Cavity QED Based on Collective Magnetic Dipole Coupling: Spin Ensembles as Hybrid Two-Level Systems

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We analyze the magnetic dipole coupling of an ensemble of spins to a superconducting microwave stripline structure, incorporating a Josephson junction based transmon qubit. We show that this system is described by an embedded Jaynes-Cummings model: in the strong coupling regime, collective spin-wave excitations of the ensemble of spins pick up the nonlinearity of the cavity mode, such that the two lowest eigenstates of the coupled spin wave–microwave cavity–Josephson junction system define a hybrid two-level system. The proposal described here enables new avenues for nonlinear optics using optical photons coupled to spin ensembles via Raman transitions. The possibility of strong coupling cavity QED with magnetic dipole transitions also opens up the possibility of extending quantum information processing protocols to spins in silicon or graphene, without the need for single-spin confinement.

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As compared to the coupling of a single emitter, the strength of optical excitations out of an ensemble of two-level emitters is enhanced by the square root of the number of emitters ($\sqrt{N_s}$). This collective enhancement of light-matter coupling in free space has played a central role in quantum memory and repeater protocols [1]. In the context of cavity quantum electrodynamics (QED), the $\sqrt{N_s}$ enhancement comes at the expense of the desirable nonlinearity of the coupled cavity-emitter system [2]. Nevertheless, strong electric-dipole coupling of a number of diverse systems including interband excitons [3], intersubband plasmons [4], and cold-atomic ensembles [5] to high quality (Q) factor optical cavities have been demonstrated: the signature of strong coupling for these systems is the appearance of vacuum Rabi splitting of two dressed modes, each with a harmonic spectrum. Direct magnetic dipole coupling of spins to cavity modes, on the other hand, have been totally ignored, even though collective excitations out of an ensemble of $\sim 10^6$ spins could easily reach a corresponding linear strong coupling regime using the superconducting microstrip (SCM) cavities recently realized in the context of circuit-QED experiments [6,7].

In this Letter, we describe how two-level hybrid-spin qubits can be defined using magnetic dipole coupling of an ensemble of spins to SCM cavities with a built-in nonlinear element such as a transmon qubit [8]. We consider a geometry where the transmon qubit, with ground and excited states denoted by $|a\rangle$ and $|b\rangle$, is introduced at an electric-field maximum of the SCM cavity. In contrast, an ensemble of spins is placed at a location where the magnetic field is maximum (Fig. 1). The Hamiltonian of the combined system in the interaction picture is given by

$$\hat{H} = \hbar g_c(\hat{\sigma}_{ba}\hat{a}_c e^{-i\delta t} + \text{H.c.}) + \hbar g_m \sum_{i=1}^{N_s} (\hat{\sigma}^i_- \hat{a}_c e^{-i\Delta t} + \text{H.c.}), \tag{1}$$

where $\hat{\sigma}_{ba} = |b\rangle\langle a|$ and $\hat{\sigma}^i_-$ denotes the spin lowering operator of the ith spin. $\hat{a}_c$ is the annihilation operator of the SCM cavity mode. g_c (g_m) denotes the electric (magnetic) dipole coupling strength of the transmon qubit (single spin) to the SCM cavity mode. We assume here that the transmon qubit (spins) is (are) red detuned from the cavity mode by δ (Δ).

Before proceeding, we highlight the recent work discussing a very similar scenario where an ensemble of polar

