Testing the Topological Nature of the Fractional Quantum Hall Edge

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We carry out numerical diagonalization for much larger systems than before by restricting the fractional quantum Hall (FQH) edge excitations to a basis that is exact for a short-range interaction and very accurate for the Coulomb interaction. This enables us to perform substantial tests of the predicted universality of the edge physics. Our results suggest the possibility that the behavior of the FQH edge is intrinsically nonuniversal, even in the absence of edge reconstruction, and therefore may not bear a sharp and unique relation to the nature of the bulk FQH state.

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The interior of a fractional quantum Hall (FQH) system [1] is gapped, but massless excitations exist at its edge, which constitutes a realization of a one-dimensional electron liquid described generically by the Tomonaga-Luttinger theory [2]. Much attention has been focused on the edge physics since the work of Wen [3], where it was conjectured that the exponent describing the long distance, low energy physics of the chiral (unidirectional) FQH edge is a unique ''topological'' quantum number for any given FQH state, independent of details, just as the quantum Hall resistance. Our understanding of the ordinary one-dimensional liquids is largely based on the method of bosonization, which exploits a one-to-one correspondence between the fermionic and bosonic Fock spaces in one dimension, and identifies a relationship between the operators on these spaces; specifically, the fermionic field operator $\hat{\psi}(x)$ is related to the bosonic field operator $\hat{\phi}(x)$ through the expression $\hat{\psi}(x) \sim \exp[-i\hat{\phi}(x)]$, which can be established rigorously at the operator level [2]. In the absence of a similar rigorous derivation for the electron field operator at the edge of a FQH system, Wen formulated an effective field theory approach (EFTA) [3] wherein he postulated that the electron operator at the edge of the $1/m$ FQH state, defined by Hall resistance quantization at $R_H = h/(1/m)e^2$, is given by

$$\hat{\psi}(x) \sim e^{-i\sqrt{m}\hat{\phi}(x)}. \quad (1)$$

Antisymmetry under exchange quantizes m to an odd integer value, independent of parameters other than the quantized Hall resistance, which leads to universal properties for the edge physics. Experiments involving lateral tunneling of an electron from an ordinary Fermi liquid into the edge of a FQH system allow for a direct test of this assertion. For the fractions $\nu = n/(2np + 1)$, a generalization of Eq. (1) predicts the tunnel current to be of the form $I \sim V^{\alpha}$ with a universal value of $\alpha = 2p + 1$ for the exponent. Ingenious experiments [4–7] have determined the edge exponent from the I-V characteristics for this geometry. While they establish the existence of non-Fermi liquid (Tomonaga-Luttinger) behavior, with an exponent different from a one-dimensional Fermi liquid ($\alpha = 1$), they also show discrepancy from the EFTA prediction. Specifically, the measured exponents at filling factors $\nu = 1/3$, $2/5$, and $3/7$ are $\sim$2.7, 2.3, and 2.1, respectively [4–7], to be compared with the EFTA prediction of 3.0; furthermore, the experimental exponent varies continuously with the filling factor, and thus is not determined solely by the quantized Hall conductance.

A number of theoretical papers have addressed this inconsistency [7–20]. Some of these suggest that the disagreement is due to edge reconstruction, which produces several counter-propagating edge modes (for which the exponent is not universal) [15–18,21], while some propose that the inconsistency persists even in the absence of edge reconstruction, thus pointing to a more fundamental deficiency of the EFTA [13,19]. A resolution of this issue is important in its own right, and also has obvious implications for the program of determining the nature of a bulk FQH state by probing its edge [22]; such a program presumes the existence of a unique relationship between the bulk and the edge physics and hence the universality of the latter.

Exact diagonalization studies often provide an unprejudiced, reliable, and decisive tool for testing ideas in the field of the FQH effect. For the edge physics, however, it has not been clear if the discrepancy between the finite system results and the EFTA is intrinsic or a finite size artifact; finite size corrections are more severe for the edge physics [20] because of a power law decay of correlations, in contrast to the Gaussian decay in the bulk. Unfortunately, the dimension of the Hilbert space grows exponentially with the number of electrons, making it impossible to increase the system sizes significantly in exact diagonalization studies.

