Local Capacitor Model for Plasmonic Electric Field Enhancement

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We present a local capacitor model that enables a simple yet quantitatively accurate description of lightning rod effect in nanoplasmonics. A notion of λ-zone capacitance is proposed and applied to predict the strongly induced electric field by a light source near nanoscale metal edges such as metal tip or metal gap. The enhancement factor, calculated from the local capacitor model, shows excellent agreement with more rigorous results. The λ-zone capacitor allows a blockwise treatment of nano-optical devices and constitutes a basic element of optical nanocircuits.

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The lightning rod induces strong electric field near the tip as charge tends to accumulate at a sharper edge. This lightning rod effect has received great interest recently in the study of nanoscale structures of metal with applications to diverse areas including near field spectroscopy [1,2], surface plasmons [3–5], optical antenna [6,7], surface enhanced Raman spectroscopy [8], biomedical science, and other applications [9]. When excited by light, the enhanced electric field near metal edges is generally accompanied by a surface charge oscillation and also called local plasmon. The plasmonic enhancement of electric field increases if metal edges form a gap as can be seen in a bow-tie antenna [7] or in a nanoparticle dimer [10]. This is expected since the gap structure is supposed to increase the "capacitance" of the system and subsequently the induced electric field. The meaning of capacitance in a plasmonic configuration is dubious, however, as the conventional concept of static capacitance fails to apply. Thus the quantitative account of plasmonic local field enhancement based on a capacitor concept so far has been lacking. If possible, the capability of controlling enhancement through capacitance would play a key role in nanoscale optical devices and also in the development of recently emerging optical nanocircuit concepts [11].

In this Letter, we introduce the concept of "λ-zone" capacitance for plasmonic systems and apply it to predict the strong enhancement of electric fields near subwavelength metal structures. The λ-zone capacitance, defined as a static local capacitance restricted to a wavelength-confined region, is evaluated explicitly for two typical cases of subwavelength metal structures: (1) metal slit with a narrow gap and (2) a metal tip near a metal surface or, equivalently, two metal tips forming a gap. We show that the plasmonic enhancement of electric field mainly comes from the local charging of the λ-zone capacitor where charging is done by the light-induced surface current flowing in to or out of the λ zone. The enhancement factor, calculated for two cases varying parameters such as gap size or metal thickness, shows an excellent agreement with rigorous finite difference time domain (FDTD) calculations. They are also consistent with the experimental result on the field enhancement in a metal slit [12]. More importantly, the λ-zone capacitor approach allows a blockwise treatment of nano-optical devices and constitutes a basic element of optical nanocircuits. The validity of the local capacitor model for metal structures at the skin-depth scale is also discussed.

When an electromagnetic wave impinges upon a metal, current is induced due to the high conductivity of metal. The current density resides mostly near the surface within the skin-depth region and can be replaced by an effective surface current $\vec{K}_{\mathrm{eff}} = \vec{n} \times \vec{H}_{\parallel}$ [13]. Here, $\vec{n}$ is the unit vector normal to the surface and $\vec{H}_{\parallel}$ is the tangential magnetic field just outside the surface. For simplicity, we assume metal to be a perfect conductor and address the effect of finite conductivity and skin-depth later. For a plane wave normally incident on a thick metal surface with magnetic field strength $\vec{H}_0$, we have $\vec{K}_{\mathrm{eff}} = \vec{n} \times 2\vec{H}_0$. This surface current, when blocked by a tip end or a discontinuous gap as shown in Fig. 1, accumulates charge at the edge through the charge conservation law: $\vec{\nabla} \cdot \vec{K}_{\mathrm{eff}} + \partial_t \sigma_S = 0$. Since the light-induced surface current flowing into the edge region is an alternating current, surface current responsible for charging the metal edge region is in fact restricted within a region of length scale the wavelength λ. This is our main observation leading to the definition of local capacitance. Henceforth we call the local region of surface contained within a distance λ from the edge "the λ zone," as illustrated in Fig. 1(a). More specifically, we measure the distance from the edge in the direction of transverse electric field of incoming light and consider a volume that encloses the metal edge within the distance λ [shaded cylinder in Fig. 1(a)]. The λ zone is then the metal surface surrounding the edge that stays inside the volume. The induced charge Q_{ind} by the surface current $\vec{K}_{\mathrm{eff}}$ flowing into the λ zone is

$$Q_{\mathrm{ind}} = \int_Z \sigma_S dA = \frac{1}{iw} \int_{\partial Z} \vec{K}_{\mathrm{eff}} \cdot \hat{n} dl, \tag{1}$$

