Terahertz Bloch Oscillator with a Modulated Bias

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(Received 21 December 2008; published 10 April 2009)

Electrons performing Bloch oscillations in an energy band of a dc-biased superlattice in the presence of weak dissipation can potentially generate THz fields at room temperature. The realization of such a Bloch oscillator is a long-standing problem due to the instability of a homogeneous electric field in conditions of negative differential conductivity. We establish the theoretical feasibility of stable THz gain in a long superlattice device in which the bias is quasistatically modulated by microwave fields. The modulation waveforms must have at least two harmonics in their spectra.

DOI: 10.1103/PhysRevLett.102.140405 PACS numbers: 03.65.Sq, 07.57.Hm, 72.30.+q, 73.21.Cd

An electrically biased semiconductor superlattice (SL), which operates in conditions of a single miniband transport regime, exhibits a static negative differential conductivity (NDC) due to Bloch oscillations [1]. Assuming a homogeneous electric field inside the nanostructure, the semiclassical theory predicts that a SL in the NDC state can provide small-signal gain for all frequencies below the Bloch frequency [2]. If miniband electrons can perform relatively many cycles of Bloch oscillations between scattering events, the Bloch gain profile is shaped as a familiar dispersion curve [Fig. 1(c), blue curve]. In general, the dispersive gain profile is a quantum dissipative phenomenon which is not limited to the Bloch oscillating electrons. Similar profiles with a resonant crossover from loss to gain have been observed in optical transitions of quantum cascade lasers [3] and in the microwave response of Josephson junctions [4]. Importantly, estimates for typical SLs predict a significant THz gain even at room temperature for frequencies in the vicinity of the Bloch gain maximum. Thus, the principal scheme of the cw Bloch oscillator would consist of a biased SL which is placed in a high-Q cavity tuned to a desirable THz frequency. However, even for moderate carrier densities a state with NDC is unstable against a development of space-charge instability which eventually results in the formation of high-field domains inside a long sample [5]. The formation of electric domains, which breaks the assumption of a homogeneous electric field used in [2], ultimately represents one of the major obstacles to the realization of the Bloch oscillator.

In short SLs the formation of electric domains can be suppressed. However, since a short sample can emit only a weak electromagnetic power, a stack of such SLs is required to observe the Bloch gain [6]. In our research we focus on the feasibility of the Bloch gain in a long SL. To circumvent the electric instability in a long SL it is necessary to modify the unstable Bloch gain profile in such a way that the sample exhibits a positive dynamical conductivity at low frequencies, including dc, whereas the high-frequency conductivity is still negative giving rise to the desired amplification [7,8]. To reach this goal it was suggested to use SLs with narrow gaps, where in addition to the intraband Bloch oscillations the interband Zener tunneling becomes important [9]. In a recent publication [10], we theoretically showed that the desirable modification of the Bloch gain profile still can be achieved within the standard single miniband transport regime. We utilized additional peaks in the voltage-current (VI) characteristics of a SL, which arise under the action of a strong ac field if the Bloch frequency is close to the ac field frequency or its harmonics [11]. However, the need for the use of a strong THz field as an ac pump limits the applied aspect of this suggestion.

In this Letter, we show that the unstable Bloch gain profile can be made stable by means of a low-frequency

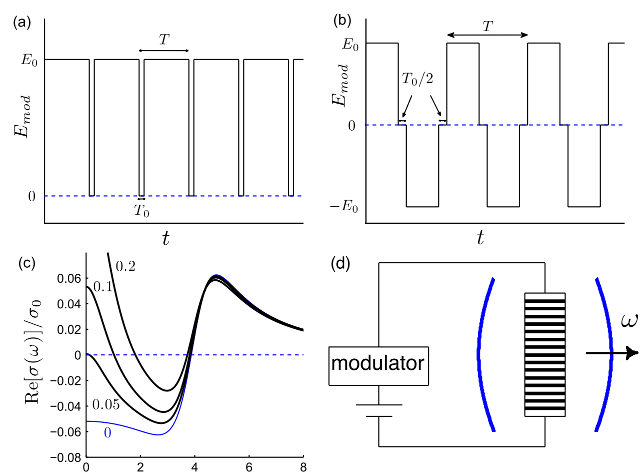

FIG. 1 (color online). (a),(b) Two waveforms of the bias modulation $E_{\rm mod}(t)$ which result in the same dynamical conductivity of a SL [Eq. (1)]. (c) Gain profiles calculated using Eq. (1) for $E_0 = 4E_{\rm cr}$ and $T_0/T =$: 0, 0.05, 0.1, 0.2. (d) Sketch of the Bloch oscillator with a modulated bias.

