Dispersion γZ-Box Correction to the Weak Charge of the Proton

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We consider elastic scattering of electrons off a proton target. The parity-violating (PV) asymmetry arises at leading order in α due to interference of γ and Z exchange. The radiative corrections to this leading mechanism were calculated in the literature and included in experimental analyses, except for γZ-box and cross-box contributions. We present here a dispersion calculation of these corrections in forward kinematics. We demonstrate that at the GeV energies of current PV experiments, such corrections are not suppressed by the small vector weak charge of the electron, as occurs in the atomic parity violation. Our results suggest that the current theoretical uncertainty in the analysis of the QWEAK experiment might be substantially underestimated, and more accurate accounts of the dispersion corrections are needed in order to interpret the PV data.

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Precision tests of the standard model at low energies provide an important tool to search for new physics and to constrain model parameters. Such tests involve high precision measurements of observables that are typically suppressed or precisely vanish in the standard model (SM). Prominent examples of such observables include the electric dipole moment and neutrino magnetic moments. Another important example of a parameter of the nucleon structure suppressed in the SM is the weak charge of the proton $Q_W^p = 1 - 4\sin^2\theta_W$. With the value of the weak mixing angle at low momentum transfers $\sin^2\theta_W(0) = 0.238\,07 \pm 0.000\,17$ [1], the SM predicts the proton weak charge of order ≈ 0.05. A precise (4%) measurement of the weak charge of the proton is the aim of the QWEAK experiment at Jefferson Lab [2].

In order to achieve the required precision in the QWEAK experiment, the radiative corrections have to be considered. This was done in various works, to mention the most important references [1,3], and the combined estimate of the theoretical uncertainty is currently 2.2%. This level of precision, coupled with a 2% measurement of the parity-violating asymmetry, would allow for a 0.3% determination of θ_W at low energies. The main difficulty in calculating the radiative corrections originates in the hadronic structure-dependent contributions from the box diagrams with the exchange of $\gamma\gamma$, ZZ, WW, and γZ, often referred to as dispersion corrections, as they involve an inclusive sum over excited intermediate states, evaluated by means of a dispersion relation. Since the parity-conserving amplitude at leading order has a $\frac{1}{Q^2}$ pole, the exchange of two photons only leads to a correction $\sim\alpha Q^2$, with $\alpha \approx \frac{1}{137}$, that can safely be neglected. The parity-violating amplitude in the one-boson exchange (OBE) approximation has no such pole, and the respective correction remains finite in the forward direction. The ZZ and WW boxes were estimated in Refs. [1,3] to give a large correction that comes from hard exchanged bosons' momenta in the loop $\sim M_Z(M_W)$, whereas low momenta contributions are suppressed by an extra power of Q^2/M_W^2. In this case, all subprocesses inside the loop can be treated perturbatively, and the contribution can be calculated reliably. The situation with the γZ box is, however, more complex, since there is generally no reason for hard exchanged momenta to dominate the loop with respect to low momenta. For atomic parity violation [3], it was observed that the γZ dispersion correction is suppressed, as the contributions from the box and the crossed box cancel, and the only nonzero term is proportional to the small vector charge of the electron thus leading to a correction below 1%. In Refs. [1,4], this argument was adopted to high energy electrons, guided by the assumption of high momentum dominance of the loop integral. Clearly, the overall theory uncertainty of 2.2% relies heavily on this cancellation mechanism. The goal of this Letter is to investigate the dispersion correction due to γZ-box graph in the kinematics of the QWEAK experiment. We will provide an explicit calculation of the box and crossed-box corrections in the framework of dispersion relations.

