Stimulated Scattering and Lasing of Intersubband Cavity Polaritons

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We present a microscopic theory describing the stimulated scattering of intersubband polariton excitations in a microcavity-embedded two-dimensional electron gas. In particular, we consider the polariton scattering induced by the interaction with longitudinal optical phonons. Our theory demonstrates the possibility of final-state stimulation for the scattering of such composite excitations, accounting for the deviations from ideal bosonicity occurring at high excitation densities. By using GaAs parameters, we predict a quantum degenerate regime and lasing without electronic population inversion in an optical pumping configuration.

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The scattering of bosons from an initial to a final state can be stimulated, i.e., enhanced, by the occupation of the final state. This property is in stark contrast to the behavior of fermions, such as electrons, whose scattering is Pauli blocked by final-state occupation. In low-energy matter, there are no elementary bosons, yet composite particles acting like bosons can be obtained when an even number of fermions are bound together, such as atoms containing an even total number of nucleons plus electrons. In condensed matter systems, the attractive interaction between two electrons can give rise to bosonic particles. Examples are Cooper pairs of electrons in metallic superconductors or Coulomb bound electron-hole pairs (excitons) in semiconductors. The strong coupling of an exciton with a microcavity photon produces the so-called exciton-polariton states, whose very small mass favors quantum degeneracy and the onset of stimulated scattering, responsible for exciton-polariton lasers [1] emitting in the near infrared.

Recently, a novel kind of cavity polariton excitations has been discovered in a microcavity-embedded two-dimensional electron gas [2] and an intense research activity is currently expanding [3–12]. These light-matter elementary excitations are the result of the strong coupling between a microcavity photon mode and the transition between two conduction subbands of a doped quantum well (QW) system. In contrast to Cooper pairs or excitons, intersubband excitations do not correspond to any bound state produced by an attractive fermion-fermion interaction. An intersubband excitation has a well-defined resonance frequency simply because the QW conduction subbands have parallel energy-momentum dispersions (see inset in Fig. 1); a sharply peaked joint optical density of states occurs already in the single-particle picture, in contrast to the case of excitons, where instead Coulomb electron-hole attraction is essential. This explains the remarkable robustness of intersubband cavity polaritons even at room temperature and the possibility to tailor their properties just by tuning the size of the QW or the density of the electron gas in the fundamental subband. Even if intersubband excitations do not correspond to any bound electronic state, they are still composed of an electron in an excited subband and a hole in the Fermi sea. Hence, we could regard them as composite bosons and expect the occurrence of stimulated scattering.

In this Letter, we present a microscopic theory of the stimulated scattering of intersubband cavity polariton excitations. In particular, we will consider the polariton scattering induced by the coupling with longitudinal optical

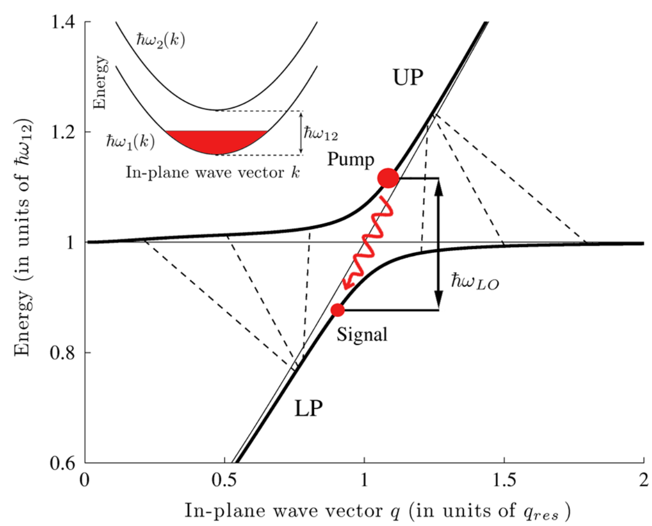

FIG. 1 (color online). A typical energy dispersion (in units of the intersubband transition energy $\hbar\omega_{12}$) of intersubband cavity polaritons versus in-plane wave vector (in units of the resonant wave vector $q_{\rm res}$). Because of the interaction with bulk LO-phonons, a polariton pumped in the upper polariton (UP) branch can scatter into a final state (signal mode) in the lower polariton (LP) branch by emitting a LO-phonon with energy $\hbar\omega_{\rm LO}$ (36 meV for GaAs). The considered modes have Hopfield coefficients $\beta_{{\rm UP},\mathbf{q}'} = \beta_{{\rm LP},\mathbf{q}} = 0.5$. The dashed lines indicate the same kind of scattering process by changing the in-plane momentum of the initial state along the upper polariton branch. Inset: the energy dispersion of conduction subbands of the doped QW versus electron wave vector k.