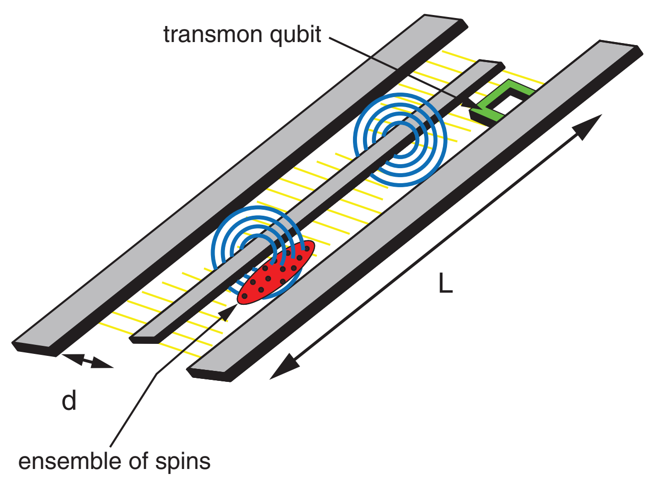

FIG. 1 (color). Schematic of the system consisting of an ensemble of spins coupled to the quantized magnetic field of a superconducting microstripline cavity with $d \sim 10$ μm and $L \sim 1$ cm. The length of the thin (orange) lines indicates the strength of the cavity-mode electric field. The thick (blue) lines that encircle the center conductor depict the magnetic field lines at the locations where their strength is maximum. The presence of a transmon qubit at an electric-field maximum ensures that the cavity has a large nonlinearity. The ensemble of spins could either be that of electrons in the silicon substrate or cold ground-state atoms trapped $\sim$10 μm above the cavity structure.

molecules is coupled to a SCM cavity containing a Josephson junction qubit [9,10]. The underlying idea for this pioneering work was the use of strong electric-dipole coupling strength of polar molecules to achieve an interface between a solid-state and an ensemble molecular qubit, with the goal of using the molecular qubit as quantum memory. The present proposal, in contrast, is based on strong coupling regime of magnetic dipole interaction in systems where electric-dipole coupling is vanishingly small: such systems have the advantage of being immune to charge noise and could constitute qubits with much longer coherence times.

A key element of the proposal we describe here is the unprecedented values of cavity-qubit electric-dipole coupling strength (g_c) achieved using transmon qubits in SCM cavities, saturating the fundamental limit $g_c = \sqrt{\alpha}\omega_c$ [8], where α and ω_c denote the fine-structure constant and the bare-cavity resonance frequency, respectively. In this regime, the condition $g_c \gg g_m\sqrt{N_s}$ is satisfied and we can start our analysis by neglecting the second term in Eq. (1): we then obtain the celebrated Jaynes-Cummings (JC) spectrum for the coupled cavity-transmon system, with an anharmonic spectrum. In the first part of our analysis, we will focus on the resonant case with $\delta = 0$.

We now turn our attention to the coupling of the *cavity-transmon molecule* to the ensemble of spins. If we choose Δ such that $|g_c - \Delta| \ll g_c$ and the cavity Q is high enough to satisfy $g_c \gg \kappa_c = \omega_c/Q$, we would only need to consider the coupling of collective excitations of the spins to the two lowest-energy eigenstates of the JC ladder: namely, the ground state $|\tilde{0}\rangle = |a, 0_c\rangle$ and the symmetric state of the single-excitation ($n = 1$) manifold $|\tilde{1}\rangle = (|a, 1_c\rangle + |b, 0_c\rangle)/\sqrt{2}$. This is a consequence of the fact that the energy separation between the lowest-energy states of the one ($n = 1$) and two ($n = 2$) photon excitation manifolds in the JC ladder is given by $\omega_c - g_c(\sqrt{2} - 1)$, which in turn implies that the detuning of the collective spin excitations from these cavity-transmon transitions is much larger than the energy scale ($g_m\sqrt{N_s}$) associated with coupling to spins. Since the eigenstates of the spin ensemble form a harmonic ladder, the coupling of the two-level system spanned by the eigenstates $|\tilde{0}\rangle$ and $|\tilde{1}\rangle$ to collective spin excitations maps on to an embedded JC ladder. The scenario we envision is depicted in Fig. 2 where the encircled states form the embedded JC model. Provided that the corresponding coupling strength, given by $g_m\sqrt{N_s}$, is larger than the decoherence rate of the spin excitation (γ_{spin}), transmon qubit (γ_{JJ}), and the SCM cavity mode (κ_c), the two lowest-energy states of the embedded JC ladder define a two-level hybrid-spin qubit. For $\delta = 0$ and $\Delta = g_c$, the Hilbert space of this qubit is spanned by the states

$$|\tilde{\tilde{0}}\rangle = |\mathcal{G}, a, 0_c\rangle$$
$$|\tilde{\tilde{1}}\rangle = \frac{1}{\sqrt{2}}|\mathcal{E}, a, 0_c\rangle + \frac{1}{2}|\mathcal{G}, b, 0_c\rangle + \frac{1}{2}|\mathcal{G}, a, 1_c\rangle, \tag{2}$$

