Instability of Anisotropic Cosmological Solutions Supported by Vector Fields

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Models with vector fields acquiring a nonvanishing vacuum expectation value along one spatial direction have been proposed to sustain a prolonged stage of anisotropic accelerated expansion. Such models have been used for realizations of early time inflation, with a possible relation to the large scale cosmic microwave background anomalies, or of the late time dark energy. We show that, quite generally, the concrete realizations proposed so far are plagued by instabilities (either ghosts or unstable growth of the linearized perturbations) which can be ultimately related to the longitudinal vector polarization present in them. Phenomenological results based on these models are therefore unreliable.

Introduction.—Observations of the cosmic microwave background (CMB) anisotropies in the WMAP experiment [1] are in overall agreement with the inflationary paradigm. However, certain features of the full sky maps seem to be anomalous in the standard picture. These anomalies include the low power in the quadrupole moment [2], the alignment of the lowest multipoles, also known as the "axis of evil" [3], and an asymmetry in power between the northern and southern ecliptic hemispheres [4]. The statistical significance of these effects has been debated in the literature. The discussion is complicated by the difficulty of quantifying the *a posteriori* probability of the effects in the different maps produced by foreground cleaning methods and in the context of statistical anisotropy where the use of the angular power spectrum as a statistic can be misleading. However, recent studies on properly masked data have shown that an anisotropic covariance matrix fits the WMAP low-ℓ data at the 3.8σ level [5,6]. The significance of the anomalous lack of large-angle correlations, together with the alignment of power, has also grown in strength with the latest data [8], with only 1 in 4000 realizations of the concordance model in agreement with the observations.

Such violations of statistical isotropy are considered at odds with the standard phase of early inflation. However, an albeit more plausible explanation of these anomalies arising from a systematic effect or foreground signal affecting the analysis is not forthcoming. This has led to a number of attempts at reconciling some of the anomalies with the standard inflationary picture through various modifications. Specifically, the alignment of lowest multipoles could be related to an *anisotropic* inflationary era, whose duration is fine-tuned so that the signature will be observed in the modes entering the horizon today, thus modifying the lowest multipoles [9,10]. An anisotropic expansion has also been considered for late time acceleration [11]. Although the present statistics of the observed supernovae do not show any evidence for the anisotropy [12], such studies are motivated by the large increase of data that is

expected in the next few years, with surveys returning many thousands of SN Ia light curves over thousands of square degrees, and by the fact that one should keep an open mind on the nature of dark energy given our present lack of understanding.

Anisotropic, but spatially homogeneous, backgrounds were classified into equivalence classes long ago by Bianchi [13]. In the presence of a cosmological constant and matter fields satisfying strong and dominant energy conditions, all Bianchi models, with the possible exception of Bianchi type IX, undergo a rapid isotropization [14]. The full formalism for cosmological perturbations in Bianchi type I backgrounds (the simplest of these classes) has been recently developed in [9,10], with an application to the case in which the only source is a slowly rolling inflaton field, which causes the isotropization as an effective cosmological constant. This is the simplest anisotropic scenario which can be associated with a later inflationary stage. In principle, such a background solution can have striking signatures, provided that the following inflationary stage is not too long. Specifically, different $a_{\ell m}$ coefficients of the CMB multipole expansion are correlated to each other, and the two gravity wave polarizations behave in a nonstandard manner and can differ from each other [9]. In the case of axisymmetric expansion (equal expansion rate in two directions), one of the two gravity wave polarizations experiences a large growth during the anisotropic era, which may result in a large B signal in the CMB [15].

Such simple models, however, do not allow for a small and controllable departure from isotropy. Indeed, the isotropization due to the (effective) cosmological constant starts from a (Kasner-type) singularity [15] and lasts only for about one *e*-fold ($\Delta t \sim H^{-1}$, where H is the expansion rate due to the cosmological constant). As a consequence, one loses predictive power on the initial conditions for the system. A prolonged anisotropic stage can be obtained by introducing some ingredients that violate the premises of Wald's theorem [14] on the rapid isotropization of Bianchi universes. This has been realized through the addition of

quadratic curvature invariants to the gravity action [16], with the use of the Kalb-Ramond axion [17], or of vector fields [18]. In this Letter, we focus on this last possibility, as it is perhaps the simplest one (at least, from a technical point of view). The evidence for an anisotropic covariance matrix reported in [5] is based on a primordial power spectrum for the perturbations which is motivated by one of such models [19]; therefore, such constructions deserve close scrutiny.