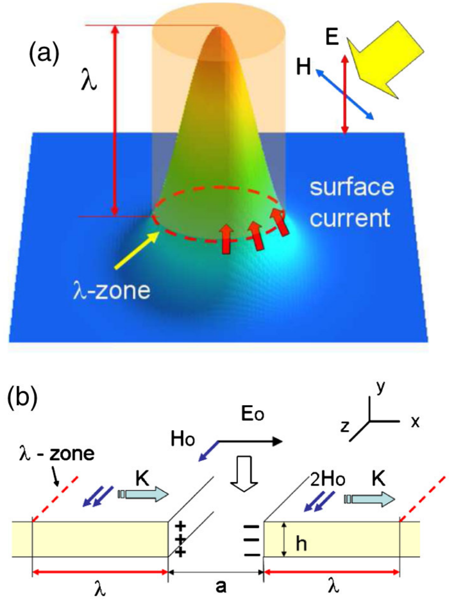

FIG. 1 (color online). (a) Schematics of the λ zone. λ zone is the surface of a tip inside the shaded cylinder of length λ. Dashed circle is the λ-zone boundary, and arrows across the boundary depict the light-induced surface current that charges the λ-zone capacitor. (b) λ zone and surface current of a metal slit for a normally incident light.

where Z and ∂Z represent the λ zone and its boundary, respectively, and $\hat{n}$ is a normal vector to the boundary. Here we assumed the harmonic time dependence e^{-iwt} for surface current and charge density σ.

In order to help in the understanding of field enhancement, we consider a metal slit of thickness h and gap width a given in Fig. 1(b). The λ zone is the surface region surrounding the edge bounded by dashed lines. If we normalize the incident light by $\vec{E}_0 = \hat{x}$ and $\vec{H}_0 = \sqrt{\epsilon_0/\mu_0}\hat{z}$, we have the induced charge per unit length $Q_{\rm ind}$,

$$Q_{\rm ind} = \frac{2}{iw}\sqrt{\frac{\epsilon_0}{\mu_0}} = \frac{\epsilon_0 \lambda}{i\pi}. \qquad (2)$$

When the gap is narrow compared to the thickness h and wavelength ($a \ll h < \lambda$), we may expect that induced charge resides mostly on the gap surface with surface charge density $\sigma_G = Q_{\rm ind}/h$ thereby forming a parallel plate capacitor. The resulting electric field, $E_x = \sigma_G/\epsilon_0 = \lambda/i\pi h$, in fact agrees with a rigorous diffraction theory calculation of electric field inside the gap using the single mode approximation and the small gap limit (not shown). This suggests that the surface current-charge scheme can be a convenient and quantitatively meaningful tool in analyzing the lightning rod effect. To predict the induced electric field for a general shape of metal, e.g., slit with any h and a at the subwavelength scale, we need to know the relation between electric field and charge. In the electrostatic limit, the overall capacitance determined by shape provides such a relation. In the case of light-induced current and charge, we show that the relation between induced charge and electric field can be practically determined by a "local capacitance" of the λ zone.

The λ-zone capacitance is defined as the ratio between the charge enclosed within the λ zone and the potential difference between gap in the electrostatic configuration. To be more specific, we first calculate the λ-zone capacitance of the exemplary slit case. The electrostatic potential around the metal slit can be found via the conformal mapping transformation [14] as given in Fig. 2,

$$z(t) = \frac{aE(t,k)}{2E(k)}, \qquad t = \sin w, \qquad z = x + iy, \qquad w = u + iv, \qquad (3)$$

where a is a constant, $E(t,k)$ is an elliptic integral of the second kind, and $E(k) = E(1,k)$. Parameter k is determined from the condition

$$\frac{a}{2} + \frac{ih}{2} = \frac{aE(1/k,k)}{2E(k)} = \frac{a}{2} + \frac{ia[E(\sqrt{1-k^2}) - K(\sqrt{1-k^2})]}{2E(k)}, \qquad (4)$$