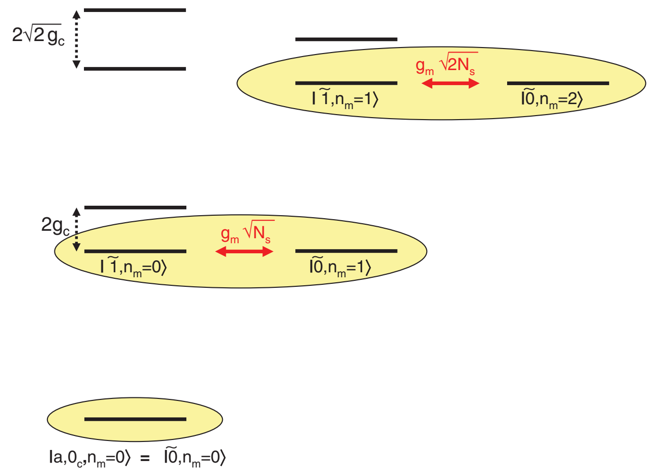

FIG. 2 (color). The level diagram depicting the lowest-energy eigenstates of the dressed cavity-transmon system along with the lowest-energy excited states of the spin ensemble. The anharmonicity of the cavity-transmon molecule is large enough to ensure that only a subset of eigenstates, highlighted in yellow, couples nonperturbatively via the collectively enhanced magnetic dipole transitions. The diagonalization of magnetic dipole interactions among these states yields an embedded Jaynes-Cummings ladder.

where $|\mathcal{G}\rangle$ and $|\mathcal{E}\rangle = (\sum_{i=1}^{N_s} \hat{\sigma}_-^i)|\mathcal{G}\rangle/\sqrt{N_s}$ denote the fully polarized ground and first excited (single collective spin flip) states of the spin ensemble. To estimate the validity of the two-level approximation, we compare the degree of the anharmonicity given by $g_m\sqrt{N_s}(2 - \sqrt{2})$ with the decoherence rates typical for the system: $\kappa_c \simeq 1 \times 10^6$ rad/sec $\simeq \gamma_{\text{JJ}} \geq \gamma_{\text{spin}}$. The single-spin magnetic dipole coupling strength is given by $g_m = \mu_B\sqrt{\mu_o(\omega_c - g_c)}/\sqrt{2\hbar V_c}$, where μ_B is the Bohr magneton, μ_o is the vacuum permeability, and V_c is the SCM cavity-mode volume. For a cavity volume $V_c = 10^{-12}$ m^2 (corresponding to a separation of $\sim$10 μm between the center conductor and the ground planes), $\omega_c = 2\pi \times 10$ GHz, we find $g_m = 1 \times 10^3$ rad/sec. This estimation implies that we would need $N_s \geq 10^6$ for the two-level approximation to be valid. A 10 μm thick bulk-doped silicon with a dopant density $n = 10^{16}$ cm^{-3} and an area of 10 μm $\times$ 100 μm would yield $N_s \sim 10^8$ spins. To ensure that the assumption $g_c \gg g_m\sqrt{N_s}$ remains valid, we need to ensure that the total effective number of spins that couple to the cavity satisfies $N_s \ll 10^{12}$.

