New Flow Regimes Generated by Mode Coupling in Buoyant-Thermocapillary Convection

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We report on a new nonlinear dynamics occurring in a confined cylindrical column filled with fluid (liquid bridge) and heated from above. We demonstrate and analyze the novel oscillatory flow state created by the interaction of two hydrothermal waves of different origins: one propagates vertically from the cold towards the hot side ($m = 0$) and another is traveling in the azimuthal direction ($m = 1$). Their interaction leads to an exotic flow structure: during a part of the oscillation period the resulting wave propagates in a given azimuthal direction, whereas during the rest of the period it moves in the opposite direction. A new bimodal flow regime is found to exist over a parameter range where these modes have comparable influence. The phase diagrams, obtained by three-dimensional nonlinear simulations, are reported. They shed light on the instability mechanism and criteria of the existence of novel states.

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The investigation of the Rayleigh-Bénard and Rayleigh-Marangoni convection has played a crucial role in the development of the nonlinear stability theory for planar spatially extended systems [1]. Recently, significant progress has been achieved in understanding the nonlinear regimes of buoyant-thermocapillary convection in cylindrical columns [2,3]. The latter problem can be considered as a paradigmatic example of nonlinear dynamics in essentially nonparallel flows. The mechanisms of instability have a universal character. Therefore, it is of a general interest, far beyond its direct applications in engineering. Because of the periodicity of the problem in the azimuthal direction, the spectrum of azimuthal wave numbers is discrete. The hydrothermal waves, which are developed due to the oscillatory instability of a steady axisymmetric flow, are characterized by integer values of the azimuthal wave number m. Typically, above the critical point, a 3D oscillatory flow starts as a finite-amplitude standing or traveling (TW) wave. When the thermal stresses increase, the nonlinear dynamics is determined mostly by the interaction of modes with $m = 1$ and $m = 2$, e.g., [2,3].

Still, the role of the simplest, axisymmetric oscillatory mode $m = 0$, is somehow elusive, and there is no agreement on that subject up to now. Xu and Davis [4] predicted the selection of $m = 0$ mode for infinitely long liquid columns with $\mathrm{Pr} > 50$. Shevtsova and Legros [5] found the instability with wave number $m = 0$ in a liquid with $\mathrm{Pr} = 105$ (in the framework of an axisymmetric problem with the interface deformed by gravity). That theoretical prediction has been confirmed by laboratory experiments (for the same liquid) by Shevtsova *et al.* [6] for the aspect ratio $\Gamma = d/R = 4/3$ in a narrow region near the critical point (d is the height and R is the radius of liquid bridge). Frank and Schwabe [7] did not find mode $m = 0$ in ground experiments for liquids with $\mathrm{Pr} = 7, 49$ and 65. The experiments in the longest ever liquid bridges, $\Gamma = 5$ and $\mathrm{Pr} = 28$, performed by Schwabe [8] in microgravity, revealed convection cells drifting from the hot towards the cold side. However, these measurements were performed above the threshold of the 3D oscillatory regime, where modes with $m \geq 1$ are present. Numerical simulations for aspect ratio close to unity $\Gamma \sim 1$ also could not reveal the $m = 0$ as being critical, e.g., [2,9,10]. In short liquid bridges the rigid walls impose a strong constraint to the appearance and spreading of axial waves.

In the present Letter, we clarify the question of the existence of axisymmetric wavy motions, and analyze the consequences of the interaction between the modes with $m = 0$ and $m = 1$ that leads to novel, highly nontrivial, flow regimes. Our analysis also explains the enigmatic flow regime with alternating directions of hot or cold spot's motion observed in our numerical simulations, as well as in the experiments of the Kawamura group [11].

Consider a three-dimensional convection in a vertical cylindrical column; see Fig. 1. The fluid volume is held between two differentially heated horizontal flat coaxial disks of radius R, separated by a distance d. The tempera-

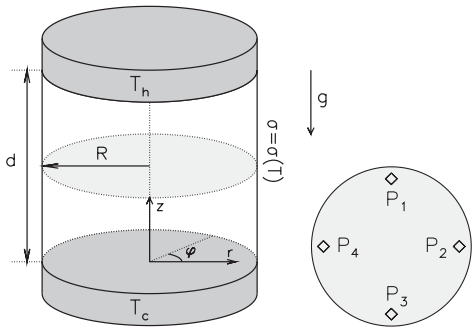

FIG. 1. Geometry of the problem. Time histories are recorded at four equidistant points P_1, P_2, P_3, P_4, in azimuthal direction for a fixed z and the same r.

tures T_h and T_c ($\Delta T = T_h - T_c > 0$) are prescribed at the upper and lower solid walls, respectively. The free surface is considered cylindrical and nondeformable. The surface tension, σ, and kinematic viscosity, ν, are linearly decreasing functions of temperature

$$\sigma(T) = \sigma_0 - \sigma_T(T - T_c), \qquad \nu(T) = \nu_0 + \nu_T(T - T_c),$$

where $\sigma_T = -\partial_T \sigma > 0$, $\nu_T = \partial_T \nu < 0$.