modulation of dc bias applied to a long SL. At least two harmonics with a proper phase difference are necessary to be present in the modulation spectrum in order to suppress the destructive low-frequency instability. The degree to which the large magnitude of THz gain near the Bloch resonance may be preserved here depends on the modulation waveform. In this respect we find the pulse modulation optimal. Since the operation of the Bloch oscillator requires that during at least some portion of the modulation period the SL is biased to the NDC state, an accumulation of charge can arise. The requirement that the charge accumulation is weak imposes a lower limit on the modulation frequency, which, however, still belongs to the microwave frequency domain. Finally, our numerical simulations show that the effect of stable THz gain persists if boundary conditions in the SL device are taken into account.

We consider a SL under the action of an electric field $E(t) = E_{\rm mod}(t) + E_\delta \cos\omega t$, where $E_\delta \cos\omega t$ is a weak probe field with the frequency ω fixed by an external circuit (external cavity) and $E_{\rm mod}(t)$ is a periodic modulation of the bias [Fig. 1(d)]. We suppose that the modulation frequencies are incommensurate with ω. To understand the main idea of our suggestion let us first consider $E_{\rm mod}(t)$ in the forms of pulses as shown in Figs. 1(a) and 1(b). For both the asymmetric (a) and symmetric (b) waveforms the amplitude of modulation E_0 is such that during each period T the SL spends most of the time $T - T_0$ in the NDC region of the VI characteristic and only for a short fraction of the period $T_0/T \ll 1$ the bias is switched off. To calculate the gain profile one needs to know the real part of the dynamical conductivity $\sigma_r(\omega) \equiv \mathrm{Re}[\sigma(\omega)]$. The gain corresponds to $\sigma_r(\omega) < 0$. We assume that the switching is faster than both T_0 and $T - T_0$ and transients after the switching can be neglected $T - T_0, T_0 \gg \tau$ (τ is the intraband relaxation time) [12]. We find for both sequences of pulses

$$\sigma_r(\omega) = \left(1 - \frac{T_0}{T}\right)\sigma_r^{\rm KSS}(\omega) + \frac{T_0}{T}\sigma_r^{\rm free}(\omega), \qquad (1)$$

where

$$\sigma_r^{\rm KSS}(\omega) = \frac{j_{\rm dc}(eE_0 d + \hbar\omega) - j_{\rm dc}(eE_0 d - \hbar\omega)}{2\hbar\omega} ed, \qquad (2)$$

$$\sigma_r^{\rm free}(\omega) = \sigma_0 \frac{1}{1 + \omega^2\tau^2} \qquad (3)$$

are the dynamical conductivity of a biased SL and free-electron absorption in an unbiased SL, respectively. Here $j_{\rm dc}(eE_{\rm dc}d)$ is the Esaki-Tsu characteristic [1,13]

$$j_{\rm dc}(eE_{\rm dc}d) = j_p \frac{2eE_{\rm dc}d\tau/\hbar}{1 + (eE_{\rm dc}d\tau/\hbar)^2}, \qquad (4)$$

where $j_p = eN\Delta d I_1/4\hbar I_0$ is the peak current corresponding to the critical field $E_{\rm cr} = \hbar/ed\tau$, $I_{1,2}(x)$ are the modified Bessel functions of the argument $x = \Delta/2k_B T_e$, N is the density of electrons in the miniband of the width Δ, d is the SL period, T_e is the temperature and $\sigma_0 = 2j_p/E_{\rm cr}$ is the Drude conductivity of the SL. Note that the finite difference form [14] given by Eq. (2) is equivalent [13] to the original Ktitorov *et al.* (KSS) formula [2] for a single relaxation time.