The qubit defined by the computational states $|\mathcal{G}\rangle$ and $|\mathcal{E}\rangle$ brings up a number of benefits, such as collective enhancement of coupling to external coherent fields, the possibility of working with spin systems with weak spin-orbit interaction, and avoiding the requirement to isolate a single spin. The drawback of such a qubit is that it is defined in a two-dimensional subspace of a harmonic system: the discussion we presented shows that this limitation could be overcome by adiabatically tuning the collective spin excitations into resonance with a nonlinear cavity and realizing the mapping

$$|\psi\rangle = \xi|\mathcal{G}, a, 0_c\rangle + \beta|\mathcal{E}, a, 0_c\rangle \rightarrow \xi|\tilde{\tilde{0}}\rangle + \beta|\tilde{\tilde{1}}\rangle. \quad (3)$$

This tuning could be realized by changing the external flux bias applied to the transmon qubit: a tuning range exceeding 1 GHz has been demonstrated experimentally [11]. Furthermore, this tuning mechanism can be implemented on time scales $\sim$1 nsec [12]. To effect the mapping described by Eq. (3), we need to ensure that the tuning of the transmon qubit is realized on time scales long compared to $g_m\sqrt{N_s}$ and short compared to κ_c.

Using the anharmonic spin-wave excitations as a qubit would require that the ensemble of spins is either completely polarized or is in a mixture of dark states that could not be further polarized due to the symmetry [13–15]. We emphasize that the initial temperature of the spins need not be low: the same strong cavity coupling utilized in defining the hybrid qubit could be used to cool down the spin ensemble into a mixture of dark states. This would be achieved ideally in the limit where the cavity Q is reduced and the spin excitation is transferred to phonons via nonradiative cavity dissipation. Provided that the cavity or substrate temperature satisfies $T_c \leq 70$ mK, it should be possible to ensure that the spins are in a mixture of dark states with mean number of collective excitations much smaller than unity. However, the inhomogeneous broadening due to different collective coupling strengths associated with different dark states may become a limitation if the degree of spin polarization is not high [15]. A temporary reduction in cavity Q could be achieved by laser irradiation that induces Ohmic losses.

Coupling of spins to the cavity mode would require that the Zeeman splitting ω_z satisfies $\omega_z = \omega_c - g_c$, which in turn implies the presence of an effective magnetic field of $B \sim 0.5$ T. Such magnetic fields will substantially reduce the cavity Q. Ideally, this problem can be remedied by inducing the electron spin splitting via polarized nuclear spins of the host material: it is well known that dynamical polarization techniques lead to effective Overhauser fields that can exceed 2 T and that survive for more than 1 min after active polarization is turned off. Since these effective fields are predominantly due to Fermi-contact hyperfine interaction, cavity Q would remain unaffected.

Alternatively, we could envision a local external field of $B \sim 1$ T that vanishes at the site of the transmon qubit. Such an external field will reduce the cavity Q and the condition $g_c \gg g_m\sqrt{N_s} > \kappa_c$ may no longer be satisfied. However, provided that $Q > 100$, $g_c \gg \kappa_c$ is still satisfied and it is possible to overcome the limitation of a lossy cavity: to this end, we assume that the transmon qubit is detuned from the cavity mode in a way to ensure that it is resonant with the electron spins ($\Delta = \delta + g_c^2/\delta$). In the limit $\hbar|\delta| \gg$ all other energy scales, we use a Schrieffer-Wolff transformation to eliminate the first term in Eq. (1) and find the transformed Hamiltonian to lowest order

$$\hat{H} = \hbar\frac{g_m g_c}{\Delta}\sum_{i=1}^{N_s}(\hat{\sigma}_-^i\hat{\sigma}_{ab} + \text{H.c.}), \quad (4)$$

where we assumed that δ is large enough to ensure that $|\delta - \Delta| \ll |\Delta|$. The transformed Hamiltonian is also of the JC form since the collective spin lowering operator $(\sum_{i=1}^{N_s}\hat{\sigma}_-^i)/\sqrt{N_s}$ approximates a bosonic creation operator in the limit $N_s \gg 1$ for low excitation manifolds. The condition for strong coupling in this case is given by $g_c g_m\sqrt{N_s}/\Delta > \kappa_c g_c^2/\delta^2, \gamma_{\text{spin}}, \gamma_{\text{JJ}}$: in this limit, the cavity mode acts as a quantum bus and its virtual excitations mediate the long distance JC interaction between the transmon qubit and the collective spin excitations.