In this Letter we report on microscopic tests of the EFTA by diagonalizing the Coulomb Hamiltonian in a truncated space of composite fermion (CF) edge excitations, which makes it possible to investigate much larger systems than before. Specifically, we consider the edge excitations of the $1/3$ FQH state in the disk geometry, and the truncated space contains all states of the form

$$\Psi_\alpha^M = \prod_{j<k}(z_j - z_k)^2 \Phi_\alpha^{M^*}, \qquad M = M^* + N(N-1), \quad (2)$$

where $z_j = x_j - iy_j$ denotes the electron coordinates as a complex number, M is the total angular momentum of the sate, and $\Phi_\alpha^{M^*}$ are all lowest Landau level states (labeled by $\alpha = 1, \cdots, D^*$) at total angular momentum M^*. The dimension of this basis space is much smaller than the full dimension of the lowest Landau level states at M, allowing investigation of systems with as many as 45 particles. The restriction to this basis is equivalent to restricting composite fermions [23] to their lowest Λ level (also known as composite fermion Landau level). More accurate results can be obtained by systematically enlarging the space of states in the standard manner [24] to include CF excitations to higher Λ levels (i.e., by including in the set $\{\Phi_\alpha^{M^*}\}$ states occupying higher Landau levels, constructing $\{\Psi_\alpha^{M^*}\}$ as in Eq. (1), and projecting them onto the lowest Landau level), but that will not be necessary for our present purposes. The lowest-Λ-level approximation for composite fermions is known to be excellent, and we have also confirmed its accuracy explicitly for edge excitations for systems with six and seven particles, for which exact results are available, both with and without a confinement potential. Also, Ψ_α^M are the only states that survive if we add to the Coulomb interaction an appropriate infinitely strong short-range interaction that annihilates states containing electronic pairs with angular momenta equal to unity; our results below are exact for this model. Therefore, our truncated Hilbert space ought to capture the topological nature, if it exists, of the edge physics. We believe that this model actually gives the best chance for universal behavior; admixture with higher Λ levels can only spoil it [13].

We consider a system of two dimensional electron gas in disk geometry. The neutralizing background has uniformly distributed positive charge contained in a disk Ω_N of radius $R_N = \sqrt{2N/\nu}$ for a system of N particles at filling factor ν; the positively charged disk is separated by a distance d from the electron disk (we quote lengths in units of the magnetic length $l = \sqrt{\hbar c/eB}$). The electrons are approximately confined to the same radius because of charge neutrality in the interior. This system is modeled by the following realistic Hamiltonian:

$$H = E_K + V_{ee} + V_{eb} + V_{bb}, \quad (3)$$

where the terms on the right-hand side represent the kinetic, electron-electron, electron-background, and background-background Coulomb interaction energies, respectively. At large magnetic fields only the lowest Landau level states are occupied; hence, the kinetic energy $\hbar\omega_c/2$ (where $\omega_c \equiv eB/m_b c$ is the cylcotron frequency) is a constant and will not be considered explicitly.

The wave functions Ψ_α^M are in general not orthogonal, and we use the method of composite-fermion diagonalization (CFD) [24] to orthogonalize them by the Gram-Schmidt procedure, evaluate the Hamiltonian matrix elements, and diagonalize it to obtain the eigenvalues and eigenvectors. All matrix elements and scalar products needed for this purpose are evaluated by the Monte Carlo method, as explained elsewhere in the literature [24]. While sufficiently accurate energy spectrum requires ~10–20 million Monte Carlo iterations, the spectral weights require ~200 million iterations for each eigenstate. These numbers do not vary significantly with N, but the computation time increases exponentially with N and ΔM, limiting our study to systems with $N = 45$ for energy, and to $N = 27$ for spectral weights; the energies were calculated for $\Delta M = 1$–8 and the spectral weights for $\Delta M = 1$–4.

Using the CF diagonalization procedure, we compute the spectra of edge excitations of the $1/3$ state, shown in Fig. 1, for several parameters in the range $N = 9$–45, $d = 0$–2.5, and $\Delta M = 0$–8. Our large system calculations confirm an earlier study [15] that for d larger than a critical separation of approximately 1.5–2.0 magnetic lengths the FQH edge undergoes a reconstruction, resulting in multiple counter propagating edge modes for which the EFTA edge exponent is not quantized, thus providing a possible explanation for the observed nonuniversal behavior. However, the important question remains whether universality occurs in the absence edge reconstruction, as is the case for sufficiently small d.

