Cooling Carbon Nanotubes to the Phononic Ground State with a Constant Electron Current

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We present a quantum theory of cooling of a mechanical resonator using back action with a constant electron current. The resonator device is based on a doubly clamped nanotube, which mechanically vibrates and acts as a double quantum dot for electron transport. Mechanical vibrations and electrons are coupled electrostatically using an external gate. The fundamental eigenmode is cooled by absorbing phonons when electrons tunnel through the double quantum dot. We identify the regimes in which ground-state cooling can be achieved for realistic experimental parameters.

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Cooling mechanical resonators has recently attracted considerable interest, as it allows ultrasensitive detection of mass [1–4], of mechanical displacements [5], and of spin [6]. An appealing prospect is to cool the mechanical resonator to its phononic ground state. This achievement would open the possibility to create and manipulate nonclassical states at the macroscopic scale and to study the transition from the classical to the quantum regime [7–9].

The lowest phononic occupation number achieved so far has been experimentally realized by cooling down the resonator in a dilution fridge [10]. Another promising approach is to employ back action, which consists of coupling mechanical oscillations to visible or microwave photons [11–19]. Recently, it has been theoretically proposed [20–22] and experimentally demonstrated [10] that back action cooling can be achieved by coupling mechanical resonators to the constant electron current through electronic nanodevices, such as normal-metal and superconducting single-electron transistors. This approach is appealing because it is easy to implement in a dilution fridge as compared to techniques based on photons. Within this approach, however, modest occupations of the phononic ground state have been predicted [19–22]. In particular, using an analogy with laser cooling of atoms [23], back action cooling by constant electron current in these systems is essentially analogous to Doppler cooling [21].

In this Letter, we theoretically demonstrate ground-state cooling of a mechanical nanotube resonator using constant electron current. Specifically, the nanotube is employed both as the mechanical resonator and the electronic device through which the current flows. In addition, we consider the device layout in which the nanotube acts as a double quantum dot (DQD). This setup allows us to access an analogous regime of sideband cooling of the oscillator [23]. Calculations are carried out by including the coupling of the resonator to the thermal noise of the electrodes and the effect of electronic dephasing inside the DQD. For realistic device parameters the temperature is lowered by a factor of about 100. Moreover, we identify the regime in which the oscillator ground state can reach more than 90% occupation.

The device layout is sketched in Fig. 1(a). The DQD system is obtained by locally depleting a semiconducting nanotube with gate T [24–27]. The dot on the right is suspended, so it can mechanically oscillate. The nanotube is electrically contacted to two electrodes and the electrochemical potential of the two dots is controlled with the gates L and R. The DQD is voltage biased in order to have three relevant quantum states of single-electron transport, $|0\rangle$, $|L\rangle$, $|R\rangle$; see Fig. 1(b). Here, $|L\rangle$ and $|R\rangle$ correspond to the excess electron localized on the left and right dot with energy ϵ_L and ϵ_R, respectively. State $|0\rangle$, at zero energy, corresponds to when both states are unoccupied [28–30]. We denote by $\Delta = (\epsilon_R - \epsilon_L)/\hbar$ the frequency difference between $|L\rangle$ and $|R\rangle$. The coherent dynamics of the DQD is given by the Hamiltonian

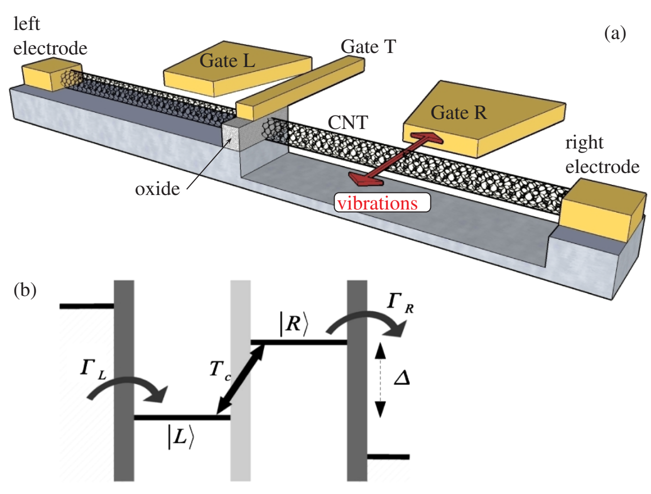

FIG. 1 (color online). (a) Sketch of the double quantum dot, carbon-nanotube (CNT) device. The dot on the right is suspended, and mechanically oscillates. (b) The single-electron levels $|L\rangle$ and $|R\rangle$, at (tunable) frequency difference Δ, are coherently coupled with tunneling rate T_c.

$$H_{\rm DQD} = \hbar[\epsilon_R|R\rangle\langle R| + \epsilon_L|L\rangle\langle L| - T_c(|L\rangle\langle R| + |R\rangle\langle L|)], \tag{1}$$

where T_c is the tunneling rate between the dots.