The principal advantage of ensemble spin qubits over that of transmon qubits is their potentially much longer decoherence times, which render them ideal candidates for use as memory elements for transmon qubits. By tuning the transmon qubit such that the final detuning of the spin-ensemble transitions from the cavity and transmon resonances is $\sim$1 GHz, we could ensure that the cavity and transmon induced decoherence of the spin-ensemble qubit is reduced by a factor $\sim 10^6$. This reduction would ensure that spins in pure silicon-28 with no hyperfine interaction and ultrasmall spin-orbit coupling could constitute a long-coherence-time quantum memory. In GaAs, the ensemble spin approach will enable conversion of quantum information carried by the spin or transmon system to that of a propagating photon. As compared to GaAs single-electron spin qubits, ensemble spins have the advantage of reduced hyperfine decoherence [16]. It should be emphasized, however, that spin-orbit induced decoherence eventually becomes prominent in bulk GaAs structures and limits the spin coherence time to $\sim 1\ \mu$sec [17].

Since the two lowest-energy collective excitations of a spin ensemble coupled to a nonlinear cavity define an anharmonic two-level system, it is possible to directly manipulate quantum information represented by such a hybrid qubit. Given the impressive advances in coupling transmon qubits via a common SCM cavity mode, the most straightforward method for effecting one- or two-qubit gates would be to map the spin-ensemble qubit onto the hybrid transmon–spin-ensemble qubit: $|\psi\rangle = \xi|\mathcal{G}, a\rangle + \beta|\mathcal{E}, a\rangle \rightarrow \xi|\mathcal{G}, a\rangle + \frac{\beta}{\sqrt{2}}(|\mathcal{G}, b\rangle + |\mathcal{E}, a\rangle)$. Two such hybrid qubits, based on two distant spin ensembles each coupled to a different transmon qubit, could then interact via a virtual excitation of the common cavity mode [8,18]. The measurement of the hybrid-spin qubit state could be carried out by mapping its state onto the transmon qubit; single-shot measurement of transmon qubits could be realized using Josephson bifurcation amplifiers [19].

Another interesting system for which the formalism discussed here applies is an ensemble of cold atoms trapped 10 μm above the SCM cavity structure. At such distances, the adverse effects of the solid interface on ground-state atoms can be avoided [20]. Without the need for strong external magnetic fields, the magnetic dipole coupling of the hyperfine transition of the atomic ensemble to the nonlinear cavity mode will then lead to an anharmonic energy level diagram for the collective atomic (hyperfine) spin excitations. In addition to providing a long-lived memory, a cigar-shaped atomic ensemble coupled to a cavity-transmon system would enable near-unity efficiency conversion of a microwave photon to an optical photon that can be collimated using a low numerical aperture lens [1].

Finally, we emphasize that the hybrid two-level system described by $\{|\tilde{\tilde{0}}\rangle, |\tilde{\tilde{1}}\rangle\}$ constitutes a new paradigm for nonlinear optics. In particular, we consider Raman coupling of the states $|\mathcal{G}\rangle$ and $|\mathcal{E}\rangle$ with a weak few-photon pulse and a nonperturbative coupling laser: this system is known to exhibit electromagnetically induced transparency and an ultraslow group velocity of the propagating weak few-photon field. The presence of strong coupling of the spin ensemble to the nonlinear microwave cavity will then lead to large dissipation-free (optical) nonlinear phase shifts, provided that the effective single-photon nonlinearity $g_m\sqrt{N_s}(2-\sqrt{2})$ is smaller than the width of the transparency window [21,22]. Realization of such a nonlinear optical device could be achieved either by using cold-atomic gases or an ensemble of solid-state impurities such as nitrogen-vacancy centers in diamond.

In conclusion, we demonstrate that collective enhancement of magnetic dipole coupling of an ensemble of spins would lead to the strong coupling regime of cavity QED. The presence of a transmon qubit in the SCM cavity would lead to the realization of an embedded JC model where the harmonic emitter (spin) system picks up the nonlinearity of the cavity. In the context of quantum information processing, our findings generalize the conclusions previously drawn for polar molecules to a larger class of systems, ranging from trapped ground-state atomic gases to spins in bulk silicon or graphene.

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