Macroscopic Quantum Electrodynamics and Duality

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We discuss under what conditions the duality between electric and magnetic fields is a valid symmetry of macroscopic quantum electrodynamics. It is shown that Maxwell's equations in the absence of free charges satisfy duality invariance on an operator level, whereas this is not true for Lorentz forces and atom-field couplings in general. We prove that derived quantities such as Casimir forces, local-field corrected decay rates, as well as van der Waals potentials are invariant with respect to a global exchange of electric and magnetic quantities. This exact symmetry can be used to deduce the physics of new configurations on the basis of already established ones.

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In the past, studies of phenomena of quantum electrodynamics (QED) have often been restricted to purely electric systems, because effects associated with magnetic properties are considerably smaller for materials occurring in nature. Two developments have recently triggered an increased interest in such magnetic effects: The first was the suggestion [1] and subsequent fabrication [2] of artificial metamaterials with controllable electric permittivity ε and magnetic permeability μ, where left-handed materials (LHMs) with negative real parts of ε and μ are of particular interest. As had been pointed out already in 1968 [3], the basis vectors of an electromagnetic wave propagating inside such a medium form a left-handed triad, implying negative refraction. Motivated by the progress in metamaterial fabrication, researchers have intensively studied their potentials, leading to proposals of a perfect lens with subwavelength resolution [4] as well as cloaking devices [5] and predictions of an unusual behavior of the decay of one or two atoms in the presence of LHMs [6,7].

Another, closely related motivation for considering magnetic systems was due to the fact that dispersion forces [8] have gained an increasing influence on micromechanical devices where they often lead to undesired effects such as stiction [9]. The question naturally arose whether LHMs could be exploited to modify or even change the sign of dispersion forces. Forces on excited systems might indeed be influenced by LHMs [10]. Ground-state forces are not as easily manipulated because they depend on the medium response at all frequencies, whereas the Kramers-Kronig relations imply that LHMs can only be realized in limited frequency windows. However, the controllable magnetic properties available in metamaterials can still have a large impact on dispersion forces: The dispersion forces between electric and magnetic atoms [11] or bodies [12] differ both in sign and power laws from those between only electric ones. Searching for repulsive dispersion forces, interactions of electric or magnetic atoms [13], plates [14,15] and atoms with plates [16,17] have been studied; more complex problems such as atom-atom interactions in the presence of a magnetoelectric bulk medium [18], plate [19] or sphere [20] have also been addressed. Reductions or even sign changes of the forces have been predicted for such scenarios and have been attributed primarily to large permeabilities rather than left-handed properties.

Metamaterials have thus considerably increased the parameter space at one's disposal for manipulating phenomena of QED. An efficient use of this new freedom requires the formulation of general statements of what might be achieved and what is impossible in principle. Working in this direction, upper bounds for the strength of attractive and repulsive Casimir forces have been formulated [15] and it has been proven that the force between two mirror-symmetric purely electric bodies is always attractive [21]. In the present Letter, we establish another such general principle on the basis of the duality of Maxwell's equations under an exchange of electric and magnetic fields [22,23], also known as electric or magnetic reciprocity within a generalized framework of classical electrodynamics [24]. In particle physics, duality has been discussed as a symmetry of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory [25]. We will prove its validity in the context of macroscopic QED [6,8] and show that under certain conditions, quantities such as decay rates and dispersion forces are invariant with respect to a global exchange of electric and magnetic properties. The parameter space to be considered in the search for optimal geometries and materials will thus be effectively halved.

