# Scaling Theory of Magnetoresistance and Carrier Localization in $Ga_{1-x}Mn_xAs$

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We compare experimental resistivity data on $Ga_{1-x}Mn_xAs$ films with theoretical calculations using a scaling theory for strongly disordered ferromagnets. The characteristic features of the temperature dependent resistivity can be quantitatively understood through this approach as originating from the close vicinity of the metal-insulator transition. However, accounting for thermal fluctuations is crucial for a quantitative description of the magnetic field induced changes in resistance. While the noninteracting scaling theory is in reasonable agreement with the data, we find clear evidence for interaction effects at low temperatures.

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Over the past decade, $Ga_{1-x}Mn_xAs$ has been the most studied ferromagnetic semiconductor [1,2]. The carrier-mediated ferromagnetism in this material makes it attractive for proof-of-concept spintronic devices. They are also good candidates for device integration with the technologically III–V semiconductors such as GaAs, where band gap engineering allows systematic modulation of the carrier density in heterostructure devices. It is important within this general context to develop a fundamental understanding of the interplay between carrier transport and magnetism in $Ga_{1-x}Mn_xAs$.

One of the basic, but least understood, properties of $Ga_{1-x}Mn_xAs$ is the temperature and magnetic field dependence of its resistivity [3]. For typical $Ga_{1-x}Mn_xAs$ samples the resistivity increases with decreasing temperature above the Curie temperature $T_C$, but then it suddenly drops below $T_C$. This results in a resistivity peak at $T_C$, a feature that gradually broadens and shifts towards higher temperatures; with an increasing external magnetic field (see Fig. 1). The resistivity shows another upturn at lower temperatures as well. In less resistive samples, the aforementioned feature is less pronounced and one observes a broad shoulder rather then a resistivity peak.

There have been a number of attempts aimed at explaining the resistivity peak [3], invoking various mechanisms such as scattering off critical fluctuations [4,5], the formation of magnetic polarons [6,7], ''dynamical'' mean-field calculations [8], or the interplay with universal conductance fluctuations [9]. Nevertheless, these theoretical approaches have been successful only in addressing particular ranges [10] or qualitative aspects [11] of the data, and a theoretical framework that could quantitatively explain all major characteristic features observed in $Ga_{1-x}Mn_xAs$ has not been available so far [12].

Here we present an approach which is able to *quantitatively* account for all essential features of the experimental data on more resistive samples: (i) the gradually increasing resistance as the temperature is lowered towards $T_C$; (ii) the pronounced peak precisely at $T_C$; (iii) the upturn in resistance at low temperatures, together with the finite resistance intercept for metallic samples; (iv) the precise amount with which an external magnetic field depletes the resistance peak at $T_C$ and shifts towards higher temperatures; (v) the ''noncrossing'' constraint of the experimental data [12]. These characteristic features are clearly visible

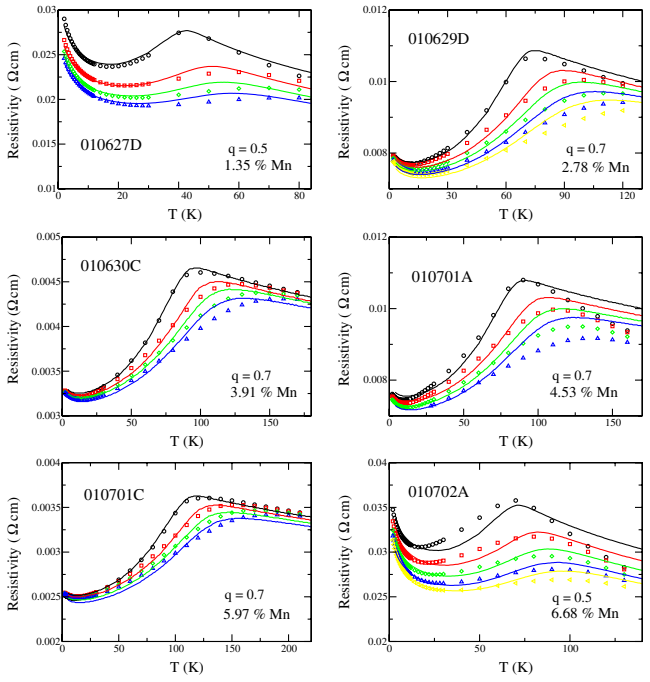


FIG. 1 (color online). Comparison between the experimental data and the theoretical results at magnetic fields $H = 0$, 3, 6, and 9 T. Dots represent experimental data, solid lines are theoretical fits.