phonons (LO-phonons), which is typically the most important scattering channel affecting semiconductor intersubband transitions, while Coulomb interactions are known to produce only moderate renormalization effects [13]. Starting from the fermionic Hamiltonian for the QW electronic system and by using an iterative commutation procedure, we are able to determine the phonon-induced polariton scattering for an arbitrary number of excitations in the initial and final intersubband cavity polariton modes. Our results indeed prove the possibility of final-state stimulation of the intersubband cavity polariton scattering. Our theory also provides the deviations from perfect bosonicity, occurring at high excitation densities. We apply our results to the case of a GaAs system with realistic losses and consider the case of intersubband cavity polariton lasing under resonant optical pumping.

We consider the Hamiltonian $H = H_{\rm lm} + H_{\rm phon}$, where $H_{\rm lm}$ is the light-matter term for the cavity system, while $H_{\rm phon}$ describes the coupling to bulk phonons via the Fröhlich interaction [14]. Namely,

$$\begin{aligned} H_{\rm lm} = & \sum_{\mathbf{k},j=1,2} \hbar\omega_j(k) c^\dagger_{j,\mathbf{k}} c_{j,\mathbf{k}} + \sum_{\mathbf{q}} \hbar\omega_{\rm cav}(q) a^\dagger_{\mathbf{q}} a_{\mathbf{q}} \\ & + \sum_{\mathbf{k},\mathbf{q}} \hbar\chi(q) a^\dagger_{\mathbf{q}} c^\dagger_{1,\mathbf{k}} c_{2,\mathbf{k}+\mathbf{q}} + \hbar\chi(q) a_{\mathbf{q}} c^\dagger_{2,\mathbf{k}+\mathbf{q}} c_{1,\mathbf{k}}, \\ H_{\rm phon} = & \sum_{\mathbf{q},q_z} \hbar\omega_{\rm LO}(\mathbf{q},q_z) d^\dagger_{\mathbf{q},q_z} d_{\mathbf{q},q_z} + \sum_{\substack{\mathbf{k},\mathbf{q},q_z \\ i,j=1,2}} \hbar C^{ij}_{\mathbf{q},q_z} d_{\mathbf{q},q_z} c^\dagger_{i,\mathbf{k}+\mathbf{q}} c_{j,\mathbf{k}} \\ & + \hbar C^{ij}_{\mathbf{q},q_z} d^\dagger_{\mathbf{q},q_z} c^\dagger_{j,\mathbf{k}} c_{i,\mathbf{k}+\mathbf{q}}, \end{aligned} \tag{1}$$

where $c^\dagger_{j,\mathbf{k}}$, $a^\dagger_{\mathbf{q}}$, and $d^\dagger_{\mathbf{q},q_z}$ are the creation operators, respectively, for an electron in the QW conduction subband j with in-plane wave vector $\mathbf{k}$, a cavity photon with in-plane wave vector $\mathbf{q}$, and a LO-phonon with three-dimensional wave vector $(\mathbf{q}, q_z)$. Their respective energies are $\hbar\omega_j(k)$, $\hbar\omega_{\rm cav}(q)$, and $\hbar\omega_{\rm LO}(\mathbf{q}, q_z) = \hbar\omega_{\rm LO}$ (the wave vector dependence of the LO-phonon energy is negligible), and their phases are chosen in order to make the coupling coefficients $\chi(q)$ and $C^{ij}_{\mathbf{q},q_z}$ real. Being all the interactions spin conserving, we omit the spin degree of freedom for the electrons. Here, for simplicity, we consider the case of a single QW [15] embedded in the microcavity. The photon polarization is meant to be transverse magnetic (TM) in accordance with the selection rules of QW intersubband transitions. Note that, neglecting the conduction band nonparabolicity, the second subband dispersion is such that $\omega_2(k) = \omega_1(k) + \omega_{12}$, as depicted in the inset of Fig. 1. Moreover, for typical photonic wave vectors $\mathbf{q}$, we can safely approximate $\omega_j(|\mathbf{k} + \mathbf{q}|) \simeq \omega_j(k)$. The elementary excitations of the light-matter Hamiltonian $H_{\rm lm}$ are the two polariton branches, whose creation operators are