We begin by verifying duality for macroscopic QED in the absence of free charges and currents. We group the fields into dual pairs $(\sqrt{\varepsilon_0}\hat{\boldsymbol{E}}, \sqrt{\mu_0}\hat{\boldsymbol{H}})$, $(\sqrt{\mu_0}\hat{\boldsymbol{D}}, \sqrt{\varepsilon_0}\hat{\boldsymbol{B}})$ and $(\sqrt{\mu_0}\hat{\boldsymbol{P}}, \sqrt{\varepsilon_0}\mu_0\hat{\boldsymbol{M}})$, so that Maxwell's equations read

$$\boldsymbol{\nabla}\cdot\begin{pmatrix}\sqrt{\mu_0}\hat{\boldsymbol{D}}\\ \sqrt{\varepsilon_0}\hat{\boldsymbol{B}}\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}, \tag{1}$$

$$\boldsymbol{\nabla}\times\begin{pmatrix}\sqrt{\varepsilon_0}\hat{\boldsymbol{E}}\\ \sqrt{\mu_0}\hat{\boldsymbol{H}}\end{pmatrix}+\frac{\partial}{\partial t}\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}\sqrt{\mu_0}\hat{\boldsymbol{D}}\\ \sqrt{\varepsilon_0}\hat{\boldsymbol{B}}\end{pmatrix}=\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{0}\end{pmatrix} \tag{2}$$

with

$$\begin{pmatrix} \sqrt{\mu_0}\hat{\boldsymbol{D}} \\ \sqrt{\varepsilon_0}\hat{\boldsymbol{B}} \end{pmatrix} = \frac{1}{c}\begin{pmatrix} \sqrt{\varepsilon_0}\hat{\boldsymbol{E}} \\ \sqrt{\mu_0}\hat{\boldsymbol{H}} \end{pmatrix} + \begin{pmatrix} \sqrt{\mu_0}\hat{\boldsymbol{P}} \\ \sqrt{\varepsilon_0}\mu_0\hat{\boldsymbol{M}} \end{pmatrix}. \qquad (3)$$

Maxwell's equations are invariant under the general SO(2) duality transformation

$$\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix}^{\star} = \mathcal{D}(\theta)\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix}, \quad \mathcal{D}(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \qquad (4)$$

which may equivalently be expressed as a U(1) transformation when introducing complex Riemann-Silberstein fields [22]. The invariance of Maxwell's equations under this rotation can be verified by multiplying Eqs. (1)–(3) by $\mathcal{D}(\theta)$ and using the fact that $\mathcal{D}(\theta)$ commutes with the symplectic matrix in Eq. (2). Note that the grouping into dual pairs is solely due to the mathematical structure of the equations and is in contrast to the fact that $\hat{\boldsymbol{E}}$, $\hat{\boldsymbol{B}}$ and $\hat{\boldsymbol{D}}$, $\hat{\boldsymbol{H}}$ are the pairs of physically corresponding quantities.

For it to be a valid symmetry of the electromagnetic field, duality must also be consistent with the constitutive relations. In the presence of linear, local, isotropic, dispersing and absorbing media, the constitutive relations in frequency space can be given as

$$\begin{pmatrix} \sqrt{\mu_0}\hat{\boldsymbol{D}} \\ \sqrt{\varepsilon_0}\hat{\boldsymbol{B}} \end{pmatrix} = \frac{1}{c}\begin{pmatrix} \varepsilon & 0 \\ 0 & \mu \end{pmatrix}\begin{pmatrix} \sqrt{\varepsilon_0}\hat{\boldsymbol{E}} \\ \sqrt{\mu_0}\hat{\boldsymbol{H}} \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & \mu \end{pmatrix}\begin{pmatrix} \sqrt{\mu_0}\hat{\boldsymbol{P}}_N \\ \sqrt{\varepsilon_0}\mu_0\hat{\boldsymbol{M}}_N \end{pmatrix}, \qquad (5)$$

where $\varepsilon = \varepsilon(\boldsymbol{r}, \omega)$ and $\mu = \mu(\boldsymbol{r}, \omega)$ denote the relative electric permittivity and magnetic permeability of the media and $\hat{\boldsymbol{P}}_N$ and $\hat{\boldsymbol{M}}_N$ are the noise polarization and magnetization which necessarily arise in the presence of absorption. Invariance of the constitutive relations (5) under the duality transformation requires that