 

in Fig. 1, where we show our experimental results for a series of annealed and unannealed $Ga_{1-x}Mn_xAs$ samples grown by molecular beam epitaxy, with the Mn concentration ($x$) systematically varied between 0.0135 and 0.067. The temperature dependent resistivity data shown in Fig. 1 were measured in different magnetic fields inside a commercial cryostat (Quantum Design PPMS), with the magnetic field normal to the sample plane. Details of the sample growth were reported elsewhere [13].

According to our theory the temperature and magnetic field dependence of more resistive samples with $T_C < 100$ K can be quantitatively understood as originating from the vicinity of metal-insulator transition (MIT). To support this, we first remark that the features mentioned above are reminiscent of localization effects in interplay with magnetism [14,15]. In fact, most III–V ferromagnetic semiconductors are bad conductors because the charged dopants (Mn for $Ga_{1-x}Mn_xAs$) introduce large disorder [16] and can even lead to the formation of an impurity band [17]. As we show later, even for some of the annealed and relatively highly doped samples, a simple upper estimate gives $k_F l \sim 0.3$, with $k_F$ the Fermi momentum and $l$ the mean free path. This value clearly suggests that it is necessary to go beyond the weak disorder picture frequently used in the literature [18,19]. Moreover, the size of the low-temperature anomaly (always interpreted in terms of interacting disordered electrons) is clearly *correlated* with the behavior above $T_C$, and also the size of residual resistivity. The scaling theory presented here provides a natural explanation for these correlations and provides a way to extrapolate the low-temperature anomalies to the range $T \sim T_C$. The extrapolation captures not only the size but also the detailed *qualitative* and *quantitative* aspects of the magnetotransport properties in these samples.

Similar resistivity anomalies have been also observed in other types of magnetic semiconductors [20], as well as some manganites [14,15], with various semiphenomenological frameworks available for explaining these phenomena in terms of localization theory [15,21,22]. However, most of these approaches focus explicitly on the localized phase, and are unsuitable for most measured $Ga_{1-x}Mn_xAs$ samples that are not insulators, but poor metals, close to the localization transition.

In Ref. [12], we developed a scaling theory of magnetoresistance to describe transport properties of localized $Ga_{1-x}Mn_xAs$ samples. Here we extend this theory to metallic samples by making use of a slightly modified version of the scaling approach applied for disordered interacting conductors [23–25]. For noninteracting electrons, a scaling theory can be constructed in terms of the dimensionless conductance $g$ and a length scale at which electrons lose their coherence. In the presence of interactions, one needs to introduce the dimensionless interaction parameters in the triplet and singlet channels, $\gamma_t$ and $\gamma_s$, respectively [23–25]. For $Ga_{1-x}Mn_xAs$ an important simplification occurs: $Ga_{1-x}Mn_xAs$ has a very large intrinsic spin-orbit gap, $\Delta_{so} \sim 4000$ K. Furthermore, in the ferromagnetic phase and in the vicinity of the Curie temperature, $T_C$, the almost classical $S = 5/2$ spins of the Mn ions fluctuate slowly in time, so at the time scales and temperatures of interest, time reversal is also locally broken, even in the paramagnetic phase. Consequently, $Ga_{1-x}Mn_xAs$ belongs to the *unitary* class [12,26]. Then $\gamma_t$ plays no role, and $\gamma_s$ can also be set to $\gamma_s = 1$ [24,25]. As a result, the scaling of the dimensionless conductance is described by a scaling equation,

$$\frac{d\ln g}{dx} = \beta(g), \tag{1}$$

where $x = \ln(\xi)$ is a scaling variable with $\xi = \xi(T)$ a length scale, and due to the simplifications above, the $\beta$ function depends only on $g$ itself [24,27]. In three dimensions, there is a MIT characterized by $\beta(g_C) = 0$, with $g_C$ the critical conductance. For $g > g_C$ one has $\beta(g) > 0$, the conductor is metallic, and the dimensionless conductance increases with increasing system size, while $\beta(g < g_C) < 0$, and one finds an insulator.