$$p^\dagger_{\eta,\mathbf{q}} = \alpha_{\eta,\mathbf{q}} a^\dagger_{\mathbf{q}} + \beta_{\eta,\mathbf{q}} b^\dagger_{\mathbf{q}}, \tag{2}$$

where $\eta = \{LP, UP\}$ is the polariton branch index, $\alpha_{\eta,\mathbf{q}}$ and $\beta_{\eta,\mathbf{q}}$ are real Hopfield coefficients describing the light and matter component, respectively, while $\hbar\omega_\eta(q)$ are their corresponding energies (see Fig. 1). $b^\dagger_{\mathbf{q}}$ is given by the expression [3]

$$b^\dagger_{\mathbf{q}} = \frac{1}{\sqrt{N}} \sum_{\mathbf{k}} c^\dagger_{2,\mathbf{k}+\mathbf{q}} c_{1,\mathbf{k}} \tag{3}$$

with N the total number of electrons in the doped QW. In the ground state, the two-dimensional electron gas fills the fundamental subband for $k < k_F$, being k_F the Fermi wave vector. $b^\dagger_{\mathbf{q}}$ creates a bright intersubband excitation with in-plane wave vector $\mathbf{q}$, obtained when the two-dimensional electron gas absorbs one cavity photon, as it can be deduced from the light-matter coupling in Eq. (1). We are interested in calculating the polariton scattering rate induced by the emission of a LO-phonon from an initial polariton "pump" mode (branch η' and in-plane wave vector $\mathbf{q}'$) to a final "signal" mode (branch η and in-plane wave vector $\mathbf{q}$). This kind of process is pictured in Fig. 1 for the case $\eta' = \mathrm{UP}$ and $\eta = \mathrm{LP}$. In order to have a sizeable polariton-phonon interaction, both the initial and final polariton modes must have significant electronic components, quantified by $|\beta_{\mathrm{UP},\mathbf{q}'}|^2$ and $|\beta_{\mathrm{LP},\mathbf{q}}|^2$. At the same time, in order to have a good coupling to the extracavity electromagnetic field (required for pumping and detection), also the photonic components $|\alpha_{\mathrm{UP},\mathbf{q}'}|^2$ and $|\alpha_{\mathrm{LP},\mathbf{q}}|^2$ need to be significant. These conditions can be simply met when the polariton energy splitting $2\hbar\chi(q_{\rm res}) \times \sqrt{N}$ at the resonant wave vector $q_{\rm res}$ [such as $\omega_{\rm cav}(q_{\rm res}) = \omega_{12}$] is a non-negligible fraction of the LO-phonon energy (36 meV for GaAs). This situation is already realized in recent microcavity samples [5,7,10,16] with midinfrared intersubband transition frequencies. If we wish to investigate the occurrence of stimulated scattering, we need to evaluate the scattering rates for arbitrary occupation numbers m and n of, respectively, the initial and final polariton modes. The emission of a LO-phonon can induce the scattering of one polariton from the pump to the signal mode, leading to a transition from the state $p^{\dagger m}_{\eta',\mathbf{q}'} p^{\dagger n}_{\eta,\mathbf{q}} |F\rangle$ to $d^\dagger_{\mathbf{q}'-\mathbf{q},q_z} p^{\dagger m-1}_{\eta',\mathbf{q}'} p^{\dagger n+1}_{\eta,\mathbf{q}} |F\rangle$, where $|F\rangle$ is the ground state (the N-electron ground state times the photon and phonon vacuum). Therefore, we need to consider the squared normalized matrix element $\hbar^2 |V^n_m|^2$ given by