$$\begin{pmatrix} \varepsilon^{\star} & 0 \\ 0 & \mu^{\star} \end{pmatrix} = \mathcal{D}(\theta)\begin{pmatrix} \varepsilon & 0 \\ 0 & \mu \end{pmatrix}\mathcal{D}^{-1}(\theta) = \begin{pmatrix} \varepsilon\cos^2\theta + \mu\sin^2\theta & (\mu - \varepsilon)\sin\theta\cos\theta \\ (\mu - \varepsilon)\sin\theta\cos\theta & \varepsilon\sin^2\theta + \mu\cos^2\theta \end{pmatrix}. \qquad (6)$$

This condition is trivially fulfilled if $\varepsilon = \mu$ (including both free space and the perfect lens, $\varepsilon = \mu = -1$ [4]), where duality is a continuous symmetry. For media with a nontrivial impedance, the condition (6) only holds for $\theta = n\pi/2$ with $n \in \mathbb{Z}$. The presence of such media thus reduces the continuous symmetry to a discrete symmetry with four distinct members, whose group structure is that of $\mathbb{Z}_4$. For $\theta = n\pi/2$, Eqs. (5) and (6) imply the transformations

$$\begin{pmatrix} \varepsilon \\ \mu \end{pmatrix}^{\star} = \begin{pmatrix} \cos^2\theta & \sin^2\theta \\ \sin^2\theta & \cos^2\theta \end{pmatrix}\begin{pmatrix} \varepsilon \\ \mu \end{pmatrix}, \qquad (7)$$

$$\begin{pmatrix} \sqrt{\mu_0}\hat{\boldsymbol{P}}_N \\ \sqrt{\varepsilon_0}\mu_0\hat{\boldsymbol{M}}_N \end{pmatrix}^{\star} = \begin{pmatrix} \cos\theta & \mu\sin\theta \\ -\varepsilon^{-1}\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \sqrt{\mu_0}\hat{\boldsymbol{P}}_N \\ \sqrt{\varepsilon_0}\mu_0\hat{\boldsymbol{M}}_N \end{pmatrix}. \qquad (8)$$

Maxwell's equations (1) and (2), together with the constitutive relations (5) for the electromagnetic field in the absence of free charges and currents, are thus invariant under the discrete duality transformations $\theta = n\pi/2$, $n \in \mathbb{Z}$ given by Eqs. (4), (7), and (8). This is not only true for the equations of motion, but clearly must also hold on a Hamiltonian level. To see this explicitly, recall that the Hamiltonian of the medium-assisted field is given by $\hat{H}_F = \sum_{\lambda=e,m}\int d^3r\int_0^\infty d\omega\hbar\omega\hat{\boldsymbol{f}}_\lambda^\dagger(\boldsymbol{r},\omega)\cdot\hat{\boldsymbol{f}}_\lambda(\boldsymbol{r},\omega)$ [6] where the fundamental bosonic fields $\hat{\boldsymbol{f}}_\lambda$ are related to the noise terms via

$$\begin{pmatrix} \sqrt{\mu_0}\hat{\underline{\boldsymbol{P}}}_N \\ \sqrt{\varepsilon_0}\mu_0\hat{\underline{\boldsymbol{M}}}_N \end{pmatrix} = \sqrt{\frac{\hbar}{\pi c^2}}\begin{pmatrix} i\sqrt{Im\varepsilon} & 0 \\ 0 & \sqrt{Im\mu}/|\mu| \end{pmatrix}\begin{pmatrix} \hat{\boldsymbol{f}}_e \\ \hat{\boldsymbol{f}}_m \end{pmatrix}. \qquad (9)$$

Combining Eqs. (7)–(9), one finds that the fundamental fields transform as

$$\begin{pmatrix} \hat{\boldsymbol{f}}_e \\ \hat{\boldsymbol{f}}_m \end{pmatrix}^{\star} = \begin{pmatrix} \cos\theta & -i(\mu/|\mu|)\sin\theta \\ -i(|\varepsilon|/\varepsilon)\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \hat{\boldsymbol{f}}_e \\ \hat{\boldsymbol{f}}_m \end{pmatrix} \qquad (10)$$

for $\theta = n\pi/2$, so that $\hat{H}_F^{\star} = \hat{H}_F$. It is sufficient to focus on the single duality transformation $\theta = \pi/2$ as summarized in Table I, which is a generator of the whole group.