If we knew the $\beta$ function, we could compute the resistivity as follows: suppose we know the typical dimensionless conductance $g_0$ at an energy scale $T_0$ and at the corresponding microscopic length scale, $\xi_0 = \xi(T_0)$. Then the resistivity of a large three-dimensional conductor can be computed by integrating the scaling equation up to a length scale $x = \ln(\xi(T)/\xi_0)$ and cutting the system to small cubes of size $\xi = \xi(T)$ to give

$$\varrho(T, H) = \frac{h}{e^2}\frac{\xi(T) g_C}{g(\xi(T), g_0/g_C)}. \tag{2}$$

Note that here we compute the typical conductance [28]. Corrections due to universal conductance fluctuations can give a singular contribution [9]. However, these corrections are very small for magnets with a short mean free path such as $Ga_{1-x}Mn_xAs$. To compute $g(\xi)$ we need to know the beta-function. While this can and has been determined numerically for a noninteracting unitary system [12], it is not known for interacting electrons. Here we shall therefore use its asymptotic form on the metallic side, $\beta(g) = 1 - g_C^{(0)}/g$, with $g_C^{(0)}$ the critical value of the conductance to lowest order in the epsilon expansion [29]. The $1/g$ asymptotics of $\beta(g)$ is a consequence of electron-electron interaction, and is ultimately responsible for the $\sqrt{T}$ behavior of the resistivity at low $T$.

Furthermore, we need to know the connection between the scale $\xi(T)$ and the temperature $T$. This can be established by looking at the pole structure of the diffusion propagator, $D(T) \sim \xi^2(T) T z(T)$, and combining Eq. (2) with the Einstein relation, $\sigma(T) \sim (e^2/h) N(0) D(T)$, which relates the density of states $N(0)$ and the diffusion constant $D(T)$ to the conductivity [24,25]. Being in the metallic regime, with a good approximation, we can neglect the energy-dependence of the renormalization factor $z \approx 1$, and express $g(\xi)$ as $g(\xi) \propto \sigma\xi \propto N(0)\xi^3 T$ [24], that we

rewrite as

$$\left(\frac{\xi}{\xi_0}\right)^3 = \frac{g(\xi/\xi_0, g_0)}{g_0}\frac{T_0}{T}, \tag{3}$$

with $T_0$ the energy scale corresponding to the scale $\xi_0$.

With $\beta(g)$ and $\xi(T)$ at hand, we only need one more ingredient, the microscopic resistivity, $g_0 = g_0(h, t)$, with $t = T/T_C$ and $h = g\mu_B HS/T_C$ denoting the dimensionless temperature and magnetic field, respectively. In the following, we shall use the simple approximation

$$g_0(t, h) = g_0(m(t, h)) \approx \tilde{g}_0(1 + qm^2(t, h)), \tag{4}$$

where $\tilde{g}_0$ is the conductance of the unpolarized system. This approximation is well justified within a mean-field description of the scattering on spin disorder [12], but it also emerges quite naturally for other mechanisms [30]. In the present formalism, however, the precise microscopic origin of the $m$ dependence of $g_0$ is of secondary importance. The quadratic form in Eq. (4) provides a very good approximation, and from the fits we find $q \approx 0.5$–$0.7$ for all samples, in rough agreement with the results of Ref. [30]. Critical fluctuations also contribute to $g_0$. According to our estimates, while they might give a sizable contribution for samples (c) and (f), their contribution is at least 1 order of magnitude too small to explain the observed $T$ dependence for $T > T_C$ for samples (a),(b),(d), (f). Inclusion of these fluctuations in $g_0$ would shift the resistivity peak slightly above $T_C$, and give rise to a singularity in $d\varrho/dT$ (see inset of Fig. 2) [31]. As these contributions are overshadowed by the terms already present in our theory, we find it unnecessary to include them in the current quantitative analysis.