In the limit $T_0/T \to 0$, Eq. (1) describes the usual Bloch gain profile [thin blue line in Fig. 1(c)]. With an increase of T_0/T the SL spends more time in an unbiased state; therefore, the gain at low frequencies decreases and eventually the dynamical conductivity becomes positive, whereas the high-frequency gain is still preserved as shown in Fig. 1(c). The physical origin of such behavior is quite intuitive: At high frequencies around $\omega \approx \omega_B - \tau^{-1}$ ($\omega_B \equiv eE_0 d/\hbar \gg \tau^{-1}$ is the Bloch frequency), the Bloch gain $\sigma_r^{\rm KSS}(\omega) \approx -\sigma_0/4\omega\tau$ dominates the free-electron absorption $\sigma_r^{\rm free}(\omega) \approx \sigma_0/\omega^2\tau^2$. In contrast, at low frequencies $\omega\tau \ll 1$, the free-electron absorption $\sigma_r^{\rm free} \approx \sigma_0$ overcomes the Bloch gain $\sigma_r^{\rm KSS} \approx -\sigma_0/\omega_B^2\tau^2$.

The dynamic conductivity expressions Eqs. (1)–(3) directly follow from the general formula describing the dynamic conductivity of the SL in the case of arbitrary slow (quasistatic) modulation

$$\sigma_r(\omega) = \frac{j_{\rm dc}^{\rm mod}(eE_{\rm dc}d + \hbar\omega) - j_{\rm dc}^{\rm mod}(eE_{\rm dc}d - \hbar\omega)}{2\hbar\omega} ed, \qquad (5)$$

where $j_{\rm dc}^{\rm mod}$ is the dc component of the current in the SL

$$j_{\rm dc}^{\rm mod}(eE_{\rm dc}d) = \frac{1}{T}\int_0^T j_{\rm dc}[eE_{\rm mod}(t)d]dt \qquad (6)$$

that adiabatically follows to the time-dependent variations $E_{\rm mod}(t)$ via the VI characteristic (4). These formulas can be derived using two different techniques. In the first method, we employed the standard semiclassical approach based on an exact solution of the Boltzmann transport equation for a single tight-binding miniband and a constant relaxation time [13]. Next, we also found that the same dependence of $\sigma(\omega)$ on the shape of the VI characteristic can be obtained in the Tucker theory of quantum tunneling in the presence of ac electric fields [14]. Both approaches result in Eq. (5) with a cumbersome expression for the $j_{\rm dc}^{\rm mod}$, which can be further reduced [15] to the simple form of Eq. (6) in the limit of low-frequency modulations.

For the pulse modulation the spectrum of $E_{\rm mod}(t)$ consists of many harmonics of the basic frequency $\Omega(= 2\pi/T)$ and, as we have seen, the gain profile has the desirable shape with $\sigma_r(\omega \to 0) > 0$. Importantly, in the case of a monochromatic quasistatic field no high-frequency gain exists if $\sigma_r(\omega \to 0) > 0$ [15]. Therefore, we conclude that *monochromatic* low-frequency modulation cannot provide the required stabilization of the space-charge instability in the Bloch oscillator. Now we are going to show that a bichromatic modulation can already mimic the kind of fast bias switching necessary for the stabilization. We consider a modulation

$$E_{\rm mod}(t) = E_{\rm dc} + E_1 \cos\Omega t + E_2 \cos(n\Omega t + \phi), \qquad (7)$$

where n is an integer and $\Omega\tau \ll 1$. Figure 2 demonstrates the VI characteristic and gain profile for the modulation with moderate field strengths $E_{1,2}$. The amplitude E_2 of the harmonic component and the relative phase ϕ are chosen to square up the field (7). For typical values $\tau = 200$ fs and $\Omega\tau = 0.05$ the frequency $\Omega/2\pi$ is 40 GHz. This $E_{\rm mod}(t)$ creates, following Eq. (6), an additional peak in the VI characteristic of the SL [Fig. 2(a)], which resembles the resonant structures in the VI characteristic induced by a monochromatic THz field [11]. This additional peak can be utilized in a similar way as in the earlier suggestion [10] to achieve THz gain at a stable operation point. Namely, we put the working point at the part of the peak with a positive slope and then apply the difference formula (5). The result is the dispersive gain profile with a positive dynamical conductivity at low frequencies as shown in Fig. 2(b). In contrast to [10], for the suppression of the electric instability in the SL we use a bichromatic microwave field [16] instead of a monochromatic THz field. Importantly, the shape of the VI characteristic and the magnitude of gain can be additionally controlled by a variation of the relative phase ϕ. The magnitude of THz gain at bichromatic modulation is a bit lower in comparison with the case of pulse modulation [cf. Figs. 2(b) and 1(c)].