Let us next turn our attention to Lorentz forces and the coupling of the medium-assisted field to charged particles: We recall that the operator Lorentz force on a neutral body occupying a volume V can be given as [8]

$$\hat{\boldsymbol{F}} = \int_{\partial V} d\boldsymbol{A}\cdot\Big\{\varepsilon_0\hat{\boldsymbol{E}}(\boldsymbol{r})\hat{\boldsymbol{E}}(\boldsymbol{r}) + \frac{1}{\mu_0}\hat{\boldsymbol{B}}(\boldsymbol{r})\hat{\boldsymbol{B}}(\boldsymbol{r}) - \frac{1}{2}\Big[\varepsilon_0\hat{\boldsymbol{E}}^2(\boldsymbol{r}) + \frac{1}{\mu_0}\hat{\boldsymbol{B}}^2(\boldsymbol{r})\Big]\mathbf{I}\Big\} - \varepsilon_0\frac{d}{dt}\int_V d^3r\hat{\boldsymbol{E}}(\boldsymbol{r})\times\hat{\boldsymbol{B}}(\boldsymbol{r}) \qquad (11)$$

($\mathbf{I}$: unit tensor), while that on a neutral atom with polarization $\hat{\boldsymbol{P}}_A$ and magnetization $\hat{\boldsymbol{M}}_A$ reads [8,26]

$$\hat{\boldsymbol{F}} = \boldsymbol{\nabla}_A\int d^3r[\hat{\boldsymbol{P}}_A(\boldsymbol{r})\cdot\hat{\boldsymbol{E}}(\boldsymbol{r}) + \hat{\boldsymbol{M}}_A(\boldsymbol{r})\cdot\hat{\boldsymbol{B}}(\boldsymbol{r}) + \hat{\boldsymbol{P}}_A(\boldsymbol{r})\times\dot{\hat{\boldsymbol{r}}}_A\cdot\hat{\boldsymbol{B}}(\boldsymbol{r})] + \frac{d}{dt}\int d^3r\hat{\boldsymbol{P}}_A(\boldsymbol{r})\times\hat{\boldsymbol{B}}(\boldsymbol{r}). \qquad (12)$$

The coupling of one or more atoms to the medium-assisted electromagnetic field can in the multipolar coupling scheme be implemented via [8,19]

TABLE I. Effect of the duality transformation with $\theta = \pi/2$.