where $K(k)$ is the complete elliptic integral of the first kind. Then, the electrostatic potential is given by $\phi = V_0 u/\pi$ with V_0 the potential difference between gap. The total charge per unit length Q_λ of the λ zone becomes

$$Q_\lambda = \epsilon_0 \int \vec{E} \cdot d\vec{\sigma} = -\epsilon_0 \int \vec{\nabla}\left(\frac{V_0}{\pi}u\right) \cdot d\vec{\sigma} = \frac{2\epsilon_0 V_0 v_\lambda}{\pi}, \qquad (5)$$

where we have used the fact

$$\vec{\nabla} u = \hat{x}\frac{\partial u}{\partial x} + \hat{y}\frac{\partial u}{\partial y} = -\hat{x}\frac{\partial v}{\partial y} + \hat{y}\frac{\partial v}{\partial x}, \qquad d\vec{\sigma} = -\hat{z} \times d\vec{s}. \qquad (6)$$

$d\vec{s}$ is in the direction of increasing v, and the parameter v_λ determines the λ zone through

$$\frac{a}{2} + \lambda + \frac{ih}{2} = \frac{aE(\sin(\pi/2 + iv_\lambda), k)}{2E(k)}. \qquad (7)$$

Finally, we have the λ-zone capacitance per unit length:

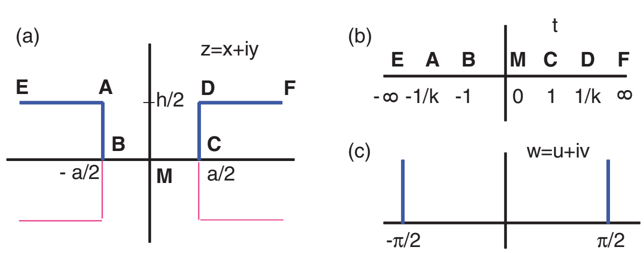

FIG. 2 (color online). Conformal mapping the exterior of a metal slit onto the t and w plane with $z = aE(t,k)/2E(k)$ and $t = \sin w$.

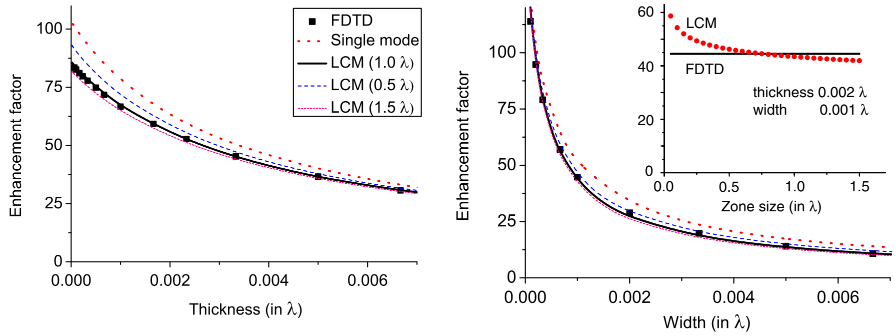

FIG. 3 (color online). Enhancement of electric field (a) with varying thickness keeping the gap size 200 nm and (b) with varying gap size keeping thickness 600 nm. Local capacitor model (LCM) with different sizes for the λ zone are compared with the FDTD result. Inset shows the zone size dependence with thickness 600 nm and width 300 nm. Since the system is scale invariant, length unit is given in λ.

$C_\lambda = 2\epsilon_0 v_\lambda/\pi$. The λ-zone capacitance in turn determines the electric field whenever the charge $Q_{\rm ind}$ is known.

In principle, there could be surface currents and charges induced by diffracted waves. An explicit diffraction theory calculation shows that diffracted waves in the front (reflection) and back (transmission) sides of a metal slit generate equal but opposite surface currents thereby leaving no net induced charge in the λ zone. Thus we neglect the contribution from diffracted waves and find the induced electric field,

$$E_{\rm ind} = \frac{Q_{\rm ind}}{aC_\lambda} = \frac{\lambda}{2iav_\lambda}. \qquad (8)$$

The total enhanced electric field appears as a superposition of incident wave and the induced field. For a large induced field, incident wave is relatively small and may be neglected. Figure 3 shows the enhancement factor (induced versus incident electric fields) with varying the thickness or the gap size. The local capacitor model is compared with a rigorous FDTD calculation and a diffraction theory using the single mode approximation. It is remarkable that the simple local capacitor model shows an excellent agreement with the rigorous FDTD result that measures electric field amplitude averaged over the midgap region. Moreover, the model does not depend significantly on the size of the λ zone [see the inset of Fig. 3(b)]. This demonstrates the validity of the λ-zone capacitor model.

