Light-Front Holography: A First Approximation to QCD

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Starting from the Hamiltonian equation of motion in QCD, we identify an invariant light-front coordinate ζ which allows the separation of the dynamics of quark and gluon binding from the kinematics of constituent spin and internal orbital angular momentum. The result is a single-variable light-front Schrödinger equation for QCD which determines the eigenspectrum and the light-front wave functions of hadrons for general spin and orbital angular momentum. This light-front wave equation is equivalent to the equations of motion which describe the propagation of spin-J modes on anti–de Sitter (AdS) space.

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One of the most important theoretical tools in atomic physics is the Schrödinger equation, which describes the quantum-mechanical structure of atomic systems at the amplitude level. Light-front wave functions (LFWFs) play a similar role in quantum chromodynamics (QCD), providing a fundamental description of the structure and internal dynamics of hadrons in terms of their constituent quarks and gluons. The light-front wave functions of bound states in QCD are relativistic generalizations of the Schrödinger wave functions of atomic physics, but they are determined at fixed light-cone time $\tau = t + z/c$—the "front form" introduced by Dirac [1]—rather than at fixed ordinary time t. A remarkable feature of LFWFs is the fact that they are frame independent; i.e., the form of the LFWF is independent of the hadron's total momentum $P^+ = P^0 + P^3$ and $\vec{P}_\perp$.

Light-front quantization is the ideal framework to describe the structure of hadrons in terms of their quark and gluon degrees of freedom. The simple structure of the light-front vacuum allows an unambiguous definition of the partonic content of a hadron. Given the LFWFs, one can compute observables such as hadronic form factors and structure functions, as well as the generalized parton distributions and distribution amplitudes which underlie hard exclusive reactions. The constituent spin and orbital angular momentum properties of the hadrons are also encoded in the LFWFs.

A key step in the analysis of an atomic system such as positronium is the introduction of the spherical coordinates r, θ, ϕ which separates the dynamics of Coulomb binding from the kinematical effects of the quantized orbital angular momentum L. The essential dynamics of the atom is specified by the radial Schrödinger equation whose eigensolutions $\psi_{n,L}(r)$ determine the bound-state wave function and eigenspectrum. In this Letter, we show that there is an analogous invariant light-front coordinate ζ which allows one to separate the essential dynamics of quark and gluon binding from the kinematical physics of constituent spin and internal orbital angular momentum. The result is a single-variable light-front Schrödinger equation for QCD which determines the eigenspectrum and the light-front wave functions of hadrons for general spin and orbital angular momentum.

Our analysis follows from recent developments in light-front QCD [2–6] which have been inspired by the AdS-CFT correspondence [7] between string states in anti–de Sitter (AdS) space and conformal field theories (CFT) in physical space-time. The application of AdS space and conformal methods to QCD can be motivated from the empirical evidence [8] and theoretical arguments [9] that the QCD coupling $\alpha_s(Q^2)$ has an infrared fixed point at low Q^2. The AdS-CFT correspondence has led to insights into the confining dynamics of QCD and the analytic form of hadronic light-front wave functions. As we have shown recently, there is a remarkable mapping between the description of hadronic modes in AdS space and the Hamiltonian formulation of QCD in physical space-time quantized on the light front. This procedure allows string modes $\Phi(z)$ in the AdS holographic variable z to be precisely mapped to the light-front wave functions of hadrons in physical space-time in terms of a specific light-front variable ζ which measures the separation of the quark and gluonic constituents within the hadron. The coordinate ζ also specifies the light-front (LF) kinetic energy and invariant mass of constituents. This mapping was originally obtained by matching the expression for electromagnetic current matrix elements in AdS space with the corresponding expression for the current matrix element using light-front theory in physical space-time [4]. More recently we have shown that one obtains the identical holographic mapping using the matrix elements of the energy-momentum tensor [6], thus providing an important consistency test and verification of holographic mapping from AdS space to physical observables defined on the light front.