It is difficult to reliably separate experimentally the magnetization of the $Ga_{1-x}Mn_xAs$ film and that of the paramagnetic substrate in high field measurements. Furthermore, only magnetization curves at small magnetic fields ($H = 50$ Oe) were available. Therefore, instead of using the experimental data, we determined the magnetization $m(t, h)$ in a finite field by performing simulations for a diluted spin system. To simulate $Ga_{1-x}Mn_xAs$ we placed magnetic ions with a given concentration at random positions of an fcc lattice following the procedure of Ref. [32]. We assumed an RKKY interaction between the Mn spins and computed the magnetization curves $m(t, h)$ by performing a Monte Carlo simulation. This procedure reproduces the $m(t, h)$ curves, which fit very nicely the experimentally measured magnetization for $h = 0$ (see Fig. 2). Note, that the curves $m(t, h)$ obtained this way have no free fitting parameters.

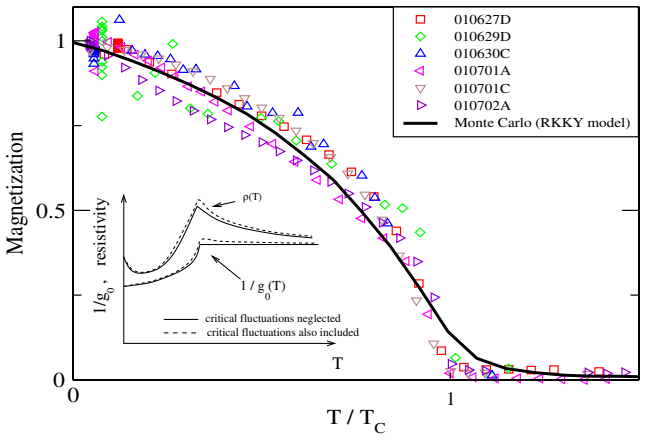


FIG. 2 (color online). Temperature dependence of the magnetization in an external in-plane magnetic field ($H = 50$ Oe) compared to our Monte Carlo results and assuming a RKKY, Mn-Mn interaction. The inset shows the effect of critical fluctuations.

We now have all ingredients to compute the magnetoresistance. The temperature and magnetic field dependence of the resistivity then originates from the temperature and magnetic field dependence of the microscopic conductance $g_0$ and that of the scale $\xi$: The correlation length $\xi$ becomes larger as $T \to 0$, and therefore the resistivity increases. This results ultimately in the low-temperature upturn of the resistivity and is also responsible for the upturn of the resistivity above $T_C$. At very low temperatures this results in a $\sim\sqrt{T}$ dependence [33]. Entering the ferromagnetic phase, or polarizing the Mn moments with an external field, on the other hand, increases $g_0$, and hence decreases the resistivity. It is the competition of these two effects that yields the major part of the resistivity anomaly at $T_C$.

Equations (2)–(4) provide a self-consistent theoretical description of the magnetoresistance in terms of three parameters for every sample, $\xi_0$, $\tilde{g}_0/g_C$, and the phenomenological parameter $q$. The latter is nearly sample independent: $q \approx 0.6$. Figure 1 shows the best fits obtained in this way for six different samples (see fitting parameters in Table I). For all samples we defined $\xi_0$ and $g_0$ as the scale and dimensionless conductance at $T_0 \equiv T_C$. The position, the shift, and the amplitude of both the resistivity maxima at $T_C$ as well as that of the low-temperature anomaly are well reproduced. This provides further evidence that for our samples both anomalies are related to the vicinity of the MIT.