Secondly, we consider a metal tip near a metal surface. For a perfect conductor, this is equivalent to the image charge system made of two tips: a metal tip and its image charge tip. The apertureless tapered tip used in near field scanning optical microscopy has the shape of a cone with rounded apex. To model the tip, we adopt the prolate spheroidal coordinates ($x = a\sinh u \sin v \cos\phi$, $y = a\sinh u \sin v \sin\phi$, $z = a\cosh u \cos v$), and specify the shape of a tip by $v = v_0$ as in Fig. 4. The electrostatic potential $\psi(v)$ with the potential difference $V_0[= \psi(v_0) - \psi(\pi + v_0)]$ between tips is

$$\psi = C_1 \ln\frac{1+\cos v}{1-\cos v}, \qquad C_1 = \frac{V_0}{2}\left[\ln\frac{1+\cos v_0}{1-\cos v_0}\right]^{-1}. \qquad (9)$$

The λ zone of a tip is illustrated in Fig. 1(a). The static charge within the λ zone is

$$\begin{aligned} Q_\lambda &= \epsilon_0 \int (-\vec{\nabla}\psi)\cdot d\vec{\sigma} \\ &= -\epsilon_0 \int_0^{2\pi} d\phi \int_0^{u_Z} dua \sinh u \sin v \frac{\partial\psi}{\partial v}\bigg|_{v=v_0} \\ &= 4\pi\epsilon_0 C_1 a(\cosh u_\lambda - 1) = \frac{4\pi\epsilon_0 C_1 \lambda}{\cos v_0}. \qquad (10) \end{aligned}$$

Here, u_λ specifies the λ zone through $a\cos v_0 + \lambda = a\cosh u_\lambda \cos v_0$ and the λ-zone boundary has a radius $\rho_0 = a\sinh u_\lambda \sin v_0$. Then the λ-zone capacitance is

$$C_\lambda = \frac{Q_\lambda}{\Delta\psi} = \frac{2\pi\epsilon_0\lambda}{\cos v_0}\left[\ln\frac{1+\cos v_0}{1-\cos v_0}\right]^{-1}. \qquad (11)$$

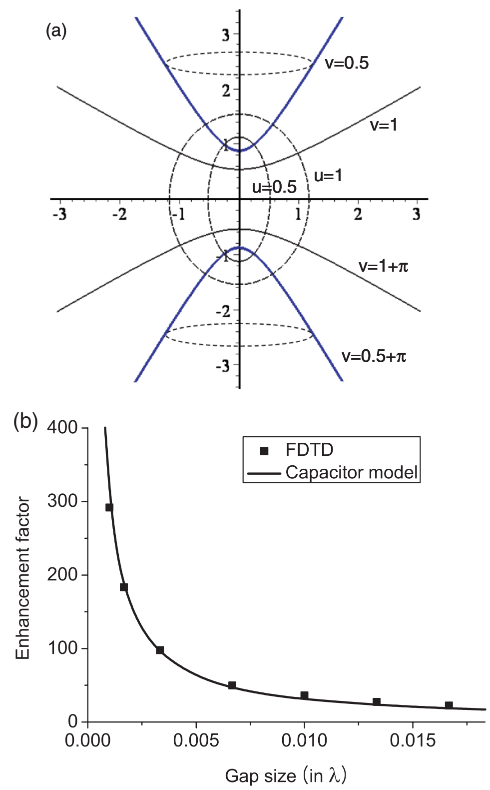

FIG. 4 (color online). (a) Prolate spheroidal coordinates. Tips are specified with constant v. (b) Enhancement of electric field with varying the gap size between tips keeping $v_0 = \pi/6$. Black squares represent FDTD results with tip-end's curvature 18.