The problem is characterized by five independent nondimensional parameters. Three of them, the Marangoni number, Ma, the Rayleigh number, Ra, and the relative variation of viscosity, R_ν, are proportional to ΔT and determined by the relations

$$\mathrm{Ma} = \frac{\sigma_T \Delta T d}{\rho_0 \nu_0 \chi}, \qquad \mathrm{Ra} = \frac{g\beta_T \Delta T d^3}{\nu_0 \chi}, \qquad R_\nu = \frac{\nu_T \Delta T}{\nu_0}, \quad (1)$$

Therefore their ratios, the dynamic Bond number,

$$\mathrm{Bo}_{\mathrm{dyn}} = \mathrm{Ra}/\mathrm{Ma} = g\beta_T \rho_0 d^2/\sigma_T$$

and the ratio R_ν/Ma, are constant. The Prandtl number is $\mathrm{Pr} = \nu_0/\chi$ and Γ was defined earlier. Here β_T is the thermal expansion coefficient, χ and ρ are the thermal diffusivity and density; subscript 0 notes quantity at reference temperature T_c.

The governing Navier-Stokes, energy and continuity equations are written in the nondimensional form (the scales for time, velocity, and pressure are $t_{\mathrm{ch}} = d^2/\nu_0$, $V_{\mathrm{ch}} = \nu_0/d$, and $P_{\mathrm{ch}} = \rho_0 V_{\mathrm{ch}}^2$):

$$\partial_t \mathbf{V} + \mathbf{V}\nabla\mathbf{V} = -\nabla P + R_\nu \cdot 2\mathbf{S} \times \nabla\Theta + (1 + R_\nu\Theta)\Delta\mathbf{V} + \vec{e}_z \mathrm{RaPr}^{-1}\Theta, \quad (2)$$

$$\partial_t \Theta + \mathbf{V}\nabla\Theta = \mathrm{Pr}^{-1}\Delta\Theta, \quad (3)$$

$$\nabla \cdot \mathbf{V} = 0, \quad (4)$$

where velocity is defined as $\mathbf{V} = (V_r, V_\varphi, V_z)$, $\Theta = (T - T_c)/\Delta T$ is the dimensionless temperature, $\mathbf{S} = \frac{1}{2} \times (\partial V_i/\partial x_k + \partial V_k/\partial x_i)$ is the strain rate tensor.

At the rigid walls no-slip conditions $\mathbf{V}(z = 0) = \mathbf{V}(z = \Gamma) = 0$, and constant temperatures $\Theta(z = 0) = 0$, $\Theta(z = \Gamma) = 1$ are imposed. The stress balance between the viscous fluid and the inviscid gas on the flat free surface ($r = 1$) is

$$(1 + R_\nu\Theta)\mathbf{S}\mathbf{e}_\mathbf{r} + \mathrm{MaPr}^{-1}(\mathbf{e}_\mathbf{z}\partial_z + \mathbf{e}_\varphi r^{-1}\partial_\varphi)\Theta = 0, \qquad V_r = 0.$$

The free surface is assumed to be thermally insulated $\partial_r\Theta(1, \varphi, z, t) = 0$. As the initial state, either a zero velocity field (for smaller Ma) or a solution obtained earlier for another value of Ma (for larger Ma) has been chosen. No dependence on initial conditions and no hysteresis have been observed in simulations presented below.

Simulations are performed for silicone oil 1cSt with $\mathrm{Pr} = 14.3$, $\mathrm{Bo}_{\mathrm{dyn}} = 3.11$, $R_\nu/\mathrm{Ma} = -6.97 \times 10^{-6}$ and aspect ratio $\Gamma = 1.8$ (keeping in mind $R = 2.5$ mm and $d = 4.5$ mm). Then the only control parameter is the Marangoni number, Ma (i.e., the temperature difference ΔT between the disks). The numerical code written in-house was previously thoroughly validated, see Melnikov *et al.* [3]. Hereafter, we examine the disturbance field: an average in the azimuthal direction $(1/2\pi)\int_0^{2\pi} q(r, \varphi, z)d\varphi$ is subtracted from its net field $q(r, \varphi, z)$ for any quantity q.