$$\frac{|\langle F| p^{n+1}_{\eta,\mathbf{q}} p^{m-1}_{\eta',\mathbf{q}'} d_{\mathbf{q}'-\mathbf{q},q_z} H_{\rm phon} p^{\dagger m}_{\eta',\mathbf{q}'} p^{\dagger n}_{\eta,\mathbf{q}} |F\rangle|^2}{\langle F| p^n_{\eta,\mathbf{q}} p^m_{\eta',\mathbf{q}'} p^{\dagger m}_{\eta',\mathbf{q}'} p^{\dagger n}_{\eta,\mathbf{q}} |F\rangle \langle F| p^{n+1}_{\eta,\mathbf{q}} p^{m-1}_{\eta',\mathbf{q}'} p^{\dagger m-1}_{\eta',\mathbf{q}'} p^{\dagger n+1}_{\eta,\mathbf{q}} |F\rangle}. \tag{4}$$

In order to evaluate Eq. (4), we can exploit the expression of the polariton operators in Eq. (2). To evaluate the matrix elements, we need to commute the destruction operators multiple times to the right side and exploit the annihilation identity $a_{\mathbf{q}}|F\rangle = b_{\mathbf{q}}|F\rangle = 0$. The cavity photons are elementary bosons obeying the standard commutation rule $[a_{\mathbf{q}}, a^\dagger_{\mathbf{q}'}] = \delta_{\mathbf{q},\mathbf{q}'}$. Instead, intersubband excitation operators are not elementary bosons and satisfy modified commutation rules. We have found that

$$[b_{\mathbf{q}}, b^{\dagger}_{\mathbf{q}'}] = \delta_{\mathbf{q},\mathbf{q}'} - D_{\mathbf{q},\mathbf{q}'},$$
$$D_{\mathbf{q},\mathbf{q}'} = \delta_{\mathbf{q},\mathbf{q}'} - \frac{1}{N}\sum_{|\mathbf{k}|<k_F} c^{\dagger}_{1,\mathbf{k}} c_{1,\mathbf{k}+\mathbf{q}-\mathbf{q}'} - c^{\dagger}_{2,\mathbf{k}+\mathbf{q}'} c_{2,\mathbf{k}+\mathbf{q}}, \tag{5}$$

where $D_{\mathbf{q},\mathbf{q}'}$ is the operator describing the deviation from the behavior of elementary bosons, originally introduced in the context of excitonic composite bosons [17]. By iteration, we have the following commutation relations

$$[D_{\mathbf{q},\mathbf{q}'}, b^{\dagger m}_{\mathbf{q}''}] = \frac{2m}{N} b^{\dagger}_{\mathbf{q}''+\mathbf{q}'-\mathbf{q}} b^{\dagger m-1}_{\mathbf{q}''}, \qquad [b_{\mathbf{q}}, b^{\dagger m}_{\mathbf{q}'}] = m b^{\dagger m-1}_{\mathbf{q}'}(\delta_{\mathbf{q},\mathbf{q}'} - D_{\mathbf{q},\mathbf{q}'}) - \frac{m(m-1)}{N} b^{\dagger}_{2\mathbf{q}'-\mathbf{q}} b^{\dagger m-2}_{\mathbf{q}'},$$
$$[b^{m}_{\mathbf{q}}, b^{\dagger}_{\mathbf{q}'}] = m(\delta_{\mathbf{q},\mathbf{q}'} - D_{\mathbf{q},\mathbf{q}'}) b^{m-1}_{\mathbf{q}} - \frac{m(m-1)}{N} b_{2\mathbf{q}-\mathbf{q}'} b^{m-2}_{\mathbf{q}}. \tag{6}$$