In these models, a vector field with a nonvanishing spatial vacuum expectation value (VEV) is responsible for the anisotropy. To our knowledge, there are three different realizations of this mechanism. The oldest one dates back to 1989 [18], and the VEV of the vector field is due to a potential $V(A_\mu A^\mu)$ involving only the vector A_μ. A more recent proposal is characterized by a nonminimal coupling $R A_\mu A^\mu$ of the vector field to the curvature [20]. For a special value of this coupling, the VEV of A_μ can have a slow roll evolution. While the original proposal of this idea [22] realizes an inflationary background through several vector fields, Ref. [23] suggested a simplified version in which an inflaton scalar field is the main source of expansion, while the vector field supports the anisotropy. A completely different class of models makes use of a Lagrange multiplier to force a spacelike fixed VEV for the vector field [19,24]. These three different implementations have been realized and studied by several authors [11,27–29].

We show that these three classes of models contain instabilities which did not emerge in previous studies. We see this from the linearized study of the perturbations around the anisotropic inflationary solutions of these models. As in all slow roll inflationary backgrounds, each mode of the perturbations is initially in the small wavelength regime (the wavelength is exponentially small at early times); as the background inflates, the wavelength becomes larger than the Hubble horizon $H_a^{-1} \simeq H_b^{-1}$ and the mode enters the large wavelength regime. This transition is dubbed horizon crossing. For the model of [18], the system of perturbations contains a ghost in the small wavelength regime. For the case of the nonminimal coupling with curvature of [22,23], and the fixed-norm case of [19], the ghost appears from some interval of time close to horizon crossing [30]. The system of linearized perturbations blows up at this moment [31].

Complete computations of cosmological perturbations are rather tedious, and, for the above models, require involved algebra. We have performed them along the lines of [9]. We first write the most general system of perturbations (both of the metric and of the vector field). We then fix the freedom of general coordinate invariance, we integrate out the nondynamical modes, and finally we study the remaining system of dynamical perturbations. The divergence of the linearized perturbations is found by solving the linearized Einstein equations. Ghosts are found from studying the kinetic matrix that couples the dynamical

perturbations in their quadratic action. The complete computation is very lengthy, and it will be reported separately [33]. The fact that the instability is related to the vector field, fortunately suggests that a partial study, with only the perturbations of this field included, can shed light on the true nature of the problem, without the need to go through the technicalities of the full computation. The results of this analysis, which are summarized in the next section, show that this is indeed the case. The significance of these results is discussed in the concluding section.

Instabilities.—Assuming a spatial VEV of the vector field aligned along the x direction, the line element is

$$ds^2 = -dt^2 + a(t)^2 dx^2 + b(t)^2 [dy^2 + dz^2]. \qquad (1)$$

We introduce the two expansion rates $H_a \equiv \dot{a}/a$, $H_b \equiv \dot{b}/b$, and we define their average H and rescaled difference h through $H \equiv (H_a + 2H_b)/3$ and $h \equiv (H_b - H_a)/3$. The inflationary expansions that we consider below are characterized by constant or slowly evolving rates. For the models we are considering, $h/H = O(B^2)$, where B is the rescaled VEV of the vector field $\langle A_x \rangle \equiv M_p a B$ [18,19,23]. Therefore, B must also be slowly rolling during the slow roll regime. We consider the phenomenologically relevant case of moderate anisotropy, $B < 1$.