Partners	Transformation	
$\hat{\boldsymbol{E}}$, $\hat{\boldsymbol{H}}$:	$\hat{\boldsymbol{E}}^{\star} = c\mu_0\hat{\boldsymbol{H}}$,	$\hat{\boldsymbol{H}}^{\star} = -\hat{\boldsymbol{E}}/(c\mu_0)$
$\hat{\boldsymbol{D}}$, $\hat{\boldsymbol{B}}$:	$\hat{\boldsymbol{D}}^{\star} = c\varepsilon_0\hat{\boldsymbol{B}}$,	$\hat{\boldsymbol{B}}^{\star} = -\hat{\boldsymbol{D}}/(c\varepsilon_0)$
$\hat{\boldsymbol{P}}$, $\hat{\boldsymbol{M}}$:	$\hat{\boldsymbol{P}}^{\star} = \hat{\boldsymbol{M}}/c$,	$\hat{\boldsymbol{M}}^{\star} = -c\hat{\boldsymbol{P}}$
$\hat{\boldsymbol{P}}_A$, $\hat{\boldsymbol{M}}_A$:	$\hat{\boldsymbol{P}}_A^{\star} = \hat{\boldsymbol{M}}_A/c$,	$\hat{\boldsymbol{M}}_A^{\star} = -c\hat{\boldsymbol{P}}_A$
$\hat{\boldsymbol{d}}$, $\hat{\boldsymbol{m}}$:	$\hat{\boldsymbol{d}}^{\star} = \hat{\boldsymbol{m}}/c$,	$\hat{\boldsymbol{m}}^{\star} = -c\hat{\boldsymbol{d}}$
$\hat{\boldsymbol{P}}_N$, $\hat{\boldsymbol{M}}_N$:	$\hat{\boldsymbol{P}}_N^{\star} = \mu\hat{\boldsymbol{M}}_N/c$,	$\hat{\boldsymbol{M}}_N^{\star} = -c\hat{\boldsymbol{P}}_N/\varepsilon$
$\hat{\boldsymbol{f}}_e$, $\hat{\boldsymbol{f}}_m$:	$\hat{\boldsymbol{f}}_e^{\star} = -i(\mu/|\mu|)\hat{\boldsymbol{f}}_m$,	$\hat{\boldsymbol{f}}_m^{\star} = -i(|\varepsilon|/\varepsilon)\hat{\boldsymbol{f}}_e$
ε, μ:	$\varepsilon^{\star} = \mu$,	$\mu^{\star} = \varepsilon$
α, β:	$\alpha^{\star} = \beta/c^2$,	$\beta^{\star} = c^2\alpha$

$$\hat{H}_{AF} = -\int d^3r[\hat{\boldsymbol{P}}_A(\boldsymbol{r})\cdot\hat{\boldsymbol{E}}(\boldsymbol{r}) + \hat{\boldsymbol{M}}_A(\boldsymbol{r})\cdot\hat{\boldsymbol{B}}(\boldsymbol{r}) + m_A^{-1}\hat{\boldsymbol{P}}_A(\boldsymbol{r})\times\hat{\boldsymbol{p}}_A\cdot\hat{\boldsymbol{B}}(\boldsymbol{r})], \tag{13}$$

when neglecting diamagnetic interactions. Using the transformation behavior given in Table I, it is immediately clear that neither the Lorentz forces on bodies or atoms nor the atom-field interactions are duality invariant on an operator level. Even for atoms and bodies at rest with time-independent fields, duality invariance is prohibited by the unavoidable noise polarization and magnetization in the constitutive relations (5).

That said, we will now show that effective quantities derived from the above operator Lorentz forces and atom-field couplings do obey duality invariance when considering atoms and bodies at rest not embedded in a medium. In particular, we will consider the Casimir force [27]

$$\boldsymbol{F} = -\frac{\hbar}{\pi}\int_0^\infty d\xi\int_{\partial V} d\boldsymbol{A}\cdot\Big\{\mathbf{G}_{ee}^{(1)}(\boldsymbol{r},\boldsymbol{r},i\xi) + \mathbf{G}_{mm}^{(1)}(\boldsymbol{r},\boldsymbol{r},i\xi) - \frac{1}{2}\mathrm{Tr}[\mathbf{G}_{ee}^{(1)}(\boldsymbol{r},\boldsymbol{r},i\xi) + \mathbf{G}_{mm}^{(1)}(\boldsymbol{r},\boldsymbol{r},i\xi)]\mathbf{I}\Big\}, \tag{14}$$

the single- and two-atom van der Waals (vdW) potentials [8,17,28]

$$U(\boldsymbol{r}_A) = \frac{\hbar}{2\pi\varepsilon_0}\int_0^\infty d\xi\Big[\alpha(i\xi)\mathrm{Tr}\mathbf{G}_{ee}^{(1)}(\boldsymbol{r}_A,\boldsymbol{r}_A,i\xi) + \frac{\beta(i\xi)}{c^2}\mathrm{Tr}\mathbf{G}_{mm}^{(1)}(\boldsymbol{r}_A,\boldsymbol{r}_A,i\xi)\Big] \tag{15}$$