The nanotube mechanical motion, considered here, is the fundamental phononic mode, which is the flexural eigenmode [31,32]. We assume a high quality factor Q so the flexural mode is to a good extent uncoupled to the other phononic modes. We denote by ω the oscillator frequency and by $a^\dagger$ and a the creation and annihilation operators of a phonon at energy $\hbar\omega$. The interaction between the mechanical motion and the conducting electrons is obtained through the capacitive coupling between the nanotube and gate R, which depends on their mutual distance. For sufficiently small vibration displacements, the electron-phonon interaction is described by the Hamiltonian $H_{e\text{-ph}} = V(a^\dagger + a)$ where

$$V = \hbar\alpha|R\rangle\langle R|, \tag{2}$$

and α is a coupling constant which scales the mechanical effect [21,22]. Cooling is achieved by gating ϵ_L below ϵ_R, so that a phonon of the nanomechanical oscillator is absorbed by electrons tunneling from left to right.

We describe the cooling dynamics by using the master equation for the density matrix ρ of electron and mechanical degrees of freedom,

$$\dot\rho = -i\omega[a^\dagger a, \rho] - i[H_{e\text{-ph}}, \rho]/\hbar + (\mathcal{L}_{\rm DQD} + \mathcal{K})\rho, \tag{3}$$

where

$$\mathcal{L}_{\rm DQD}\rho = -i[H_{\rm DQD}, \rho] + \Gamma_L/2(2s_L^\dagger\rho s_L - \rho s_L s_L^\dagger - s_L s_L^\dagger\rho) + \Gamma_R/2(2s_R\rho s_R^\dagger - \rho s_R^\dagger s_R - s_R^\dagger s_R\rho) + \mathcal{L}_d\rho \tag{4}$$

includes incoherent electron (tunnel) pumping at rate Γ_L from the left electrode into state $|L\rangle$, and (tunnel) extraction at rate Γ_R from state $|R\rangle$ into the right electrode, with $s_L = |0\rangle\langle L|$ and $s_R = |0\rangle\langle R|$ [33]. The term $\mathcal{K}\rho = \gamma_-/2(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) + \gamma_+/2(2a^\dagger\rho a - aa^\dagger\rho - \rho aa^\dagger)$ describes the thermalization of the flexural mode with the environment (other phononic modes of the nanotube and of the electrodes) at the cryogenic temperature T, where $\gamma_- = (\bar n_p + 1)\gamma_p$, $\gamma_+ = \bar n_p\gamma_p$, with $\gamma_p = \omega/Q$ and $\bar n_p = [\exp(\hbar\omega/k_BT) - 1]^{-1}$ [19,34]. Finally, $\mathcal{L}_d\rho$ is a process inducing incoherent hopping between the two dots, which we will discuss later on.

In the limit $\gamma_p \ll \alpha \ll \omega$, a rate equation for the population p_n of the phononic energy level with n excitations is derived, which reads $\dot p_n = (n+1)A_- p_{n+1} - [(n+1)A_+ + nA_-]p_n + nA_+ p_{n-1}$. Here, the terms A_+ and A_- give the rate with which the oscillator is heated and cooled, respectively, by one phonon. The rate equation describes the dynamics of a damped quantum oscillator when $A_- > A_+$, reaching the mean phonon number $\bar n_{\rm St} = A_+/\gamma_{\rm tot}$ at the cooling rate $\gamma_{\rm tot} = A_- - A_+$. In this regime, the equation describes thermalization with an effective bath at temperature $T_{\rm osc}$, which can be well below the cryogenic temperature.