Apart from $\sigma_r(\omega \to 0) > 0$, the conditions for space-charge control in a microwave driven SL with NDC include also the requirement $\Omega\tau_d > 1$ [17], where τ_d is a characteristic time of domain formation. This means that an accumulation of charge during the period of modulation T is always weak [17,18]. If the conditions $\Omega\tau_d > 1$ and $\sigma_r(\omega \to 0) > 0$ are satisfied, the VI characteristic of the SL is modified by strong microwave fields according to the quasistatic theory prediction with an assumption of spatially homogeneous fields, i.e., following Eq. (6). Experimental observation of such modifications in the case of the monochromatic microwave field has been reported in [19]. The time τ_d is of the order of the dielectric relaxation time $\epsilon\epsilon_0/|\sigma_{\rm dc}(E_{\rm dc})|$ [20], where $\sigma_{\rm dc}(E_{\rm dc})$ is the dc differential conductivity, ϵ is the relative permittivity of SL material

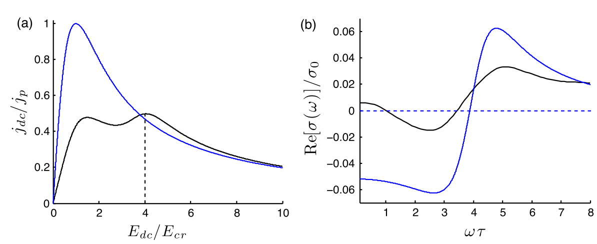

FIG. 2 (color online). (a) Esaki-Tsu characteristic (thin blue line) [Eq. (4)] and the characteristic of a SL modified by the bichromatic field [Eq. (6)] (thick black line) for $E_1 = 2.1E_{\rm cr}$, $E_2 = 1.7E_{\rm cr}$, $n = 3$, $\phi = 0$. (b) Dynamical conductivity [Eq. (5)] as a function of ω (thick black line) for the same bichromatic modulation and $E_{\rm dc} = 4E_{\rm cr}$. The thin blue curve shows the usual Bloch gain profile [Eq. (2)].

and ϵ_0 is the permittivity of vacuum. For the Esaki-Tsu characteristic [Eq. (4)] $\min[\sigma_{\rm dc}(E_{\rm dc})] = -\sigma_0/8$, and therefore we can present the condition of limited charge accumulation in the form

$$\omega_{\rm pl}^2\tau^2 \lesssim 8\Omega\tau, \qquad (8)$$

where $\omega_{\rm pl} = (2j_p ed/\hbar\epsilon_0\epsilon)^{1/2}$ is the miniband plasma frequency [21]. Since $\omega_{\rm pl}^2 \propto N\Delta$, for given electron concentration N and miniband width Δ, the inequality (8) imposes a lower limit on the modulation frequency Ω. Alternatively, for a fixed value of the modulation frequency Ω, Eq. (8) requires the use of SLs for which the product $N\Delta$ is less than some critical value.

The absorption (gain) α in units cm^{-1} is related to the scaled dynamical conductivity as

$$\alpha = \omega_{\rm pl}^2\tau^2 \frac{\sqrt{\epsilon}}{c\tau}\frac{\sigma_r(\omega)}{\sigma_0}, \qquad (9)$$

where c is the speed of light in vacuum. Comparing Eqs. (8) and (9) we see that the condition (8) imposes a restriction on the maximal value of α. For the parameters of bichromatic modulation considered in Fig. 2 and $\Omega\tau = 0.05$ we estimate $\alpha \approx -3.6\ \mathrm{cm}^{-1}$ at the gain resonance $\omega\tau \approx 2.5$. The corresponding value $\omega_{\rm pl}^2\tau^2 = 0.4$ [Eq. (8)] can be realized, for example, at room temperature in a SL with $d = 6$ nm, $\Delta = 60$ meV and $N = 6 \times 10^{15}\ \mathrm{cm}^{-3}$. The magnitude of gain can be increased by using the rectangular wave modulations. In this case, for the same basic modulation frequency $\Omega\tau = 0.05$ we get $\alpha \approx -12.8\ \mathrm{cm}^{-1}$ at the THz gain resonance shown in Fig. 1(c). It requires the use of $\simeq$10 ps dc pulses. However, a proof-of-the-principle demonstration can be performed with commercially available electric pulses of the duration $\simeq$0.1 ns. The restriction (8) can probably be relaxed in the case of lateral surface SLs [22] formed in two-dimensional electron systems. Because the domain growth rate can be suppressed in 2D systems [23], we can expect reasonably large magnitudes of THz gain even for a rather slow modulation of the bias.