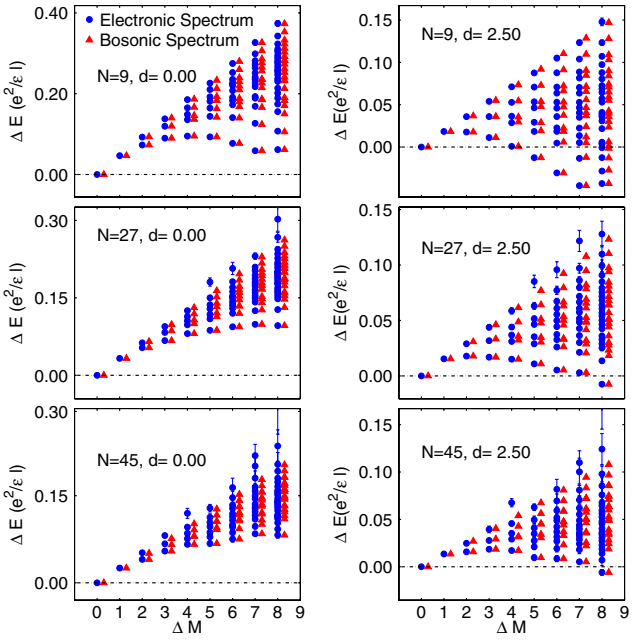

FIG. 1 (color online). Energy spectrum for the edge excitations of $\nu = 1/3$ for $N = 9$, 27, 45 particles at electron-background separations in the range $d = 0.0$ and 2.5. Blue dots indicate the energies obtained by CF diagonalization, whereas the adjacent red triangles (shifted along the x axis for clarity) show the bosonic spectra (see text for explanation). All energies are quoted in units of $e^2/\epsilon l$, and measured relative to the energy of the ground state at $\Delta M = 0$. ΔM is the angular momentum of the excited state, l is the magnetic length, and ϵ is the dielectric constant of the background semiconductor.

To address this issue we follow the pioneering work of Palacios and MacDonald [9] to test the validity of Eq. (1), upon which the notion of universality rests. Specifically, we compare certain matrix elements of the electron field operator, computed with the help of the microscopic CFD wave functions, with the predictions of the bosonized form in Eq. (1). We also consider d greater than the critical separation for completeness; here, we assume that the ground state remains at $\Delta M = 0$, which can be arranged by adding an *ad hoc* angular momentum dependent single particle energy term that strongly penalizes the edge excitations responsible for edge reconstruction, but does not change either the eigenfunctions or the energy ordering of states at a given ΔM. (This can be accomplished by adding an appropriate parabolic confinement term which adds to the total energy a term proportional to the total angular momentum.)

The so-called "spectral weights" are defined by

$$C_{\{n_l\}} = \langle\{n_l\}|\hat{\psi}^\dagger(\theta)|0\rangle/\langle 0|\hat{\psi}^\dagger(\theta)|0\rangle, \tag{4}$$

where $|\{n_l\}\rangle$ represents the bosonic state with occupation $\{n_l\}$, $|0\rangle$ is the vacuum state with zero bosons, and $\hat{\psi}^\dagger(\theta)$ is the electron creation operator at position θ along the edge circle. Here l denotes the single boson angular momentum; the total angular momentum is denoted by $\Delta M = \sum_l l n_l$ and the total energy by $\Delta E = \sum_l n_l \epsilon_l$, with ϵ_l being the energy of a single boson at angular momentum l. With the help of $\hat{\psi}^\dagger(\theta) \propto e^{i\sqrt{m}\hat{\phi}(\theta)} = \sqrt{z} e^{i\sqrt{m}\hat{\phi}_+(\theta)} e^{i\sqrt{m}\hat{\phi}_-(\theta)}$, $\hat{\phi}_+(\theta) = -\sum_{l>0}(1/\sqrt{l}) a_l^\dagger e^{il\theta} = \hat{\phi}_-^\dagger(\theta)$, it is straightforward to obtain the predictions for the spectral weights:

$$|C_{\{n_l\}}|^2 = \frac{m^{n_1+n_2+\cdots}}{n_1! n_2! \cdots 1^{n_1} 2^{n_2} \cdots}. \tag{5}$$

We note that the denominator in Eq. (4) eliminates the unknown normalization constant $\sqrt{z}$.