Terms $A_\pm$ are the sum of two contributions, the thermalization rate via the environment and the coupling to the electronic current. They read [35,36]

$$A_\pm = \frac{\gamma_\pm}{2} + \frac{2}{\hbar^2}\,\mathrm{Re}[\mathrm{Tr}\{V(\mathcal{L}_{\rm DQD} \mp i\omega)^{-1}V\rho_{\rm St}\}], \tag{5}$$

where the second term is the autocorrelation function of the operator V, Eq. (2), taken over the density matrix $\rho_{\rm St}$, which satisfies the equation $\mathcal{L}_{\rm DQD}\rho_{\rm St} = 0$. The corresponding steady state occupation number reads

$$\bar n_{\rm St} = \frac{\gamma_p\bar n_p + \gamma_0\bar n_0}{\gamma_p + \gamma_0}, \tag{6}$$

where γ_0 and $\bar n_0$ are the cooling rate and the steady state phonon population solely due to electron-phonon interaction ($\gamma_{\rm tot} = \gamma_0 + \gamma_p$). Equation (6) highlights the competition between the active cooling process due to electron transport and the thermalization with the surrounding environment, showing that cooling is more effective when $\gamma_0 \gg \gamma_p$.

We emphasize that the derived rate equation describes cooling of the oscillator deep in the quantum regime. Moreover, note that, using relation $k_BT_{\rm osc} = \hbar\omega(\ln\frac{\bar n_{\rm St}+1}{\bar n_{\rm St}})^{-1}$ and taking the semiclassical limit of Eq. (6), we recover the formal expression for the temperature obtained when the oscillator is cooled by coupling to a single-electron transistor [21,22]. However, while in the single-electron transistor setup back action cooling with constant electron current is essentially analogous of Doppler cooling [21], we access in our case the sideband cooling regime and may hence reach unit occupation of the ground state.

Figure 2 displays the contour plots of $\bar n_{\rm St}$ and cooling rate $\gamma_{\rm tot}$ as a function of the tunneling rate T_c and of the

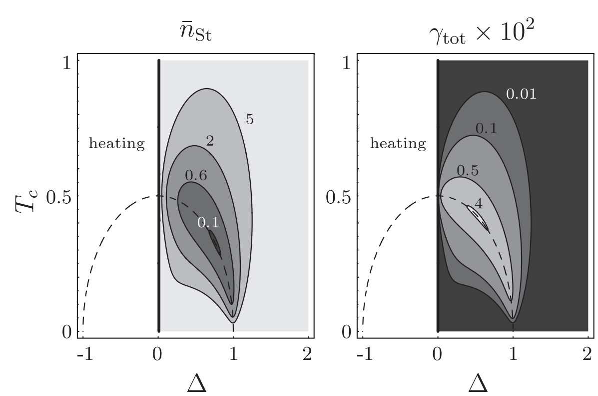

FIG. 2. Contour plot for average phonon number at steady state $\bar n_{\rm St}$ and cooling rate $\gamma_{\rm tot}$ as a function of Δ and T_c. Rates and frequencies are in units of ω. The other parameters are $\Gamma_L = 10\omega$, $\Gamma_R = 0.1\omega$, $\alpha = 0.1\omega$, $\gamma_p = 10^{-4}\omega$, and $\bar n_p = 24$. The dashed lines indicate the curve $\Delta^2 + 4T_c^2 = \omega^2$. In the "heating" region one finds $\bar n_{\rm St} > \bar n_p$.

frequency difference Δ, for $\Gamma_R \ll \omega$ and $\mathcal{L}_d = 0$. For the chosen parameters, the lowest occupation is $\bar{n}_{\rm St} \sim 0.1$ and the corresponding cooling rate is $\gamma_{\rm tot} \sim 0.03\omega$. To provide some typical numbers, for $T_c = 2\pi \times 30$ MHz, $\Delta = 2\pi \times 80$ MHz, $\Gamma_L = 2\pi \times 1$ GHz, $\Gamma_R = 2\pi \times 10$ MHz, and $\alpha = 2\pi \times 10$ MHz [37], we find that the fundamental mode of a nanotube resonator at $\omega = 2\pi \times 100$ MHz and quality factor $Q = 10^4$ can be cooled from $T = 120$ mK ($\bar{n}_p = 24$) to $T_{\rm osc} = 2$ mK ($\bar{n}_{\rm St} = 0.1$) with cooling rate $\gamma_{\rm tot} \sim 2\pi \times 3$ MHz. Note that if the initial cryostat temperature ($\bar{n}_p$) is lower, the value of $\bar{n}_{\rm St}$ will be smaller; see Eq. (6).