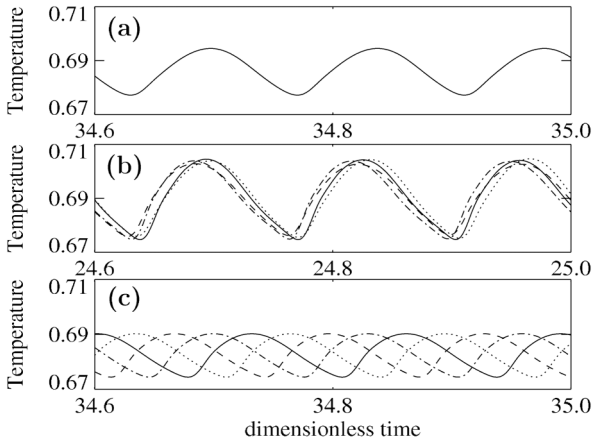

FIG. 2. Time dependence of temperature oscillations at equidistant in azimuthal direction points P_i, (a) Ma = 15 830; (b) Ma = 17 200; (c) Ma = 17 900.

The system at the chosen set of parameters displays a collection of remarkable features. The first intriguing feature is that, at the first critical point, it bifurcates from a 2D stationary solution to a 2D time-dependent one. This represents excitation of the $m = 0$ mode when the perturbation spreads along the z axis, parallel to the temperature gradient as in extended cavities or thin layers [12]. In the considered case of long liquid bridge ($\Gamma = 1.8$) the wave $m = 0$ appears as a critical one at $\mathrm{Ma}_1^{\mathrm{cr}} = 14\,160$ and persists up to $1.3\mathrm{Ma}_1^{\mathrm{cr}}$.

The same phase and amplitude of oscillations shown in Fig. 2(a) at four different points P_i is an unambiguous

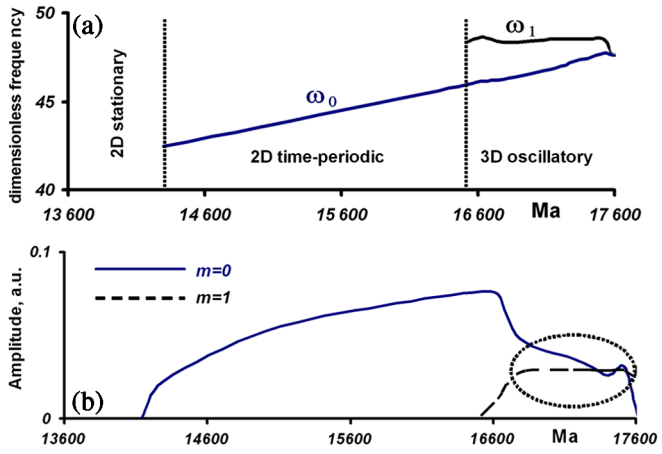

FIG. 3 (color online). Evolution of the frequency and of spatial Fourier amplitude (in arbitrary units) of the temperature field.

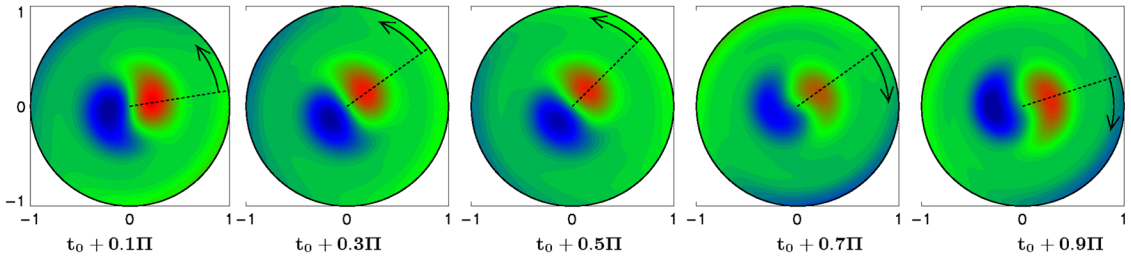

FIG. 4 (color online). Snapshots of temperature disturbance field in the horizontal cross section. Ma = 17 200.

proof of perturbations with mode $m = 0$. An analysis of its properties reveals that it is a hydrothermal wave which spreads from the cold to the hot side. The Fourier spectrum corresponds to a periodic motion with the basic frequency ω_0, which linearly grows near the instability threshold; see Fig. 3.