Elastic scattering of massless electrons off a nucleon $e(k) + N(p) \to e(k') + N(p')$ in the presence of parity violation (and in the absence of CP violation) is described by six amplitudes $f_i(\nu, t)$, $i = 1, 2, \ldots, 6$,

$$T = \frac{e^2}{-t}\bar{u}(k')\gamma_\mu\gamma_5 u(k)\bar{N}(p')\left[f_1\gamma^\mu_{(p)} + f_2 i\sigma^{\mu\alpha}\frac{\Delta_\alpha}{2M}\right]N(p) + \frac{e^2}{-t}f_3\bar{u}(k')\gamma_\mu\gamma_5 u(k)\bar{N}(p')\gamma^\mu\gamma_5 N(p)$$
$$- \frac{G_F}{2\sqrt{2}}\bar{u}(k')\gamma_\mu\gamma_5 u(k)\bar{N}(p')\left[f_4\gamma^\mu + f_5 i\sigma^{\mu\alpha}\frac{\Delta_\alpha}{2M}\right]N(p) - \frac{G_F}{2\sqrt{2}}f_6\bar{u}(k')\gamma_\mu u(k)\bar{N}(p')\gamma^\mu\gamma_5 N(p), \quad (1)$$

where only electromagnetic and weak neutral currents are considered. The amplitudes $f_{1,2,3}$ are parity-conserving (PC), and $f_{4,5,6}$ are explicitly parity-violating (PV). All six amplitudes are functions of energy $\nu = \frac{PK}{M}$ (with $K = \frac{k+k'}{2}$ and $P = \frac{p+p'}{2}$) and the elastic momentum transfer $t = \Delta^2 < 0$, with $\Delta = k - k' = p' - p$. Since we are interested in very forward scattering angles $\theta \approx 6°$ corresponding to the QWEAK kinematics [2], the number of structures that are relevant is further reduced. The magnetic terms vanish in the forward direction and can be neglected. The Gordon's identity allows us to rewrite $\bar{N}\gamma^\mu N = \frac{P^\mu}{M}\bar{N}N - \bar{N}i\sigma^{\mu\alpha}\frac{\Delta_\alpha}{2M}N \to 2P^\mu$, where we made use of the nucleon state normalization $\bar{N}N = 2M$. The parity-conserving amplitude f_3 arises due to an exchange of at least two photons or Z^0 bosons and is suppressed as $\sim\mathcal{O}(t)$, as compared to the leading PC amplitude. Finally, amplitude f_6 depends on nucleon spin and makes no contribution to observables with an unpolarized target. Only two amplitudes of relevance remain in the forward direction, and, denoting their forward values as $\tilde{f}_i \equiv f_i(\nu, t = 0)$, we obtain

$$T(t \to 0) = \frac{e^2}{-t}2\tilde{f}_1\bar{u}(k')\not{P}u(k) - \frac{G_F}{\sqrt{2}}\tilde{f}_4\bar{u}(k')\not{P}\gamma_5 u(k). \tag{2}$$

The form of the forward amplitude coincides with that for a spin-0 target [5]. To leading order in t and in Fermi constant, the parity-violating asymmetry arises from the interference of PC and PV amplitudes:

$$A^{\rm PV}(t \to 0) = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \frac{G_F t}{4\pi\alpha\sqrt{2}}\frac{{\rm Re}(\tilde{f}_1^*\tilde{f}_4)}{|\tilde{f}_1|^2}. \tag{3}$$

At tree level, the amplitudes reduce to $\tilde{f}_1^{\rm OBE} = 1$ and $\tilde{f}_4^{\rm OBE} = g_A^e Q_W^p = 1 - 4\sin^2\theta_W$, and thus the PV asymmetry gives direct access to the weak charge of the proton. Using the exact forward values of the amplitudes (the neglect of the t dependence of respective form factors) only leads to power corrections in t that can be safely neglected [1]. In the following, we will assume that all of the radiative corrections except the γZ direct and crossed boxes are known. Denoting these other corrections by $\delta_{\rm RC}$, we introduce the correction due to γZ exchange as $\delta_{\gamma Z} \equiv (\tilde{f}_4 - \tilde{f}_4^{\rm OBE})/\tilde{f}_4^{\rm OBE} - \delta_{\rm RC}$, with $\delta_{\gamma Z}$ a complex function of energy ν. Its real part contributes to the parity violation asymmetry as

$$A^{\rm PV} = \frac{G_F t}{4\pi\alpha\sqrt{2}}Q_W^p[1 + {\rm Re}\delta_{\rm RC} + {\rm Re}\delta_{\gamma Z}(\nu)]. \tag{4}$$

To calculate the real part of the γZ direct and crossed-box graphs shown in Fig. 1, we adopt a dispersion relation formalism, and we start with the calculation of the imaginary part of the direct box (the crossed-box contribution to the real part will be calculated using crossing)