In Table I we also list the values of $k_F l$ we obtain from the resistivity by assuming a spin 1/2 valence hole band of effective mass $m^* = 0.45m_e$ and a compensation of 50%. Clearly, the values obtained in this way are inconsistent with a weakly disordered free electron picture, and show a clear correlation with $\varrho(T_C)$ and the microscopic conductance $\tilde{g}_0$. There is no such simple correlation between $\varrho(T_C)$ and $x$, since $\varrho(T_C)$ is sensitive to the details of sample preparation or annealing protocol.

The fitted values of $\xi$ are smaller than the thickness of the films, $W \approx 123$ nm even at $T = 1$ K, and they are in good agreement with the values obtained in Refs. [34]. These samples are therefore clearly three dimensional from the point of view of conductance properties down to these temperatures. Also, $\xi$ remains larger than the typical

TABLE I. Characteristic parameters: $x$ is the Mn concentration, $\varrho(T_C)$ is the resistivity at $T_0 \equiv T_C$, and $k_F$ stands for the Fermi momentum obtained by assuming a compensation of 50%. We computed $k_F l$ from the Drude formula. We also show the Fermi wavelength $\lambda_F$ of the nonmagnetic system, the fitted correlation length at $T_0 = T_C$ [$\xi_0$] and at 1 K [$\xi(1\text{ K})$]. The dephasing length $\xi_{\text{Drude}}(T_C)$ is obtained using the Drude estimate. Finally we list the fitted values of $\tilde{g}_0/g_C$.

| Sample number | $x$ (%) | $T_C$ (K) | $\varrho(T_C)$ $\Omega$ cm | $k_F l(T_C)$ | $\lambda_F(T_C)$ (nm) | $\xi^{\text{Drude}}(T_C)$ (nm) | $\xi^{\text{fit}}(1\text{ K})$ (nm) | $\xi^{\text{fit}}(T_C)$ (nm) | $\tilde{g}_0/g_C$ |
|---|---|---|---|---|---|---|---|---|---|
| (a) 010627D | 1.35 | 42 | $27 \times 10^{-3}$ | 0.24 | 3.34 | 3.59 | 54.43 | 8.4 | 1.35 |
| (b) 010629D | 2.78 | 65 | $10 \times 10^{-3}$ | 0.51 | 2.64 | 4.38 | 70.94 | 8.8 | 2.85 |
| (c) 010630C | 3.91 | 90 | $4.5 \times 10^{-3}$ | 1.01 | 2.35 | 5.29 | 96.76 | 10.2 | 4.16 |
| (d) 010701A | 4.53 | 85 | $11 \times 10^{-3}$ | 0.39 | 2.24 | 3.43 | 94.03 | 10.2 | 4.01 |
| (e) 010701C | 5.87 | 110 | $3.6 \times 10^{-3}$ | 1.10 | 2.04 | 4.95 | 100.68 | 9.6 | 4.76 |
| (f) 010702A | 6.68 | 70 | $35 \times 10^{-3}$ | 0.10 | 1.97 | 1.82 | 73.62 | 8.8 | 1.70 |

Mn-Mn separation and the mean free path (both $\sim$1 nm) over the whole range of temperatures, thereby justifying the scaling approach used here. Table I also shows the theoretical estimate of $\xi^{\text{Drude}}(T) = \sqrt{D_{\text{Drude}}/T}$ where we evaluated $D_{\text{Drude}}$ by using the Drude formula and the density of states of a parabolic valence band with a renormalized mass. The values obtained in this way do not depend too much on the specific sample, and, apart from an overall factor, are in rough agreement with the values extracted from the experimentally measured magnetoresistance data.

In conclusion, we have presented a systematic study of the resistivity of various $Ga_{1-x}Mn_xAs$ samples. We have shown that even the annealed samples are very close to the MIT. The magnetic field dependence of the resistivity anomaly at $T_C$ as well as the low-temperature upturn of the resistivity can be *quantitatively* described in terms of a scaling theory, combined with Monte Carlo simulations.

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