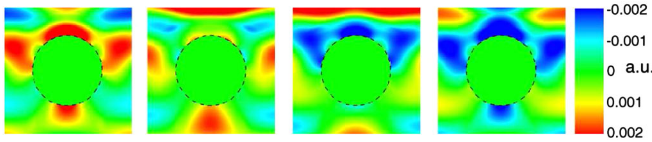

FIG. 5 (color online). Crosscut views of field component H_ϕ around the tip at $t = 0$, $\frac{1}{6}T$, $\frac{2}{6}T$, $\frac{3}{6}T$ for period T (FDTD calculation). $2H_\phi$ evaluated at the tip surface (dashed circle) represents the surface current.

The surface current flowing into the λ zone, $\vec{K} = 2\vec{H}_0 \times \hat{n}$ with $\hat{n}$ a normal vector, results in the induced charge,

$$Q_{\mathrm{ind}} = 2\frac{1}{iw}\int_{-\pi/2}^{\pi/2} \vec{K}\cdot(\hat{\phi}\times\hat{n})\rho_0 d\phi = \frac{8H_0\rho_0}{iw}. \quad (12)$$

The integration is taken along the λ-zone boundary in the front part of a tip on which light impinges. In the back side, surface current is diminished due to the shadow effect. When the tip is of subwavelength size, however, strong diffraction renders surface current flow even in the back side as shown in Fig. 5, where the surface current is $2H_\phi$ just outside the surface. We include this effect by multiplying a factor of 2 in (12). Assuming normalized incident light, we obtain the induced electric field

$$E_{\mathrm{ind}} = \frac{Q_{\mathrm{ind}}}{dC_\lambda} = \frac{2\rho_0\cos\upsilon_0}{i\pi^2 d}\left[\ln\frac{1+\cos\upsilon_0}{1-\cos\upsilon_0}\right]. \quad (13)$$

Figure 4(b) shows the field enhancement calculated by the capacitor model varying the gap distance $d = 2a\cos\upsilon_0$ but keeping the taper structure ($\upsilon_0 = \pi/6$). In doing so, the curvature of the tip end also changes gradually, but it does not affect significantly the field enhancement. The comparison with the FDTD result obtained by varying the gap size only once again shows a good agreement.

Thus far, we have neglected the finite conductivity and the skin-depth effect assuming a perfectly conducting metal. The skin depth of gold at a wavelength of 800 nm, for instance, is 25 nm and metal structures smaller or comparable to this size may not be described by a model based on a perfect conductor. To test the validity, we have computed the enhancement factor for the slit in a gold plate of thickness 10 nm which is smaller than the skin depth. The comparison with the FDTD results in Fig. 6 shows that the local capacitor model predicts enhancement reasonably well even below the skin-depth level. A better agreement can be expected if we generalize the local capacitor model to the finitely conducting case by considering surface impedance. For thickness less than 10 nm, quantum effects such as the Landau damping of plasmon could arise in a size dependent way and reduce enhancement. The inclusion of quantum effect to the local capacitor model is an important open problem.

The concept of λ zone and the local capacitor is not restricted to structures admitting analytic solutions of static

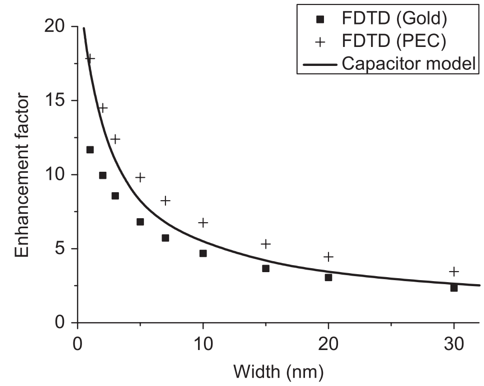

FIG. 6. Enhancement of electric field inside a slit gap in a gold plate of thickness 10 nm with varying gap width and $\lambda = 800$ nm. Black squares and crosses are FDTD results for the gold and the perfect conductor. The discrepancy between FDTD [perfect electric conductor (PEC)] and the capacitor model becomes noticeable as the gap size become larger.

potential. It can be easily extended to other structures with the help of various potential solving methods. Most of all, the concept of λ zone allows a blockwise treatment of nano-optical devices and therefore constitutes a basic element of optical nanocircuits.

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