Owing to the fact that typical photonic wave vectors q are much smaller (at least 2 orders of magnitude) than the Fermi wave vector k_F, we have $D_{\mathbf{q},\mathbf{q}'}|F\rangle \simeq 0$. We will thus assume $D_{\mathbf{q},\mathbf{q}'}|F\rangle = 0$ and neglect corrections of the order of $|\mathbf{q}-\mathbf{q}'|k_F$ due to the electrons occupying the edge of the Fermi sphere. Exploiting Eq. (6), some algebra shows that the unnormalized polaritonic matrix element $\langle F| p^{n+1}_{\eta,\mathbf{q}} p^{m-1}_{\eta',\mathbf{q}'} d_{\mathbf{q}-\mathbf{q}',q_z} H_{\mathrm{phon}} p^{\dagger m}_{\eta',\mathbf{q}'} p^{\dagger n}_{\eta,\mathbf{q}}|F\rangle$ is given by

$$(n+1)!m!\beta_{\eta,\mathbf{q}}\bar{\beta}_{\eta',\mathbf{q}'}(C^{22}_{\mathbf{q}-\mathbf{q}',q_z} - C^{11}_{\mathbf{q}-\mathbf{q}',q_z}) \sum_{\substack{l=0,\ldots,n\\ h=0,\ldots,m-1}} \binom{n}{l}\binom{m-1}{h} |\alpha_{\eta,\mathbf{q}}|^{2l}|\beta_{\eta,\mathbf{q}}|^{2(n-l)}|\alpha_{\eta',\mathbf{q}'}|^{2h}|\beta_{\eta',\mathbf{q}'}|^{2(m-1-h)} f^{n-l}_{m-h}, \tag{7}$$

where $f^{n}_{m} = \frac{n}{m}\mathcal{K}^{m-1,m}_{n+1,n-1} + \mathcal{K}^{m-1,m-1}_{n+1,n}$ and the quantity $\mathcal{K}^{n,s}_{m,r}$ is defined by the relation $n!m!\mathcal{K}^{n,s}_{m,r} = \langle F| b^{n}_{\mathbf{q}} b^{m}_{\mathbf{q}'} b^{\dagger s}_{\mathbf{q}} b^{\dagger r}_{\mathbf{q}'} b^{\dagger}_{\mathbf{Q}}|F\rangle$ with $\mathbf{Q} = \mathbf{q}(n-s) + \mathbf{q}'(m-r)$. Analogously, for the normalization factors in Eq. (4), we find

$$\langle F| p^{n}_{\eta,\mathbf{q}} p^{m}_{\eta',\mathbf{q}'} p^{\dagger m}_{\eta',\mathbf{q}'} p^{\dagger n}_{\eta,\mathbf{q}}|F\rangle = n!m! \sum_{\substack{l=0,\ldots,n\\ h=0,\ldots,m}} \binom{n}{l}\binom{m}{h} |\alpha_{\eta,\mathbf{q}}|^{2l}|\beta_{\eta,\mathbf{q}}|^{2(n-l)}|\alpha_{\eta',\mathbf{q}'}|^{2h}|\beta_{\eta',\mathbf{q}'}|^{2(m-h)} \mathcal{K}^{n-l,n-l}_{m-h,m-h-1}. \tag{8}$$

Using the commutators in Eq. (6), we get the recurrence relation

$$\mathcal{K}^{n,s}_{m,r} = \delta_{m,r}\delta_{n,s+1}\mathcal{K}^{n-1,n-1}_{m,m-1} + \delta_{m,r+1}\delta_{n,s}\mathcal{K}^{n,n-1}_{m-1,m-1} - \frac{s!r!}{n!m!N}[n(n-1)\mathcal{K}^{s,n-2}_{r,m} + m(m-1)\mathcal{K}^{s,n}_{r,m-2} + 2nm\mathcal{K}^{s,n-1}_{r,m-1}]$$

that allows us to numerically evaluate $\mathcal{K}^{m,s}_{n,r}$. After some algebra, Eq. (4) becomes

$$|V^{n}_{m}|^2 = (n+1)mB^{n}_{m}|\beta_{\eta,\mathbf{q}}\beta_{\eta',\mathbf{q}'}(C^{22}_{|\mathbf{q}-\mathbf{q}'|,q_z} - C^{11}_{|\mathbf{q}-\mathbf{q}'|,q_z})|^2,$$