The connection between light-front QCD and the description of hadronic modes on AdS space is physically compelling and phenomenologically successful. However, there are lingering questions in this approach that should be addressed. In particular, one wants to understand under what approximations (if any) a formal gauge-gravity correspondence can be established for physical QCD. This question is most important if QCD is to be described by the low energy limit of some (yet unknown) string theory in a higher dimensional space. In string theory a spin-J hadronic state is described by a spin-J field, whereas in physical QCD hadrons are composite and thus are inevitably endowed of orbital angular momentum. How can these two pictures be compatible? The mapping between string modes in AdS and LFWFs described in [4,6] is an important step, but one must also prove that our identification of orbital angular momentum is correct and compatible with the string description in terms of eigenmodes of total spin J. It is also important to understand the nature and the validity of the approximations involved in establishing a gauge-gravity correspondence to find a framework to systematically improve the results.

In this letter we will show that to a first semiclassical approximation, light-front QCD is formally equivalent to the equations of motion on a fixed AdS_5 gravitational background. To prove this, we show that the LF Hamiltonian equations of motion of QCD lead to an effective LF equation for physical modes $\phi(\zeta)$ which encode the hadronic properties. This LF equation carries the orbital angular momentum quantum numbers and is equivalent to the propagation of spin-J modes on AdS space.

We express the hadron four-momentum generator $P = (P^+, P^-, \mathbf{P}_\perp)$, $P^\pm = P^0 \pm P^3$, in terms of the dynamical fields, the Dirac field ψ_+, $\psi_\pm = \Lambda_\pm \psi$, $\Lambda_\pm = \gamma^0\gamma^\pm$, and the transverse field $\mathbf{A}_\perp$ in the $A^+ = 0$ gauge [10]

$$\begin{aligned} P^- &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp \bar{\psi}_+ \gamma^+ \frac{(i\boldsymbol{\nabla}_\perp)^2 + m^2}{i\partial^+}\psi_+ \\ &\quad + (\text{interactions}), \\ P^+ &= \int dx^- d^2\mathbf{x}_\perp \bar{\psi}_+ \gamma^+ i\partial^+ \psi_+, \\ \mathbf{P}_\perp &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp \bar{\psi}_+ \gamma^+ i\boldsymbol{\nabla}_\perp \psi_+, \end{aligned} \tag{1}$$

where the integrals are over the initial surface $x^+ = 0$, $x^\pm = x^0 \pm x^3$. The operator P^- generates LF time translations $[\psi_+(x), P^-] = i\partial\psi_+(x)/\partial x^+$, and the generators P^+ and $\mathbf{P}_\perp$ are kinematical. For simplicity we have omitted from (1) the contribution from the gluon field $\mathbf{A}_\perp$.

The Dirac field operator is expanded as

$$\begin{aligned} \psi_+(x^-, \mathbf{x}_\perp)_\alpha = \sum_\lambda \int_{q^+>0} \frac{dq^+}{\sqrt{2q^+}} \frac{d^2\mathbf{q}_\perp}{(2\pi)^3} \\ \times [b_\lambda(q) u_\alpha(q,\lambda) e^{-iq\cdot x} \\ + d_\lambda(q)^\dagger v_\alpha(q,\lambda) e^{iq\cdot x}], \end{aligned} \tag{2}$$

with u and v LF spinors [11]. Similar expansion follows for the gluon field $\mathbf{A}_\perp$. Using LF commutation relations $\{b(q), b^\dagger(q')\} = (2\pi)^3 \delta(q^+ - q'^+)\delta^{(2)}(\mathbf{q}_\perp - \mathbf{q}'_\perp)$, we find

$$\begin{aligned} P^- = \sum_\lambda \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3}\left(\frac{\mathbf{q}_\perp^2 + m^2}{q^+}\right) b_\lambda^\dagger(q) b_\lambda(q) \\ + (\text{interactions}), \end{aligned}$$

and we recover the LF dispersion relation $q^- = \frac{\mathbf{q}_\perp^2 + m^2}{q^+}$, which follows from the on shell relation $q^2 = m^2$. The LF time evolution operator P^- is conveniently written as a term which represents the sum of the kinetic energy of all the partons plus a sum of all the interaction terms.