Before studying these models, consider a massive vector field in an isotropic background [Eq. (1) with $a = b$]

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{M^2}{2} A_\mu A^\mu \right]. \qquad (2)$$

We assume that A_μ has vanishing VEV, and we decompose its fluctuations as $A_\mu = (\alpha_0, \partial_i \alpha_L + \alpha_i^T)$. The transverse vector perturbation α_i^T, satisfying $\partial_i \alpha_i^T = 0$, contains two physical modes. These modes are well behaved, and decoupled from the α_0, α_L perturbations. We disregard them in the following. For $M^2 \neq 0$, the two perturbations α_0, α_L encode one additional degree of freedom, namely, the longitudinal vector polarization. Indeed the mode α_0 is nondynamical, since it appears without time derivatives in the action, and must be integrated out. Its equation of motion, after Fourier decomposition in the spatial directions, gives $\alpha_0 = [p^2/(p^2 + M^2)]\dot{\alpha}_L$, where $p = k/a$ is the physical momentum of the mode, k the comoving momentum, and dot denotes time differentiation. Inserting this solution back into (2) we obtain the action for the dynamical mode:

$$S_{\text{longitudinal}} = \int dt d^3k a^3 \frac{p^2 M^2}{2} \left[\frac{|\dot{\alpha}_L|^2}{p^2 + M^2} - |\alpha_L|^2 \right]. \quad (3)$$

The longitudinal vector mode exists due to the mass term, so it is not a surprise that M^2 multiplies the kinetic term. We see that this mode is a ghost for $M^2 < 0$.

Let us now turn to the models of our interest. The two models [18,23] can be studied together, using [34]

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{F^2}{4} - V(A^2) + \frac{\xi}{2} R A^2 \right]. \quad (4)$$

Expanding the potential at quadratic order in A_μ, and comparing with Eq. (2), this action leads to the mass term

$$M^2 = 2\frac{\partial V}{\partial A^2} - \xi R = 2\frac{\partial V}{\partial A^2} - 6\xi(2H^2 + h^2 + \dot{H}). \quad (5)$$

The equations of motion for the rescaled VEV B obtained from (4) is $\ddot{B} + 3H\dot{B} + \mathcal{Q}B = 0$, with

$$\mathcal{Q} \equiv 2\frac{\partial V}{\partial A^2} - 2Hh - 5h^2 - 2\dot{h} + (1 - 6\xi)(2H^2 + h^2 + \dot{H}).$$

Slow roll of B requires $\mathcal{Q} \ll H^2$ (since the $3H\dot{B}$ term provides a "friction" to the motion). This is achieved in two different ways by [18,23]. Reference [18] studied solutions with constant $H_{a,b}$ in the absence of the $A^2 R$ term, $\xi = 0$. This requires $\mathcal{Q} = 0$, or, in other terms,

$$\frac{\partial V}{\partial A^2} = -H^2 + Hh + 2h^2 = -H_a H_b < 0. \quad (6)$$

This corresponds to a negative square mass in Eq. (5). From our discussion of the model (2) we therefore immediately see that the longitudinal vector polarization is a ghost in the limit of isotropic background ($B = 0$). In [23], the choice $\xi = 1/6$ is made, so that (following the idea of [22]) the $O(H^2)$ contribution is absent from $\mathcal{Q}$. Then, slow roll is achieved in the case of small anisotropy, $B \ll 1$, and for $\partial V/\partial A^2 \ll H^2$. We then see that the square mass parameter (5) is negative in this limit, indicating that the longitudinal vector polarization is a ghost in the isotropic limit in this case too. It is not immediate to prove that the same conclusion also holds when the background is anisotropic, and when gravity (background plus fluctuation) is consistently included [35]. However, the complete computations of [33] show that this is the case for the models under consideration.

Finally, let us discuss the stability of model [19],

$$S = \int d^4 x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{F^2}{4} + \lambda(A^2 - m^2) - V_0 \right]. \quad (7)$$

In this case, the rescaled VEV B is forced to be constant, and equal to m/M_p by the Lagrange multiplier λ. One then finds a background solution with constant expansion rates satisfying $H_b = (1 + B^2)H_a$ [19]. We decompose the vector field in VEV plus fluctuations, $A_\mu = \langle A_\mu \rangle + (\alpha_0, \alpha_1, \partial_i \alpha + \alpha_i)$ where the index $i = 2, 3$ spans only the coordinates of the y-z plane, and $\partial_i \alpha_i = 0$. The equation of motion for λ, once expanded at the linearized level in the perturbations, reads $B\alpha_1 = 0$. This equation identically vanishes if the background is isotropic ($B = 0$), while it eliminates one of the vector perturbations for $B \neq 0$. Therefore, contrary to the previous study, we cannot consider the isotropic limit in this case.

