetic inertial mode problem, our model does not apply (without modification) to magnetars. It is nevertheless interesting to consider these systems. Given typical magnetar parameters ($P \sim 10$ s, $T \sim 10^9$ K), we would not expect these objects to exhibit large glitches. Yet, they do [2]. Perhaps these glitches involve a larger fraction I_s/I_c?

A key question for the future concerns the presence or absence of this kind of instability in a more realistic neutron star model. It is clear that both magnetic tension and crustal shear stresses may alter the inertial modes that we have considered here significantly. What is not at all clear is whether the modes of a more complex multifluid system will suffer an analogous instability. The answer to this question requires a detailed mode calculation. This problem is a serious challenge, but there is some evidence that this kind of instability may operate in a magnetized star [28].

The results presented in this Letter are promising, but we are still far away from a complete understanding of this new mechanism. Future work needs to consider more detailed neutron star models. We need to understand the local mutual friction parameters and the nature of vortex pinning. We should also make more detailed attempts at understanding the observations. Finally, we need to understand the nonlinear development of the instability. This problem can perhaps be studied with numerical simulations [29]. Ultimately, one would hope to arrive at a truly quantitative model for pulsar glitches.

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