It is convenient to define a light-front Lorentz invariant Hamiltonian $H_{\mathrm{LF}} = P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2$ with eigenstates $|\psi_H(P^+, \mathbf{P}_\perp, S_z)\rangle$ and eigenmass $\mathcal{M}_H^2$, the mass spectrum of the color-singlet states of QCD [10]

$$H_{\mathrm{LF}}|\psi_H\rangle = \mathcal{M}_H^2|\psi_H\rangle. \tag{3}$$

A state $|\psi_H\rangle$ is an expansion in multiparticle Fock states $|n\rangle$ of the free LF Hamiltonian: $|\psi_H\rangle = \sum_n \psi_{n/H}|n\rangle$, where a one parton state is $|q\rangle = \sqrt{2q^+} b^\dagger(q)|0\rangle$. The Fock components $\psi_{n/H}(x_i, \mathbf{k}_{\perp i}, \lambda_i^z)$ are independent of P^+ and $\mathbf{P}_\perp$ and depend only on relative partonic coordinates: the momentum fraction $x_i = k_i^+/P^+$, the transverse momentum $\mathbf{k}_{\perp i}$ and spin component λ_i^z. Momentum conservation requires $\sum_{i=1}^n x_i = 1$ and $\sum_{i=1}^n \mathbf{k}_{\perp i} = \mathbf{0}$. The LFWFs $\psi_{n/H}$ provide a *frame-independent* representation of a hadron which relates its quark and gluon degrees of freedom to their asymptotic hadronic state.

We compute $\mathcal{M}^2$ from the hadronic matrix element $\langle\psi_H(P')|H_{\mathrm{LF}}|\psi_H(P)\rangle = \mathcal{M}_H^2\langle\psi_H(P')|\psi_H(P)\rangle$, expanding the initial and final hadronic states in terms of its Fock components. The computation is much simplified in the light-cone frame $P = (P^+, M^2/P^+, \vec{0}_\perp)$ where $H_{\mathrm{LF}} = P^+P^-$. We find

$$\begin{aligned} \mathcal{M}_H^2 = \sum_n \int [dx_i][d^2\mathbf{k}_{\perp i}] \sum_q \left(\frac{\mathbf{k}_\perp^2 + m_q^2}{x_q}\right)|\psi_{n/H}(x_i, \mathbf{k}_{\perp i})|^2 \\ + (\text{interactions}), \end{aligned} \tag{4}$$

plus similar terms for antiquarks and gluons ($m_g = 0$). The integrals in (4) are over the internal coordinates of the n constituents for each Fock state with phase space normalization

$$\sum_n \int [dx_i][d^2\mathbf{k}_{\perp i}]|\psi_{n/H}(x_i, \mathbf{k}_{\perp i})|^2 = 1. \tag{5}$$

The LFWF $\psi_n(x_i, \mathbf{k}_{\perp i})$ can be expanded in terms of $n-1$ independent position coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \ldots, n-1$, so that $\sum_{i=1}^n \mathbf{b}_{\perp i} = 0$. We can also express (4) in terms of the internal coordinates $\mathbf{b}_{\perp j}$ with $\mathbf{k}_\perp^2 \to -\nabla_{\mathbf{b}_\perp}^2$. The normalization is defined by

$$\sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j}|\psi_{n/H}(x_j, \mathbf{b}_{\perp j})|^2 = 1. \tag{6}$$

To simplify the discussion we will consider a two-parton hadronic bound state. In the limit $m_q \to 0$