We remark that cooling is most efficient for tunneling rates $T_c \sim \omega$ and for $\Delta \sim \sqrt{\omega^2 - 4T_c^2}$ (see dashed line in Fig. 2). Insight into this result can be gained by analyzing the transport processes in the basis of the bonding and antibonding states of the DQD, $|-\rangle = \cos\theta|L\rangle + \sin\theta|R\rangle$, $|+\rangle = -\sin\theta|L\rangle + \cos\theta|R\rangle$, respectively, with $\tan\theta = 2T_c/(\Delta + \epsilon)$ and with $\epsilon = \sqrt{\Delta^2 + 4T_c^2}$ the distance in frequency between the bonding and antibonding state. In this basis there are two relevant physical processes, which lead to a change by one phonon due to electron transport through the DQD. They consist of the sequential occupation of the states $|0, n\rangle \to |\pm, n\rangle \to |\mp, n \mp 1\rangle \to |0, n \mp 1\rangle$, as illustrated in Fig. 3. Both processes are resonant when the condition $\epsilon = \omega$ is satisfied, which correspond to the dashed curve in Fig. 2. The resonator is hence cooled provided that the rate of the cooling process in Fig. 3(a) is faster than the heating process in Fig. 3(b), which is satisfied for $\Delta > 0$.

A simple analytical result for the cooling rate γ_0 due to electron-phonon coupling can be found in the limit of large Γ_L and reads

$$\gamma_0 \simeq \frac{\alpha^2}{\omega^2}\left(\frac{4T_c^2\Gamma_R}{\Delta^2 + 2T_c^2 + \frac{\Gamma_R^2}{4}}\right)\frac{\omega^3\Delta(2T_c^2 + \Gamma_R^2 + \omega^2)}{\omega^2(\Delta^2 + 4T_c^2 + \frac{5}{4}\Gamma_R^2 - \omega^2)^2 + \Gamma_R^2(\Delta^2 + 2T_c^2 + \frac{\Gamma_R^2}{4} - 2\omega^2)^2} + O\left(\frac{1}{\Gamma_L}\right) \tag{7}$$

showing that, for $\omega \gg \Gamma_R$, it is maximum when $\epsilon \simeq \omega$. Note that $\bar{n}_0$, in Eq. (6), reads

$$\bar{n}_0 \simeq \frac{\Gamma_R^2 + 4(\Delta - \omega)^2}{16\Delta\omega} + O\left(\frac{1}{\Gamma_L}\right). \tag{8}$$

This equation has the same form found in sideband cooling of trapped ions under specific conditions [35,36]. The same equation (for other physical parameters) was derived in [16,17,19,34], where cooling of nanomechanical resonators with photons was mapped to sideband cooling of trapped ions (the detailed theory can be found in [18]). In our case, small values $\bar{n}_0 \ll 1$ are achieved for $\omega \gg \Gamma_R$. The minimum value $\bar{n}_0 = \Gamma_R^2/16\omega^2$ is reached for $\Delta = \omega$, independent of the value of T_c. However, γ_0 is not maximum for $\Delta = \omega$ as for other schemes [16,17,19,34] and optimal cooling is found as a compromise between maximizing γ_0 and minimizing $\bar{n}_0$; see Eq. (6) and Fig. 2. In particular, for $\gamma_0 \gg \gamma_p$, then $\bar{n}_{\rm St} \simeq \bar{n}_0 + (\bar{n}_p - \bar{n}_0)\gamma_p/\gamma_0$ and the resonator can be cooled to the ground state.

We now discuss how the above cooling efficiency is affected by electronic dephasing inside the DQD (i.e., loss of coherence other than incoherent tunneling from and to the electrodes). We model this mechanism setting $\mathcal{L}_d\rho = \Gamma_d(\sigma_z\rho\sigma_z - \rho)$ in Eq. (4), where $\sigma_z = |R\rangle\langle R| - |L\rangle\langle L|$ and Γ_d is the dephasing rate [38]. We find that its effect on the cooling dynamics is to lower the efficiency by broadening the mechanical resonances. The resonator can be cooled when $\Gamma_d < \omega$ and smaller values of Γ_d give higher cooling efficiencies. Figure 4 displays $\bar{n}_{\rm St}$ and $\gamma_{\rm tot}$ for $\Gamma_d = 0.05\omega$, while the other parameters are the same as in Fig. 2. For these parameters, the resonator is cooled from $\bar{n}_p = 24$ to $\bar{n}_{\rm St} \sim 0.5$ (corresponding to $T_{\rm osc} \sim 4$ mK for $\omega = 2\pi \times 100$ MHz).