and

$$U(\boldsymbol{r}_A,\boldsymbol{r}_B) = -\frac{\hbar}{2\pi\varepsilon_0^2}\int_0^\infty d\xi\,\mathrm{Tr}\Big\{\alpha_A(i\xi)\alpha_B(i\xi)\mathbf{G}_{ee}(\boldsymbol{r}_A,\boldsymbol{r}_B,i\xi)\cdot\mathbf{G}_{ee}(\boldsymbol{r}_B,\boldsymbol{r}_A,i\xi) + \alpha_A(i\xi)\frac{\beta_B(i\xi)}{c^2}\mathbf{G}_{em}(\boldsymbol{r}_A,\boldsymbol{r}_B,i\xi)\cdot\mathbf{G}_{me}(\boldsymbol{r}_B,\boldsymbol{r}_A,i\xi) + \frac{\beta_A(i\xi)}{c^2}\alpha_B(i\xi)\mathbf{G}_{me}(\boldsymbol{r}_A,\boldsymbol{r}_B,i\xi)\cdot\mathbf{G}_{em}(\boldsymbol{r}_B,\boldsymbol{r}_A,i\xi) + \frac{\beta_A(i\xi)}{c^2}\frac{\beta_B(i\xi)}{c^2}\mathbf{G}_{mm}(\boldsymbol{r}_A,\boldsymbol{r}_B,i\xi)\cdot\mathbf{G}_{mm}(\boldsymbol{r}_B,\boldsymbol{r}_A,i\xi)\Big\} \tag{16}$$

(α, β: atomic polarizability, magnetizability) and the atomic decay rate [6,29]

$$\Gamma_n(\boldsymbol{r}_A) = \frac{2}{\hbar\varepsilon_0}\sum_{k<n}\Big[\boldsymbol{d}_{kn}\cdot\mathrm{Im}\mathbf{G}_{ee}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega_{nk})\cdot\boldsymbol{d}_{nk} + \frac{\boldsymbol{m}_{kn}}{c}\cdot\mathrm{Im}\mathbf{G}_{mm}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega_{nk})\cdot\frac{\boldsymbol{m}_{nk}}{c}\Big] \tag{17}$$

($|n\rangle$: initial atomic state, ω_{nk}: atomic transition frequencies; $\boldsymbol{d}_{nk}$, $\boldsymbol{m}_{nk}$: electric, magnetic dipole matrix elements). Here, $\mathbf{G}^{(1)}$ is the scattering part of the classical Green tensor, where a left index e, m indicates that $\mathbf{G}$ is multiplied by $i\omega/c = -\xi/c$ or $\boldsymbol{\nabla}\times$ from the left and a right index e, m denotes multiplication with $i\omega/c = -\xi/c$ or $\times\overleftarrow{\boldsymbol{\nabla}}'$ from the right. The Casimir force and the single-atom vdW force are the ground-state averages of the above operator Lorentz forces, while the atomic potentials and rates follow from the atom-field coupling.

To prove the duality invariance of the above quantities (14)–(17), we note that the Casimir force depends solely on the classical Green tensor

$$\Big[\boldsymbol{\nabla}\times\frac{1}{\mu(\boldsymbol{r},\omega)}\boldsymbol{\nabla}\times - \frac{\omega^2}{c^2}\varepsilon(\boldsymbol{r},\omega)\Big]\mathbf{G}(\boldsymbol{r},\boldsymbol{r}',\omega) = \boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}'), \tag{18}$$