where B^{n}_{m} is a bosonicity factor depending on the coefficients $\mathcal{K}^{n,s}_{m,r}$. Its expression is cumbersome, but it can be obtained putting together Eqs. (4), (7), and (8). Such a quantity depends on the Hopfield coefficients and on excitation numbers m and n normalized to the total number of electrons N in the ground state. In the inset of Fig. 2, we report B^{0}_{m} versus m/N obtained by a numerical evaluation of our recursive relations. For normalized excitation densities $\frac{m+n}{N}$ smaller than 0.1, we find that B^{n}_{m} is well approximated by the formula $B^{n}_{m} \simeq 1 - \zeta\frac{m+n}{N}$, where ζ depends on the Hopfield coefficients of the polariton modes and varies from 0 for pure photonic excitations to 1 for pure matter ones. Using the Fermi golden rule and calling $A^{q_z}_{\mathbf{q}}(\omega)$ the spectral function of a LO-phonon with three-dimensional wave vector $(\mathbf{q}, q_z)$, we have

$$\Gamma^{m,n}_{\mathrm{sc}} = 2\pi\sum_{q_z}\int d\omega |V^{n}_{m}|^2 A^{q_z}_{\mathbf{q}-\mathbf{q}'}(\omega)\delta(\omega_{\eta}(q) - \omega_{\eta'}(q') + \omega),$$

where $\Gamma^{m,n}_{\mathrm{sc}}$ is the number of polaritons per unit time scattered from the pump mode (with occupancy m) into the final signal mode (with occupancy n).

Using a Lorentzian shape of width Γ_{LO} for the phonon spectral function and neglecting the LO-phonon dispersion, we thus obtain

$$\Gamma^{m,n}_{\mathrm{sc}} = \frac{m}{S}(n+1)B^{n}_{m}|\beta_{\eta,\mathbf{q}}|^2|\beta_{\eta',\mathbf{q}'}|^2\frac{\omega_{\mathrm{LO}}}{\Gamma_{\mathrm{LO}}}\frac{4e^2L_{\mathrm{QW}}F_{\sigma}}{\epsilon\hbar}. \tag{9}$$

This expression contains the effect of final-state stimulation through the $(n+1)$ term and the deviations from ideal bosonic behavior through the bosonicity factor B^{n}_{m}. The other parameters in the formula are S the sample surface, L_{QW} the QW length, and F_{σ} a form factor (depending on $\sigma = L_{\mathrm{QW}}|\mathbf{q}-\mathbf{q}'|$) describing the overlap between the conduction subband and the phonon envelope wave functions [14]. For typical QW widths and photonic wave vectors, $F_{\sigma} \simeq 0.1$. For GaAs LO-phonons, the ratio $\frac{\omega_{\mathrm{LO}}}{\Gamma_{\mathrm{LO}}} \approx 100$. In Fig. 2, we report the calculation of the spontaneous in-scattering rate $\Gamma^{m,0}_{\mathrm{sc}}$ (i.e., $n=0$, unoccupied final state) for the process shown in Fig. 1 for a GaAs system with $\hbar\omega_{12} = 150$ meV (midinfrared), $L_{\mathrm{QW}} = 10$ nm, $N/S = 10^{12}$ cm^{-2}. In order to have a buildup of the occupation number of the final state and to enter the regime of stimulated scattering, the spontaneous in-scattering rate $\Gamma^{m,0}_{\mathrm{sc}}$ must be compared with the polariton damping rate given by the formula