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \frac{\mathbf{k}_\perp^2}{x(1-x)} |\psi(x, \mathbf{k}_\perp)|^2 + (\text{interactions})$$
$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\mathbf{b}_\perp \psi^*(x, \mathbf{b}_\perp)(-\nabla^2_{\mathbf{b}_\perp})\psi(x, \mathbf{b}_\perp)$$
$$+ (\text{interactions}). \quad (7)$$

It is clear from (7) that the functional dependence for a given Fock state is given in terms of the invariant mass

$$\mathcal{M}_n^2 = \left(\sum_{a=1}^n k_a^\mu\right)^2 = \sum_a \frac{\mathbf{k}^2_{\perp a} + m_a^2}{x_a} \to \frac{\mathbf{k}_\perp^2}{x(1-x)}, \quad (8)$$

the measure of the off-mass shell energy $\mathcal{M}^2 - \mathcal{M}_n^2$. Similarly, in impact space the relevant variable for a two-parton state is $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$. Thus, to first approximation LF dynamics depends only on the boost invariant variable $\mathcal{M}_n$ or ζ and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$: $\psi(x, \mathbf{k}_\perp) \to \phi(\zeta)$. We choose the normalization of the LF mode $\phi(\zeta) = \langle\zeta|\phi\rangle$

$$\langle\phi|\phi\rangle = \int d\zeta |\langle\zeta|\phi\rangle|^2 = 1. \quad (9)$$

We write the Laplacian operator in (7) in circular cylindrical coordinates (ζ, φ) with $\zeta = \sqrt{x(1-x)}|\mathbf{b}_\perp|$: $\nabla^2_\zeta = \frac{1}{\zeta}\frac{d}{d\zeta}(\zeta\frac{d}{d\zeta}) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$, and factor out the angular dependence of the modes in terms of the $SO(2)$ Casimir representation L^2 of orbital angular momentum in the transverse plane: $\phi(\zeta, \varphi) \sim e^{\pm iL\varphi}\phi(\zeta)$. Expressing the LFWF $\psi(x, \zeta)$ as a product of the LF mode $\phi(\zeta)$ and a prefactor $f(x)$, $\psi(x, \zeta) = \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}} f(x)$, we find

$$\mathcal{M}^2 = \int d\zeta \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}}$$
$$+ \int d\zeta \phi^*(\zeta) U(\zeta)\phi(\zeta)$$
$$= \int d\zeta \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta), \quad (10)$$

where the complexity of the interaction terms in the QCD Lagrangian is summed up in the addition of the effective potential $U(\zeta)$, which is then modeled to enforce confinement at some IR scale. The LF eigenvalue equation $H_{\rm LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle$ is thus a LF wave equation for ϕ

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = \mathcal{M}^2\phi(\zeta), \quad (11)$$

an effective single-variable light-front Schrödinger equation which is relativistic, covariant, and analytically tractable. From (4) one can readily generalize the equations to allow for the kinetic energy of massive quarks [5].

As the simplest example we consider a baglike model [12] where partons are free inside the hadron and the interaction terms effectively build confinement. The effective potential is a hard wall: $U(\zeta) = 0$ if $\zeta \leq 1/\Lambda_{\rm QCD}$ and $U(\zeta) = \infty$ if $\zeta > 1/\Lambda_{\rm QCD}$. However, unlike the standard bag model [12], boundary conditions are imposed on the boost invariant variable ζ, not on the bag radius at fixed time. If $L^2 \geq 0$ the LF Hamiltonian is positive definite $\langle\phi|H_{\rm LF}|\phi\rangle \geq 0$ and thus $\mathcal{M}^2 \geq 0$. If $L^2 < 0$ the LF Hamiltonian is unbounded from below and the particle "falls towards the center" [13]. The critical value corresponds to $L = 0$. The mode spectrum follows from the boundary conditions $\phi(\zeta = 1/\Lambda_{\rm QCD}) = 0$, and is given in terms of the roots of Bessel functions: $\mathcal{M}_{L,k} = \beta_{L,k}\Lambda_{\rm QCD}$. Since in the conformal limit $(U(\zeta) \to 0)$ Eq. (11) is equivalent to an AdS wave equation, the hard-wall LF model discussed here is equivalent to the hard-wall model of Ref. [14]. Likewise a two-dimensional oscillator with effective potential $U(\zeta) \sim \zeta^2$ is equivalent to the soft-wall model of Ref. [15] which reproduces the usual linear Regge trajectories.