The degree of homogeneity of the electric field inside the SL also depends on the boundary conditions defined by the attached electric contacts. Since the THz probe field is weak and bias modulation is quasistatic, we can use the standard drift-diffusion model [17] with typical boundary conditions [24]. A SL with nonlinear conductivity, which follows from the Esaki-Tsu dependence (4), is placed between two Ohmic buffer regions of equal conductivity σ_b [24]. The ac voltage across the device is periodically varied as shown in Figs. 1(a) and 1(b). We numerically calculated the degree of homogeneity as $r = \langle[E_{\rm max}(t) - E_{\rm min}(t)]/E_0\rangle_t$, where $E_{\rm max}(t)$ and $E_{\rm min}(t)$ are the maximum and minimum of the electric field in the whole SL and the time-averaging $\langle\ldots\rangle_t$ is performed over a long time interval of sustained field oscillations. Figure 3 shows the values of r in the σ_b-$\omega_{\rm pl}^2$ plane for a very long

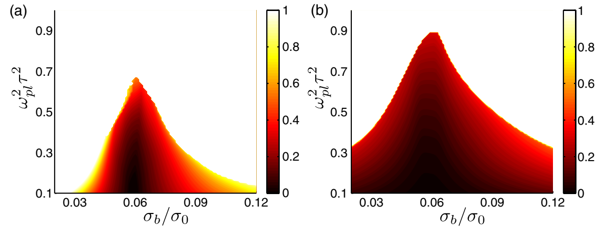

FIG. 3 (color online). Regions of a spatially homogeneous electric field (red) in the σ_b-$\omega_{\rm pl}^2$ plane in the case of (a) asymmetric and (b) symmetric pulse voltages with $\Omega\tau = 0.05$ and $T_0/T = 0.1$. The SL has the length $500d$.

SL. In the indicated dark regions of small r values the electric field inside the SL can be considered as spatially homogeneous. We also verified that for these parameters spatially-averaged gain profiles [25] are practically indistinguishable from those calculated earlier [Fig. 1(c)] within the homogeneous field approximation. If the buffer conductance coincides with the SL conductance ($\sigma_b/\sigma_0 \approx 0.06$ in Fig. 3), the upper boundary of the homogeneous field region in $\omega_{\rm pl}^2\tau^2$ is even a bit larger than expected from Eq. (8). For mismatched conductances smaller values of $\omega_{\rm pl}^2$ are required to achieve a desirable homogeneity. Interestingly, in the case of symmetric modulation the SL is less sensitive to the boundary effects [cf. Figs. 3(a) and 3(b)]. We attribute it to the fact that the time-averaged current induced by the symmetric voltage is zero both in the buffers and in the SL independently of the value of σ_b and thus, on average, there should be no accumulation or depletion of charges at the boundaries. For shorter SLs we found that the difference between the effects of symmetric and asymmetric waveforms becomes much less prominent and, as expected, the field stays homogeneous for a much wider range of σ_b. We underline that an observation of the Bloch gain certainly requires a gradual change of conductance between the active SL region and the metallic contacts.

In summary, we have theoretically shown that a proper quasistatic modulation of the bias in a Bloch oscillator results in THz Bloch gain in conditions of positive differential conductivity. The waveform of the modulation should include two distinct parts: Time interval of almost constant bias responsible for the THz gain due to the excitation of Bloch oscillations and a virtually unbiased interval during which the dominant free-carrier absorption effectively suppresses the undesirable low-frequency instability. Spectrum of the modulation must have at least two harmonics with a controlled difference of phases.

We conclude with two remarks. First, if the probe frequency ω coincides with a particular low-order harmonic or half-integer harmonic of the modulation frequency, a parametric gain [26] can provide an additional contribution to $\sigma(\omega)$ of the SL. Specifically, a magnitude of the parametric gain will depend on the phase of the probe field. Second, along with the semiconductor SLs our results can be directly applied to such semi- and superconducting microstructures as dilute nitride alloys [27] and small Josephson junctions [28]. It has already been found that, despite a very different physical origin, the small Josephson junctions in fact do demonstrate Esaki-Tsu VI characteristics and an ac response similar to the single miniband superlattice [28].

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