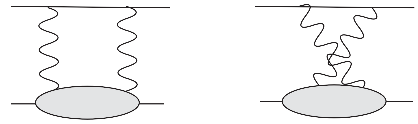

FIG. 1. Box and crossed-box diagrams.

$${\rm Im}\, T_{\gamma Z} = -\frac{G_F}{\sqrt{2}}\frac{e^2}{(2\pi)^3}\int\frac{d^3\vec{k}_1}{2E_1}\frac{l_{\mu\nu}W^{\mu\nu}}{Q^2(1 + Q^2/M_Z^2)}, \tag{5}$$

where $Q^2 = -(k - k_1)^2$ denotes the virtuality of the exchanged photon and Z (in the forward direction, they carry exactly the same Q^2), and we explicitly set the intermediate electron on-shell. In the c.m. of the (initial) electron and proton, $E_1 = (s - W^2)/2\sqrt{s}$, with s the full c.m. energy squared and W the invariant mass of the intermediate hadronic state. Note that, for on-shell intermediate states, the exchanged bosons are always spacelike. The leptonic tensor is given by $l_{\mu\nu} = \bar{u}(k')\gamma_\nu\not{k}_1\gamma_\mu(g_V^e + g_A^e\gamma_5)u(k)$. In the case of the elastic hadronic intermediate state, the needed structure $A(e) \times V(p)$ always contains the explicit factor of Q_W^p or g_V^e. Correspondingly, the correction $\delta_{\gamma Z}$ is not enhanced with respect to the small tree-level coupling and is generally small, in accordance with [1]. We therefore turn to the inelastic contribution. In the forward direction, the imaginary part of the doubly virtual "Compton scattering" ($\gamma^* p \to Z^* p$) amplitude is given in terms of the structure functions $\tilde{F}_{1,2,3}(x, Q^2)$, with $x = \frac{Q^2}{2Pq}$ the Bjorken variable. Making use of the gauge invariance of the leptonic tensor, we have $W^{\mu\nu} = 2\pi\{-g^{\mu\nu}\tilde{F}_1 + \frac{P^\mu P^\nu}{Pq}\tilde{F}_2 + i\epsilon^{\mu\nu\alpha\beta}\frac{P_\alpha q_\beta}{Pq}\tilde{F}_3\}$. Contracting the two tensors, one obtains after a little algebra

$$\begin{aligned}{\rm Im}\delta_{\gamma Z}(\nu) = \frac{\alpha}{8Q_W^p}\int_{W_\pi^2}^{s}\frac{dW^2}{(Pk)^2}\int_0^{Q^2_{\rm max}}\frac{dQ^2}{1 + \frac{Q^2}{M_Z^2}} \\ \times\{g_A^e[\tilde{F}_1 + A\tilde{F}_2] - Bg_V^e\tilde{F}_3\},\end{aligned} \tag{6}$$

with $W_\pi^2 = (M + m_\pi)^2$ the pion production threshold, and the Q^2 integration is constrained below a maximum value $Q^2_{\rm max} = \frac{(s-M^2)(s-W^2)}{s}$ as a condition of on-shell intermediate states for an imaginary part calculation. The kinematical factors in Eq. (6) are $A = \frac{2Pk_1 Pk}{Q^2 Pq} - \frac{P^2}{2Pq}$ and $B = \frac{(P,k+k_1)}{2Pq}$, respectively. In order to write down the dispersion relation for the function $\delta_{\gamma Z}(\nu)$, we consider its behavior under crossing. Crossing corresponds to a CP transformation applied to a part of the amplitude, so that it relates the original reaction $e^-(k) + N(p) \to e^-(k') + N(p')$ to the reaction $e^+(-k') + N(p) \to e^+(-k) + N(p')$. The requirement that the crossed reaction be described by the same invariant amplitudes taken at the crossed kinematics $\nu \to -\nu$ ($K \to -K$ with P unchanged) imposes a constraint on the form of its ν dependence. The tensor that multiplies the amplitude $\tilde{f}_4$ is even under crossing (being an axial vector). Under C parity applied to the electron part, the tree-level contribution to $\tilde{f}_4$ is also even, and as a function of ν the OBE amplitude is an even function—as it is observed, in fact, since it depends only on the elastic momentum transfer that is unchanged under crossing. At one-loop order, exchange of two vector currents (the electromagnetic and the vector parts of the neutral current) leads to C-even behavior, and vector-axial vector exchange to C-odd behavior. Correspondingly, the part of $\mathrm{Im}\delta_{\gamma Z}$ that contains g_A^e is an odd function of ν, whereas the one with g_V^e is even. We distinguish then the two contributions $\delta_{\gamma Z_V}$ and $\delta_{\gamma Z_A}$ that obey dispersion relations of two different forms:

$$\mathrm{Re}\delta_{\gamma Z_A}(\nu) = \frac{2\nu}{\pi}\int_{\nu_\pi}^{\infty}\frac{d\nu'}{\nu'^2-\nu^2}\,\mathrm{Im}\delta_{\gamma Z_A}(\nu'),$$
$$\mathrm{Re}\delta_{\gamma Z_V}(\nu) = \frac{2}{\pi}\int_{\nu_\pi}^{\infty}\frac{\nu' d\nu'}{\nu'^2-\nu^2}\,\mathrm{Im}\delta_{\gamma Z_V}(\nu'). \quad (7)$$

This is just another formulation of the mechanism of the cancellation between the box and crossed-box graphs observed in [3]. For small values of ν that are relevant for parity violation in atoms, the explicit factor of ν in front suppresses $\mathrm{Re}\delta_{\gamma Z_A}$. Instead, no cancellation occurs for $\mathrm{Re}\delta_{\gamma Z_V}$ which is, however, suppressed by the small value of $g_V^e = -1 + 4\sin^2\theta_W$. As was already mentioned, numerically $\mathrm{Re}\delta_{\gamma Z_V} \lesssim 0.65\%$ [1,3], and we will neglect this contribution in the following. The result of Eq. (7) can be considered as a sum rule, since it represents the quantity that is to be measured (the proton's weak charge plus corrections to it) through other observables [parity-violating deep inelastic scattering (PVDIS) structure functions], and this relation does not rely on any assumption, other than the neglect of higher order radiative corrections.

In the absence of any detailed experimental data on PVDIS structure functions, to provide estimates of $\mathrm{Re}\delta_{\gamma Z_A}(\nu)$, we need a phenomenological model for W^2 and Q^2 dependence of $\tilde{F}_{1,2}$. Since we need input from low to high values in both variables, one has to distinguish two different contributions: nucleon resonance contributions in the intermediate states and a high energy nonresonant part. In Ref. [6], the color dipole picture and vector meson dominance (CDP + VMD) approach (see, e.g., [7]) was combined with phenomenology input to successfully describe world high energy data on the parity-conserving DIS structure functions $F_{1,2}$ from low to high Q^2 (but at low Bjorken x). Although no data are available to directly judge whether or not the PVDIS structure functions $\tilde{F}_{1,2}$ would follow exactly the same pattern, we can expect that they may be similar. In CDP, the interaction of the high-energetic photon with the proton target occurs in two steps. First, the photon fluctuates into a quark-antiquark pair that forms a color dipole. Second, this dipole interacts with the target by means of exchange of gluons, as shown in Fig. 2.

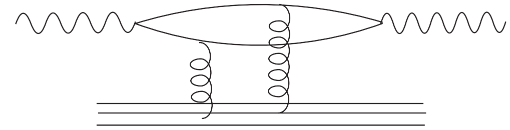

FIG. 2. Representative diagram in the color dipole picture.