In order to obtain the spectral weights from our electronic spectra, it is natural to identify the vacuum state $|0\rangle$ with the ground state of interacting electrons at $\nu = 1/m$, denoted by $|\Psi_0^N\rangle$, and the field operator has the standard meaning of $\hat{\psi}^\dagger(\theta) = \sum_l \eta_l^*(\theta) c_l^\dagger \equiv \sum_l \psi_l^\dagger(\theta)$, where $c_l^\dagger$ and c_l are creation and annihilation operators for an electron in the angular momentum l state. The denominator of Eq. (4) corresponds to

$$\langle 0|\hat{\psi}^\dagger(\theta)|0\rangle = \frac{\langle\Psi_0^{N+1}|\hat{\psi}_{L_0}^\dagger(\theta)|\Psi_0^N\rangle}{\sqrt{\langle\Psi_0^{N+1}|\Psi_0^{N+1}\rangle\langle\Psi_0^N|\hat{\psi}_{L_0}(\theta)\hat{\psi}_{L_0}^\dagger(\theta)|\Psi_0^N\rangle}},$$

where $|\Psi_0^N\rangle$ is the ground state of N interacting electrons at $\nu = 1/m$, and $L_0 = mN$. The numerator is similarly defined as

$$\langle\{n_l\}|\hat{\psi}^\dagger(\theta)|0\rangle = \frac{\langle\Psi_{\{n_l\}}^{N+1}|\hat{\psi}_L^\dagger(\theta)|\Psi_0^N\rangle}{\sqrt{\langle\Psi_{\{n_l\}}^{N+1}|\Psi_{\{n_l\}}^{N+1}\rangle\langle\Psi_0^N|\hat{\psi}_L(\theta)\hat{\psi}_L^\dagger(\theta)|\Psi_0^N\rangle}}.$$

Here, we have

$$\hat{\psi}_L^\dagger|\Psi_0^N\rangle = \mathcal{N}_L \mathcal{A}[z_{N+1}^L e^{-|z_{N+1}|^2/4} \Psi_0^N(z_1, z_2 \ldots, z_N)],$$

where $\mathcal{A}$ is the antisymmetrization operator, $\mathcal{N}_L$ is the normalization constant, and $L \equiv L_0 + \Delta M$ is the angular momentum of added electron.

The wave function $\Psi_{\{n_l\}}^{N+1}$, the electronic counterpart of the bosonic state $|\{n_l\}\rangle$, represents an excited state at total angular momentum $M = \Delta M + mN(N+1)/2$, which should also be related to the total angular momentum of the N particle ground state through $M = L + mN(N-1)/2$. At each ΔM, there are in general many eigenstates. Following Ref. [15] we identify ϵ_l, the energy of a single boson with angular momentum l, with the lowest energy at $l = \Delta M$ in the calculated spectrum. Using the equations