We are interested in the quadratic action for the perturbations. It is easy to see that the perturbations α_i decouple. We are then left with a quadratic action containing α_0 and α. α_0 is nondynamical in this case too, and can be integrated out, leading to the quadratic action

$$\delta^2 S = \frac{1}{2} \int dt d^3 k \, a b^2 p_T^2 (p_L^2 - 2H_a H_b)$$

$$\times \left[\frac{|\dot{\alpha}|^2}{p_L^2 + p_T^2 - 2H_a H_b} - |\alpha|^2 \right], \quad (8)$$

where p_L and p_T are the components of the physical momentum along the x direction and in the perpendicular plane, respectively. We see the presence of a ghost close to horizon crossing (H_a and H_b are constant, while the physical momentum exponentially decreases, $p_L \propto a^{-1}$, $p_T \propto b^{-1}$). Moreover, the kinetic term for α vanishes when $p_L^2 = 2H_a H_b$. As a consequence, the solution diverges at that moment (α diverges logarithmically, while α_0 linearly [33]). This exact problem appears also in the full computation [33] (also for the models [22,23]): all linearized modes diverge close to horizon crossing.

Discussion.—We start by stressing the limits of our computation. The above results have been obtained for standard kinetic terms for the vector field. Since the U(1) symmetry is already broken by the potential term, there is no special reason for this choice. Indeed, works on Lorentz violating vector fields study generalized kinetic terms of the type $\mathcal{L} \supset -\beta_1 \nabla^\mu A^\nu \nabla_\mu A_\nu - \beta_2 (\nabla_\mu A^\mu)^2 - \beta_3 \nabla^\mu A^\nu \nabla_\nu A_\mu$. The standard kinetic term corresponds to $\beta_1 = -\beta_3 = 1/2$, $\beta_2 = 0$. Reference [18] only discusses the case of a standard kinetic term. We have studied perturbations in this model for arbitrary β_i coefficients. We find that the ghost is absent for $\beta_1 + \beta_2 + \beta_3 \neq 0$. However, in this case one of the perturbations is a tachyon in the early time (small wavelength) regime [33]. Reference [28] showed that the model [19] is unstable in the case of $\beta_1 + \beta_2 + \beta_3 \neq 0$ [36]. For the nonminimal coupling to the curvature, all the studies done so far are limited to standard kinetic terms, and therefore our computations have also been restricted to this case. A second limitation is related to the fact that we have performed specific computations only for the three models [18,19,23]. However, these models are "prototypes" for the three different ways of obtaining the nonvanishing spatial VEV explicitly realized in the literature. We expect that the issues raised here also arise in all models for which the anisotropy is obtained in one of these three ways.

Because of these limitations, we do not claim to rule out all possible models of anisotropic expansion through vector fields. Nonetheless, the issues we have found are specific to models with vector fields, and should be checked in the stability analysis of all these models. Already the presence of a ghost indicates that the theory is valid only below some energy Λ (since the vacuum decays in ghost-nonghost quanta with a rate that diverges in the UV). In the models we have studied, there is the additional problem that the linearized solutions diverge close to horizon crossing. The computation of cosmological perturbations, and the associated phenomenology, requires that the subhorizon regime is under control (this is the reason why it is commonly argued that inflation probes physics at the en-

ergy scale H). While it is possible that the full nonlinear solutions are not diverging, and that the models could admit a UV completion without ghosts, the phenomenological predictions that have been derived from the linearized computations for these models are invalid. Although our concrete studies have produced a negative outcome, we hope that some specific constructions will eventually overcome the issues found here.

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