Correspondingly, the color dipole-proton scattering cross section is the only universal nonperturbative ingredient, and all of the information about the external particles is contained in the hadronic ($q\bar{q}$) wave functions of the virtual photon or vector meson, depending on the reaction under consideration. In our case, it is the wave function of the Z boson, and its vector part is the same as for a virtual photon of equal virtuality (which is the case due to forward kinematics), except for the quark flavor dependence. This flavor dependence goes as $\sum_q e_q^2$ for the $\gamma\gamma$ DIS structure functions and as $\sum_q e_q g_V^q$ for the γZ PVDIS structure functions, provided we restrict ourselves to the lightest flavors (since all intermediate states have to be real particles, production of a heavy quark-antiquark pair requires very high energy), and the dependence on quark masses can be neglected in the photon and Z wave functions. By taking the SM values, $\sum_{q=u,d,s} e_q^2 = \frac{2}{3}$, whereas $\sum_{q=u,d,s} e_q g_V^q = \frac{2}{3}[1 + (1 - 4\sin^2\theta_W)] \approx \frac{2}{3}$. We also remind the reader that the very first PVDIS measurement on the deuteron target [8] was used to confirm the parton model: In the scaling limit, the isoscalar structure functions of DIS and PVDIS depend on the same combination of the quark distribution functions that cancel out in the asymmetry, leaving only kinematical factors and couplings. This similarity is the case, as well, for the $\Delta(1232)$ resonance, as shown in Ref. [9].

We conclude that the assumption of the similarity of the interference γZ structure functions to the usual ones with two virtual photons is supported within the parton model and CDP at high energy and at low energies, at least for the most important $\Delta(1232)$ resonance. We will use this assumption to provide a realistic estimate for the dispersion correction to the proton weak charge.

Alternatively to the DIS structure functions, one can use the transverse and longitudinal virtual photon cross sections $\sigma_{T,L}$ related to $F_{1,2}$ as (see, e.g., Ref. [10] for details) $\sigma_T = \frac{4\pi^2\alpha}{Pq}F_1$ and $\sigma_L = \frac{4\pi^2\alpha}{Pq}[(\frac{1}{2x} + \frac{M^2}{Pq})F_2 - F_1]$. We construct the cross sections from nucleon resonances and a high energy (Regge) part:

$$\sigma_{T,L}(W^2, Q^2) = \sum_R \frac{\sigma_R \Gamma_R \Gamma_R^\gamma M_R^2}{(W^2 - M_R^2)^2 + M_R^2\Gamma_R^2} F_{T,L}^2(Q^2) + \sigma_{T,L}^{\mathrm{Regge}}(W^2, Q^2). \quad (8)$$

The relative strength of the resonances in Breit-Wigner form and the high energy Regge part that contains the Pomeron and the ρ exchanges are matched at $Q^2 = 0$ to

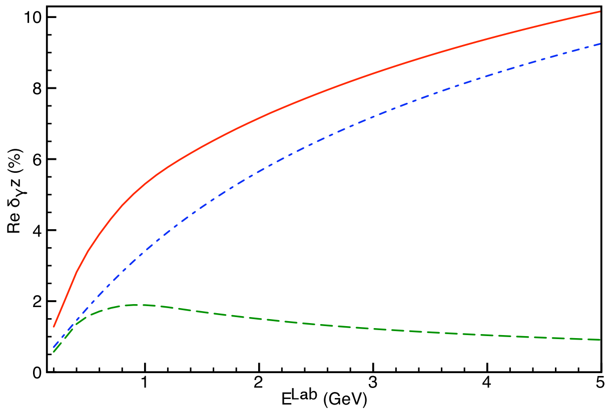

FIG. 3 (color online). Results for Re$\delta_{\gamma Z_A}$ as a function of energy. The contributions of nucleon resonances (dashed line), the Regge part (dashed-dotted line), and the sum of the two (solid line) are shown.

fit the real Compton data (for the parameters, see [11]). Reference [6] gives full W^2 and Q^2 dependence of the longitudinal and transverse virtual photon cross sections $\sigma_{L,T}^{\rm Regge}(W^2, Q^2)$, and we refer the reader to that reference for further details. For the resonances, to depart from the real photon point, transition form factors are used. The latter are to some extent known for a number of resonances, and we assume a dipole form $F_T(Q^2) = \frac{1}{(1+Q^2/\Lambda^2)^2}$ and $F_L(Q^2) = \frac{Q/\Lambda}{(1+Q^2/\Lambda^2)^{2.5}}$, with $\Lambda \approx 1$ GeV.

It is important to note that the model described above is necessary only because the experimental data on total cross sections for absorption of real and virtual photons only exist in limited intervals of the variables W^2 and Q^2, and the described model interpolates the data points to intermediate values. The model is predominantly designed to describe low energies (resonance part) and very high energies, where the parameters of the two components are known from a direct comparison to data. However, the presence of the Regge ρ exchange ($q\bar{q}$ exchange in the t channel may mimic the parton model's handbag diagram) along with the pure CDP Pomeron (two gluon exchange) allows one to access the intermediate range of x, as well. We will address the extent to which the model of Eq. (8) works in this intermediate range of x and relate the uncertainty induced by using electromagnetic DIS data in place of PVDIS data in upcoming work.