while vdW forces and decay rates also depend on α, β, $\hat{\boldsymbol{d}}$ and $\hat{\boldsymbol{m}}$. While the transformation behavior of the latter quantities under duality follows immediately from that of ε, μ, $\hat{\boldsymbol{P}}_A$ and $\hat{\boldsymbol{M}}_A$ (see Table I), the transformed Green tensor, which is the solution to Eq. (18) with ε and μ exchanged, can be determined as follows: We first note that Maxwell's equations (1) and (2) together with the constitutive relations (5) are uniquely solved by [6]

$$\underline{\hat{\boldsymbol{E}}}(\boldsymbol{r},\omega) = -\varepsilon_0^{-1}\int d^3r'\mathbf{G}_{ee}(\boldsymbol{r},\boldsymbol{r}',\omega)\cdot\underline{\hat{\boldsymbol{P}}}_N(\boldsymbol{r}',\omega) - c\mu_0\int d^3r'\mathbf{G}_{em}(\boldsymbol{r},\boldsymbol{r}',\omega)\cdot\underline{\hat{\boldsymbol{M}}}_N(\boldsymbol{r}',\omega), \tag{19}$$

$$\underline{\hat{\boldsymbol{B}}}(\boldsymbol{r},\omega) = -c\mu_0\int d^3r'\mathbf{G}_{me}(\boldsymbol{r},\boldsymbol{r}',\omega)\cdot\underline{\hat{\boldsymbol{P}}}_N(\boldsymbol{r}',\omega) - \mu_0\int d^3r'\mathbf{G}_{mm}(\boldsymbol{r},\boldsymbol{r}',\omega)\cdot\underline{\hat{\boldsymbol{M}}}_N(\boldsymbol{r}',\omega), \tag{20}$$

$$\underline{\hat{\boldsymbol{D}}}(\boldsymbol{r},\omega) = -\frac{\varepsilon(\boldsymbol{r},\omega)}{c}\int d^3r'\mathbf{G}_{em}(\boldsymbol{r},\boldsymbol{r}',\omega)\cdot\underline{\hat{\boldsymbol{M}}}_N(\boldsymbol{r}',\omega) - \int d^3r'[\varepsilon(\boldsymbol{r},\omega)\mathbf{G}_{ee}(\boldsymbol{r},\boldsymbol{r}',\omega) - \boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')]\cdot\underline{\hat{\boldsymbol{P}}}_N(\boldsymbol{r}',\omega), \tag{21}$$

$$\underline{\hat{\boldsymbol{H}}}(\boldsymbol{r},\omega) = -\frac{c}{\mu(\boldsymbol{r},\omega)}\int d^3r'\mathbf{G}_{me}(\boldsymbol{r},\boldsymbol{r}',\omega)\cdot\underline{\hat{\boldsymbol{P}}}_N(\boldsymbol{r}',\omega) - \int d^3r'\Big[\frac{\mathbf{G}_{mm}(\boldsymbol{r},\boldsymbol{r}',\omega)}{\mu(\boldsymbol{r},\omega)} + \boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')\Big]\cdot\underline{\hat{\boldsymbol{M}}}_N(\boldsymbol{r}',\omega). \tag{22}$$

The invariance of Maxwell's equations implies that this solution remains valid after applying the duality transformation. Taking duality transforms of Eqs. (19) and (20), the unknown transformed Green tensor appears on the right-hand side of these equations, whereas the transformations of all other quantities occurring in the equations can be determined with the aid of Table I. After using Eqs. (19)–(22) to express the resulting fields on the left-hand side in terms of $\underline{\hat{\boldsymbol{P}}}_N$ and $\underline{\hat{\boldsymbol{M}}}_N$ and equating coefficients, one obtains the following transformation rules:

$$\mathbf{G}^{\star}_{ee}(\mathbf{r},\mathbf{r}',\omega) = \mu^{-1}(\mathbf{r},\omega)\mathbf{G}_{mm}(\mathbf{r},\mathbf{r}',\omega)\mu^{-1}(\mathbf{r}',\omega) + \mu^{-1}(\mathbf{r},\omega)\boldsymbol{\delta}(\mathbf{r}-\mathbf{r}'), \quad (23)$$