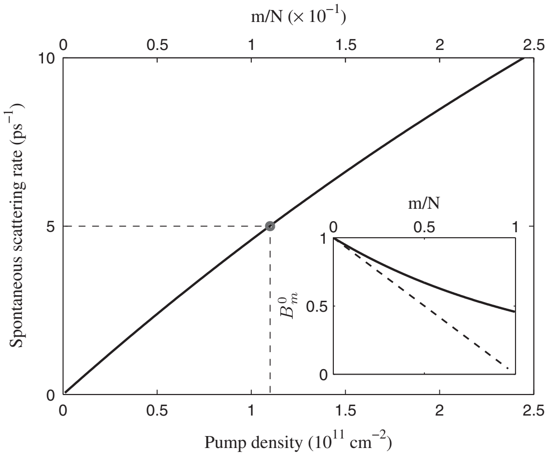

FIG. 2. Spontaneous scattering rate $\Gamma_{\rm sc}^{m,n=0}$ from the pumped polariton to the signal polariton mode for the process depicted in Fig. 1 versus the pump polariton density m/S. The electron density in the ground state is $N/S = 10^{12}$ cm^{-2}. In the considered range of excitation densities, $m/N < 0.25$, i.e., much smaller than the onset of electronic population inversion. Other GaAs parameters are given in the text. Inset: the solid line represents the bosonicity factor $B_m^{n=0}$ versus m/N for the pump and signal polariton modes considered in Fig. 1. For elementary bosons, B_m^n is always 1. The dashed line is the same quantity for pure matter excitations. For $m/N \ll 1$, deviations from perfect bosonicity are negligible even in the case with only one QW [15]. For $n \ll m$ (signal much smaller than pump), $B_m^n \simeq B_m^{n=0}$ and the stimulated scattering rate is $\Gamma_{\rm sc}^{m,n} \simeq (n+1)\Gamma_{\rm sc}^{m,n=0}$.

$$\Gamma_{\eta,q}^{\rm loss} \simeq |\alpha_{\eta,\mathbf{q}}|^2\Gamma_{{\rm cav},q}^{\rm loss} + |\beta_{\eta,\mathbf{q}}|^2\Gamma_{12}^{\rm loss}, \tag{10}$$

where $\Gamma_{{\rm cav},q}^{\rm loss}$ is the damping rate for the cavity mode (due to the finite mirror transmission) and $\Gamma_{12}^{\rm loss}$ is the intersubband excitation damping rate due to nonradiative processes. Neglecting the pump depletion (relevant only above an eventual stimulation threshold), we can write two rate equations for the signal and pump mode occupation numbers, namely

$$\frac{dn}{dt} = \Gamma_{\rm sc}^{m,n} - \Gamma_{\eta,q}^{\rm loss}n, \quad \frac{dm}{dt} = \frac{\kappa I_{\rm pump}S}{\hbar\omega_{\eta',q'}} - \Gamma_{\eta',q'}^{\rm loss}m, \tag{11}$$

where κ is the polariton absorption coefficient at the pump frequency and $I_{\rm pump}$ the optical pump intensity. From the steady-state solution for n, we can calculate the threshold pump density $m_{\rm thr}/S$ to have a lasing instability. For $n \ll m$, $B_m^n \simeq B_m^0$ and $\Gamma_{\rm sc}^{m,n} \simeq (n+1)\Gamma_{\rm sc}^{m,0}$. The threshold pump polariton density $m_{\rm thr}/S$ is then given by the equation $\Gamma_{\rm sc}^{m_{\rm thr},0} = \Gamma_{\eta,q}^{\rm loss}$. The steady-state solution for m gives the threshold pumping intensity versus the polariton threshold density $I_{\rm pump}^{\rm thr} = \frac{\Gamma_{\eta',q'}^{\rm loss}\hbar\omega_{\eta'}(q')}{\kappa} m_{\rm thr}/S$. For a realistic value $\Gamma_{\eta,q}^{\rm loss} = \Gamma_{\eta',q'}^{\rm loss} = 5$ ps^{-1}, we obtain a threshold density for the pump mode of 1.1×10^{11} cm^{-2}, i.e., $m/N = 0.11$, as indicated in Fig. 2. With a polariton absorption coefficient $\kappa = 0.4$ [10], this gives a threshold pump intensity of 3.5×10^4 W/cm^2. This is approximately 2 orders of magnitude smaller of what required to achieve electron population inversion [18].

Note that the mechanism described here is different from the standard phonon-assisted lasing based on stimulated Raman photon scattering [19]: in such a traditional case, the stimulation concerns the photon field. In our case, it is the *polariton* field to be stimulated and the pump creates real polariton excitations.

In conclusion, we have derived a theory for the stimulated scattering of intersubband cavity polariton excitations of a dense two-dimensional electron gas. The intersubband cavity polariton excitations are composite bosons arising from the strong light-matter coupling and are not associated to any bound electronic states. We have shown exactly how the bosonicity of these excitations is controlled by the density of the two-dimensional electron gas in the ground state. The present theory could pave the way to the experimental demonstration of fundamental quantum degeneracy phenomena and unconventional lasing devices without population inversion based on composite bosons with controllable properties and interactions.

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