We present results of the dispersion calculation in Fig. 3. It can be seen that, starting from $E_{\rm lab} \approx 1$ GeV, the high energy (Regge) contribution dominates the contribution from the resonances. This is the consequence of a relatively slow convergence of the dispersion integral for the Regge part, while the resonances drop very fast. In the presented calculation, the upper limit of the integration over ν' was chosen to be 500 GeV, although the $1/\nu'^2$ weighting ensures the convergence already at lower values. While at very low energies the correction is indeed very small, at the 1.16 GeV energy of the QWEAK experiment the correction is 5.7%. More specifically, QWEAK aims at comparing the measured weak charge of the proton $(4\pi\alpha\sqrt{2}/G_F t)A^{\rm PV}$ to its value as given in the SM, $Q_W^p[1 + \delta_{\rm RC} + {\rm Re}\delta_{\gamma Z}]$, and from this comparison draw conclusions about the new physics contributions. The current estimate of the uncertainty due to the corrections in the square brackets is 2.2%, and this estimate relies on the assumption that $\delta_{\gamma Z}$ is highly suppressed ($\leq$0.65%). As explained above, this estimate is taken over from low energy estimates for parity violation in atoms and is not based on any microscopic calculation. Although the numbers presented here are themselves model-dependent, our calculation shows that the γZ-box diagrams can be almost an order of magnitude larger than it was believed to date, and this result suggests larger possible theoretical errors for the QWEAK experiment. If the uncertainty in the dispersion correction is to be comparable to the proposed 2% experimental error in $A^{\rm PV}$, one may need to calculate the dispersion γZ correction (that we think is near 6%) to a fractional accuracy of order 30%. Alternatively, uncertainties in these dispersion corrections could provide a limit on the precision of a standard model test. Since the calculation uses the PVDIS structure functions as input, it would be extremely helpful to have experimental data on PVDIS to check the model adopted here.

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[1] J. Erler, A. Kurylov, and M. J. Ramsey-Musolf, Phys. Rev. D **68**, 016006 (2003).

[2] W. T. H. Van Oers (QWEAK Collaboration), Nucl. Phys. **A790**, 81 (2007).

[3] W. J. Marciano and A. Sirlin, Phys. Rev. D **27**, 552 (1983); **29**, 75 (1984); **31**, 213(E) (1985).

[4] M. J. Ramsey-Musolf, Phys. Rev. C **60**, 015501 (1999).

[5] M. Gorchtein and C. J. Horowitz, Phys. Rev. C **77**, 044606 (2008).

[6] G. Cvetic, D. Schildknecht, B. Surrow, and M. Tentyukov, Eur. Phys. J. C **20**, 77 (2001); G. Cvetic, D. Schildknecht, and A. Shoshi, Eur. Phys. J. C **13**, 301 (2000).

[7] L. Stodolsky, Phys. Rev. Lett. **18**, 135 (1967); V. N. Gribov, Sov. Phys. JETP **29**, 483 (1969); **30**, 709 (1970); J. J. Sakurai and D. Schildknecht, Phys. Lett. **40B**, 121 (1972); L. V. Gribov, E. M. Levin, and M. G. Ryskin, Phys. Rep. **100**, 1 (1983); N. N. Nikolaev and B. G. Zakharov, Z. Phys. C **49**, 607 (1991).

[8] C. Y. Prescott *et al.*, Phys. Lett. **77B**, 347 (1978); **84B**, 524 (1979); R. N. Cahn and F. J. Gilman, Phys. Rev. D **17**, 1313 (1978).

[9] K. Matsui, T. Sato, and T.-S. H. Lee, Phys. Rev. C **72**, 025204 (2005).

[10] D. Drechsel, S. S. Kamalov, and L. Tiator, Phys. Rev. D **63**, 114010 (2001).

[11] N. Bianchi *et al.*, Phys. Rev. C **54**, 1688 (1996).