$$\mathbf{G}^{\star}_{em}(\mathbf{r},\mathbf{r}',\omega) = -\mu^{-1}(\mathbf{r},\omega)\mathbf{G}_{me}(\mathbf{r},\mathbf{r}',\omega)\varepsilon(\mathbf{r}',\omega), \quad (24)$$

$$\mathbf{G}^{\star}_{me}(\mathbf{r},\mathbf{r}',\omega) = -\varepsilon(\mathbf{r},\omega)\mathbf{G}_{em}(\mathbf{r},\mathbf{r}',\omega)\mu^{-1}(\mathbf{r}',\omega), \quad (25)$$

$$\mathbf{G}^{\star}_{mm}(\mathbf{r},\mathbf{r}',\omega) = \varepsilon(\mathbf{r},\omega)\mathbf{G}_{ee}(\mathbf{r},\mathbf{r}',\omega)\varepsilon(\mathbf{r}',\omega) - \varepsilon(\mathbf{r},\omega)\boldsymbol{\delta}(\mathbf{r}-\mathbf{r}'). \quad (26)$$

The duality invariance of dispersion forces and decay rates follows immediately. Using Eqs. (23) and (26) and noting that the δ function does not contribute to the scattering part of the Green tensor, it is seen that the Casimir force (14) on a body is unchanged when globally exchanging ε and μ, provided that the body is located in free space. The duality invariance of the vdW potentials (15) and (16) also follows from the transformation rules (23)–(26). This invariance with respect to a simultaneous exchange $\varepsilon \leftrightarrow \mu$ and $\alpha \leftrightarrow \beta/c^2$ again only holds if $\varepsilon(\mathbf{r}_{A/B}) = \mu(\mathbf{r}_{A/B}) = 1$. In contrast to the Casimir force, this does not mean that the atom has to be located in vacuum, but merely implies that for atoms embedded in media, local-field corrections must be included via the real-cavity model in order to insure invariance [30].

Duality invariance can be used to obtain the full functional dependence of dispersion forces in given scenarios on the atomic and medium parameters from knowledge of the respective dual scenario. For instance, it has recently been shown that in the retarded limit the vdW potential of two polarizable atoms reads $U(r_{AB}) = -1863\hbar c\alpha_A\alpha_B\varepsilon^2/[64\pi^3\varepsilon_0^2\sqrt{\varepsilon\mu}(2\varepsilon+1)^4 r_{AB}^7]$ when including local-field corrections [30]. Making the replacements $\alpha \to \beta/c^2$, $\varepsilon \leftrightarrow \mu$, one can immediately infer $U(r_{AB}) = -1863\hbar c\mu_0^2\beta_A\beta_B\mu^2/[64\pi^3\sqrt{\varepsilon\mu}(2\mu+1)^4 r_{AB}^7]$ for magnetizable atoms. The utility of this principle becomes even more apparent for complex problems like the interaction of two atoms in the presence of a magnetoelectric object [19,20]. Finally, using the fact that two purely electric, mirror-symmetric bodies always attract [21], we can immediately conclude that so do two purely magnetic ones.

In addition, Eqs. (23) and (26) imply the duality invariance of the decay rate (17) since the δ functions do not contribute to the imaginary part of the Green tensor; again, local-field corrections have to be included for atoms embedded in media. This symmetry can be exploited, e.g., to obtain magnetically driven spin-flip rates of atoms in specific environments from known electric-dipole driven decay rates.

In conclusion, we have shown that dispersion forces on atoms and bodies as well as decay rates of atoms are duality invariant, provided that the bodies are located in free space at rest and that local-field corrections are taken into account when considering (stationary) atoms embedded in a medium. The established symmetry operation of globally exchanging electric and magnetic body and atom properties is a powerful tool for obtaining new results on the basis of already established ones. The invariance can easily be extended to other effective quantities of macroscopic QED such as frequency shifts, heating rates or energy transfer rates.

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