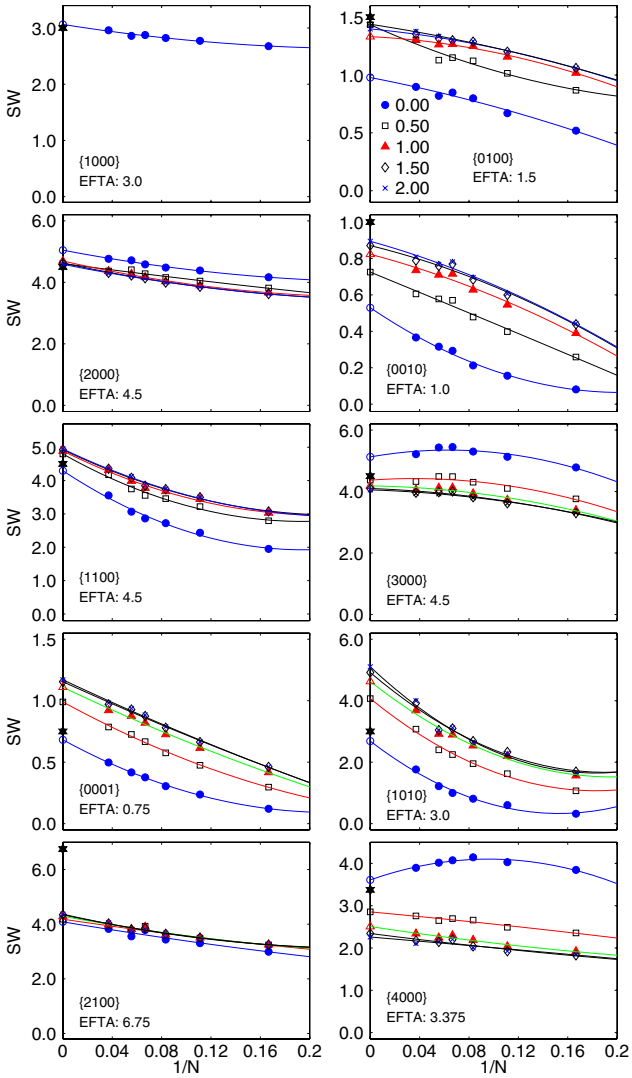

FIG. 2 (color online). N dependence of spectral weights (SW) $|C_{\{n_l\}}|^2$ for several states $\{n_l\}$, indicated on each panel, and various separations d (quoted in units of the magnetic length in the {0100} panel). The EFTA prediction from Eq. (1) is indicated by a star on the y axis, with the value also given on each panel. The points on the y axis are determined by a quadratic fit to the finite N results.

$\sum_l l n_l = \Delta M$ and $E_{\{n_l\}} = \sum_l n_l \epsilon_l$, the energies of the all bosonic states $\{n_l\}$ can now be obtained (see Fig. 1), which can then be identified with the corresponding electronic states. We note that in the lowest Λ level subspace, the numbers of electronic and bosonic states are equal at each ΔM, so a one-to-one correspondence between the two sets of states can be established from their energy ordering. For small systems (for example, $N = 9$ in Fig. 1), the CFD spectra and the bosonic spectra are very close to each other, which explicitly confirms the interpretation of the lowest branch as the single boson branch. The agreement between the electronic and bosonic spectra becomes less accurate with increasing N or ΔM, but still remains adequate for the low energy states, which will be our focus. (The higher energy states of the spectra shown in Fig. 1 mix with higher Λ level excitations of composite fermions, not considered in our model.)

Figure 2 shows the squared spectral weights for different excited states as a function of N and d. A quadratic fit extrapolates the result to the thermodynamic limit $1/N = 0$. The EFTA predictions from Eq. (1) are also shown in each panel. These plots demonstrate that the spectral weights are nonuniversal: they in general depend on d and do not extrapolate to the EFTA value. For the {1000} excitation the thermodynamic result agrees with the predicted result of 3.0 for all d, but the agreement is not meaningful because in this case our truncated space contains a single (center-of-mass) excitation, the wave function for which is independent of interactions (within our model). For many other cases, including single boson states such as {0100} and {0010}, the deviation from the EFTA value is significant.

Another model for edge confinement [15] restricts the single particle angular momentum to a maximum value of $l_{\max} = 3(N-1) + l_0$ (where l_0 is a small integer), which may be a reasonable approximation for the sharp confinement produced by cleaved edge overgrowth [7]. However, angular momentum conservation shows that in this model the actual spectral functions identically vanish for $\Delta M > l_0$, resulting in an even more substantial disagreement with the EFTA.

In summary, our studies of a realistic model of the $1/m$ FQH edge show that while the energy spectrum is fairly well described in terms of chiral bosons, the ansatz in Eq. (1) is not strictly valid even for an unreconstructed edge in the thermodynamic limit, raising the possibility that the edge physics of the $1/m$ FQH state is intrinsically nontopological. Similar considerations are likely to apply for other FQH states as well, which are believed to have more complex edges. The problem of how the actual electron field operator is related to the bosonic field operator remains unresolved, however. Following Ref. [19] one may abandon the antisymmetry requirement and try an expression of the type $\hat{\psi}(x) \sim e^{-i\sqrt{\alpha}\hat{\phi}(x)}$ with arbitrary α; we have found that no single value of α gives a satisfactory description of all spectral weights obtained from numerical diagonalization. A nonlocal relation between the electron and boson operators [13] is another possibility, not explored here.

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