Far above the first bifurcation, at Ma $>$ 17 600, the flow state is an azimuthal traveling wave with the mode $m = 1$ spreading in counterclockwise direction with frequency ω_1. The oscillations, shown at different azimuthal points P_i in Fig. 2(c) have a constant phase shift and constant amplitude with time.

A novel type of flow organization, shown in Fig. 2(b), is found in the intermediate range, at 16 520 $<$ Ma $<$ 17 600. This type of oscillations does not correspond to either $m = 0$ or traveling/standing wave. The temporal Fourier spectrum reveals two basic frequencies [see Fig. 3(a)]: ω_0 for the mode $m = 0$ and ω_1 for the mode $m = 1$ as well as their combinations. On the plot of spatial amplitudes in Fig. 3(b), this region is shown by dotted elliptical curve.

Five snapshots in Fig. 4 outline the temporal behavior of the enigmatic flow regime. The first three snapshots show that the structure of the temperature field corresponds to traveling wave "$m = 1$" propagating in the counterclockwise direction. However, after 2/3 of a period, $\Pi = \frac{1}{2\pi\omega_1}$, the spots start rotating in the opposite, clockwise direction (see direction of arrows). For the first time we report a new, earlier unknown flow behavior; see movie in the supplementary material [13].

In addition to the swing oscillations, amazing flow patterns are observed on the interface. In far supercritical area (Ma = 17 900) the inclined TW with the $m = 1$ propagates in azimuthal direction. In the case of mode coupling, a collection of different flow patterns is observed. Snapshots of temperature disturbance field on the unrolled interface for this case are shown in Fig. 5 (Ma = 17 200). As a general trend, the cells move from the hot to the cold side. Starting from almost four symmetrical cells [Fig. 5(a)] they transform to a pattern typical for azimuthal TW [Fig. 5(b)]. Further, [Fig. 5(c)] each spot splits into triple cells and wave in the upper part becomes inclined while cells on the cold side are almost straight. The most remarkable situation is shown in Fig. 5(d), when cells near cold and hot walls have a different inclination angle.

The nature of the observed "swing regime" can be understood by the analysis of the corresponding trajectory in the phase space. Near the second critical point, when azimuthal hydrothermal wave bifurcates from 2D state, the temperature disturbances can be presented as

$$\Theta = \sum_{-\infty}^{\infty} \theta_m(r, z, t) \exp(im\varphi) + \text{c.c.} \tag{5}$$

Obviously, mode $m = 1$ is present in the flow organization shown in Fig. 4. To examine its properties the temperature field is decomposed in real and imaginary parts at fixed horizontal position $z = z^*$

$$\begin{aligned} A_1(z^*, t) &= \int_0^1 r dr \int_0^{2\pi} \Theta(r, z^*, \phi, t) e^{-i\phi} d\phi \\ &= \int \Theta(r, t) \cos(\varphi) r dr d\varphi - i \int \Theta(r, t) \\ &\quad \times \sin(\varphi) r dr d\varphi \\ &= \mathrm{Re}[A_1(t)] + i\mathrm{Im}[A_1(t)]. \end{aligned}$$

The evolution of the real, $\mathrm{Re}(A_1)$, and imaginary, $\mathrm{Im}(A_1)$, parts with time is shown in Fig. 6 for the classical traveling wave $m = 1$, which exists in supercritical regime at Ma $>$ 17 900, and for *the new type* state at Ma = 17 200.

The observation of the displacement of the prescribed temperature with time, (e.g. the position of a point on trajectory) is performed from 2D steady state, i.e., at the point "O" where $\mathrm{Re}(A_1) = \mathrm{Im}(A_1) = 0$. In the case of TW, see Fig. 6(a), during one period of oscillation this point circumscribes a circle with a center in point "O",

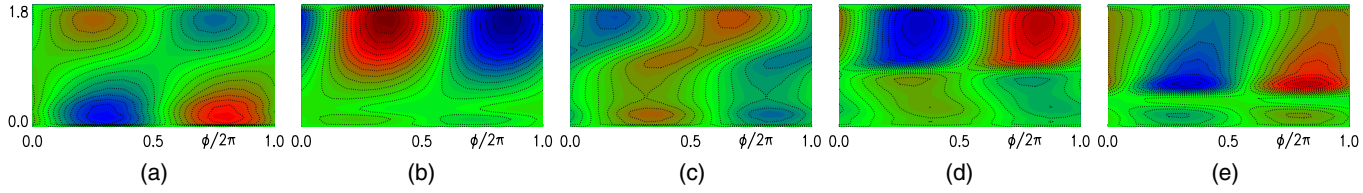

FIG. 5 (color online). Snapshots of temperature disturbance field on the unrolled interface; Ma = 17 200.