We now summarize the relevant conditions for ground-state cooling. The cooling mechanism is based on the resonant absorption of a phonon by electron transport through the DQD. We have shown that the resonance

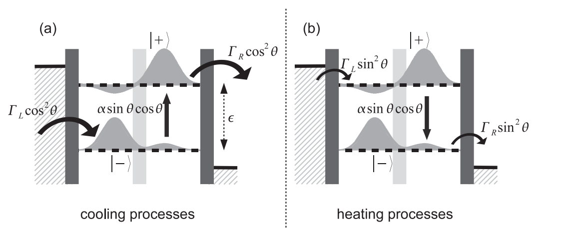

FIG. 3. Dominant cooling (a) and heating (b) processes in the parameter region of optimal cooling in Fig. 2. States $|-\rangle$ and $|+\rangle$ denote the bonding and antibonding states at frequency difference $\epsilon = \sqrt{\Delta^2 + 4T_c^2}$. The rates are reported in the new basis, with $\tan\theta = 2T_c/(\Delta + \epsilon)$. For $\Delta > 0$, then $\tan^2\theta < 1$ and the processes in (a) are faster, so that the resonator is cooled.

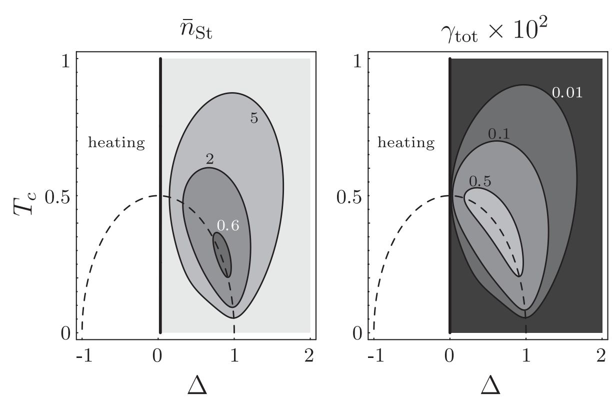

FIG. 4. Same as in Fig. 2 with $\Gamma_d = 0.05\omega$.

condition is found by tuning Δ and T_c such that $\omega = \sqrt{\Delta^2 + 4T_c^2}$, provided that $\Delta > 0$. Resonant enhancement of phonon absorption can be achieved when the lifetimes of the electronic states inside the DQD are long as compared to the period of the mechanical oscillations, i.e., Γ_R, $\Gamma_d \ll \omega$ [39]. We remark that this condition cannot be achieved for an individual quantum dot, where the resonance is thermally broadened by the Fermi-Dirac distribution of the electrons in the electrodes.

Most parameters of the proposed setup can be tuned. The frequency ω depends on the length of the nanotube section that is suspended, while the frequency difference Δ, the tunneling rates T_c, Γ_L, and Γ_R, can be controlled by the external gates [40]. However, little is known on electron dephasing in nanotubes: the rate Γ_d remains to be measured and it is not clear whether Γ_d can be tailored so to fulfill $\Gamma_d < \omega$. Note that the proposed cooling scheme can be applied to other device layouts. For instance, our calculations can be employed for mechanical resonators electrostatically coupled to fixed DQDs, microfabricated with metal or semiconducting material [10].

The effective temperature achieved by cooling could be probed by measuring the amplitude of the oscillation fluctuations as a function of the oscillator frequency. One possible strategy is to monitor the microwave signal reflected off an external LC tank that is coupled to the DQD [10]. The signal might be improved by switching the barriers along the nanotube to higher transmissions once the phononic mode has been cooled down. There would be a short time window ($<1/\gamma_p$) for measurements before the effective temperature increases.

In conclusion, we have shown that a high-Q flexural mode of a carbon nanotube can be cooled to the ground state with constant electron currents. The possibility to manipulate such macroscopic objects using matter waves open novel avenues for quantum manipulation of this kind of mesoscopic systems. An interesting outlook is to use these concepts for realizing quantum reservoir engineering, in the spirit of [9], in order to achieve other kinds of nonclassical states.

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