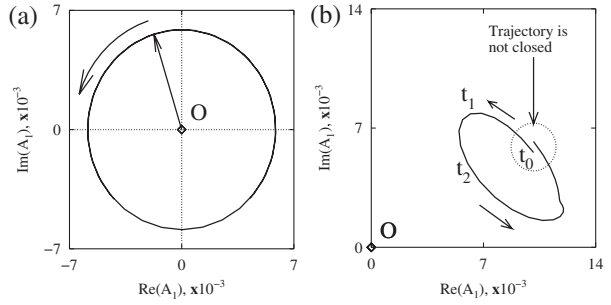

FIG. 6. Trajectories of dynamic system during one "fast" period in the case of (a) traveling wave at Ma = 17 900 and (b) "new type" flow state when Ma = 17 200.

moving in counterclockwise direction. In the Fig. 6(b), the observations are still performed from the point "O", but the motion of the dynamic system generates an elliptical trajectory that is no longer centered at "O", but is distant from it. The nonlinear interaction with the mode $m = 0$ shifts the center of the small pseudocycle from the origin.

The shift is so strong, that though the rotation around the center of the "ellipse" is permanently counterclockwise, the observer can notice that during a part of the cycle ($t_0 \rightarrow t_1$) the system of spots moves in a given direction, while in another part of the cycle, it moves in the opposite direction ($t_1 \rightarrow t_2$).

Complexity of the system is that at the end of a single period the system does not return to its initial point, but slightly shifts. In the following cycle the system starts from a different azimuthal position. Within the next cycles the system performs similar rotations at a fixed rate and simultaneously translates in a counterclockwise direction, also at a fixed rate, see Fig. 7(a). If we continue observation in time, we can see that the trajectory develops further to reach a helicoidal form.

Finally, on the long time scale, the trajectory of the system forms a torus (doughnut) produced by spiral shapes; see Fig. 7(b). The time of formation of the doughnut shape is about 30 times larger than the period of a single coil. The torus is a superposition of two motions: a fast cyclic motion along the orbit "of index zero" and slow motion around the origin. Both original periodic structures,

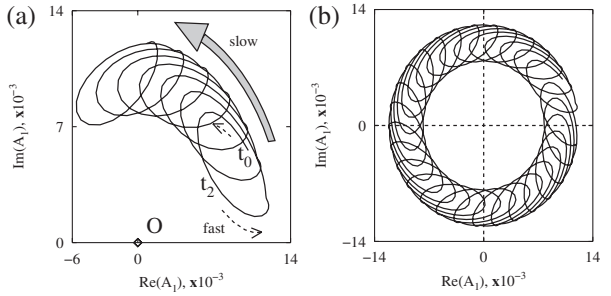

FIG. 7. (a) The progress of multicycles trajectory of dynamic system when Ma = 17 200; (b) the final view of the trajectory of the system.

i.e., $m = 0$ axial wave and $m = 1$ azimuthally traveling $m = 1$ wave, are fast. The frequency of a slowly varying stationary wave is the difference of the two frequencies $\omega_{\text{slow}} = \omega_1 - \omega_0$. So, the trajectories shown in Fig. 7 can be presented as (a_i is the corresponding amplitude)

$$A_1(t) \approx [a_0 + a_1 \exp(i\omega_1 t) + a_{-1} \exp(-i\omega_1 t) + a_2 \exp(2i\omega_1 t) + \ldots] \exp(i\omega_{\text{slow}} t). \quad (6)$$

The diagram of observed regimes is shown in Fig. 3. Note that near the point $\text{Ma}_2^{\text{cr}} \approx 16\,520$, where the quasiperiodic regime is born, the amplitude $A_1(t)$ is small, but its evolution described by (6) is essentially anharmonic. Near the right border of the quasiperiodicity interval, Ma ≈ 17 600, the frequencies ω_0 and ω_1 become very close. In that region, where $m = 1$ prevails, the flow looks as a TW with slowly changing amplitude. Swing regime is found for $1.77 < \Gamma < 2$ when $m = 1$ (TW) and $m = 0$ coexist.

To conclude, we have reported an example of a simple system which leads to complexity near the second critical point Ma_2^{cr}.

Note that the novel phenomenon is generic, because it describes a generic bifurcation from an axisymmetric oscillatory regime and therefore it should be interesting to a much wider community of physicists than just the experts in the field of thermocapillary convection.

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[13] See EPAPS Document No. E-PRLTAO-102-072915 for a movie of a new, earlier unknown flow behavior. For more information on EPAPS, see http://www.aip.org/pubservs/epaps.html.