We are now in a position to find out if the first-order approximation to light-front QCD discussed above admits an effective gravity description. To examine this question it is useful to study the structure of the equation of motion of p forms in AdS space, which for $p = 0$ and $p = 1$ represent spin 0 and spin 1 states, respectively. A p form in AdS is a totally antisymmetric tensor field $\Phi_{\ell_1\ell_2\cdots\ell_p}$ of rank p which couples to an interpolating operator $\mathcal{O}$ of dimension $d - p$ at the AdS boundary. Fermionic modes will be described elsewhere. In tensor notation the equations of motion for a p form are expressed as the set of $p + 1$ coupled equations [16]

$$[z^2\partial_z^2 - (d+1-2p)z\partial_z - z^2\partial_\rho\partial^\rho - (\mu R)^2 + d + 1 - 2p]\Phi_{z\alpha_2\cdots\alpha_p} = 0,$$
$$[z^2\partial_z^2 - (d-1-2p)z\partial_z - z^2\partial_\rho\partial^\rho - (\mu R)^2]\Phi_{\alpha_1\alpha_2\cdots\alpha_p} = 2z(\partial_{\alpha_1}\Phi_{z\alpha_2\cdots\alpha_p} + \partial_{\alpha_2}\Phi_{\alpha_1 z\cdots\alpha_p} + \cdots), \quad (12)$$

where μ is a $d+1$-dimensional mass, $\rho = 0, 1, \cdots, d-1$ and R is the AdS$_{d+1}$ radius.

Consider the plane-wave solution $\Phi_P(x, z)_{\alpha_1\cdots\alpha_p} = e^{-iP\cdot x}\Phi(z)_{\alpha_1\cdots\alpha_p}$, with four-momentum P_μ, invariant hadronic mass $P_\mu P^\mu = \mathcal{M}^2$ and spin indices α along the $3+1$ physical coordinates. For string modes with all indices along the Poincaré coordinates, $\Phi_{z\alpha_2\cdots\alpha_p} = \Phi_{\alpha_1 z\cdots\alpha_p} = \cdots = 0$, the coupled differential equations (12) reduce to the homogeneous wave equation

$$[z^2\partial_z^2 - (d-1-2p)z\partial_z + z^2\mathcal{M}^2 - (\mu R)^2]\Phi_{\alpha_1\cdots\alpha_p} = 0, \quad (13)$$

with conformal dimension Δ given by the relation $(\mu R)^2 = (\Delta - p)(\Delta - d + p)$.

Thus when the polarization indices are chosen along the physical $3+1$ Poincaré coordinates, the p-form equation

(12) becomes homogeneous and its polarization structure decouples; i.e., it is independent of the kinematical polarization structure of the indices. Thus it also describes the dynamics of a spin $J = p$ mode in AdS $\Phi(x, z)_{\mu_1 \cdots \mu_J}$, which is totally symmetric in all its indices. To prove this, consider the AdS wave equation (13) for a scalar mode Φ ($p = 0$), and define a spin-J field $\Phi_{\mu_1 \cdots \mu_J}$ with shifted dimensions: $\Phi_J(z) = (z/R)^{-J}\Phi(z)$, and normalization [17]

$$R^{d-2J-1} \int_0^{z_{\max}} \frac{dz}{z^{d-2J-1}} \Phi_J^2(z) = 1. \tag{14}$$

The shifted field Φ_J obeys the equation of motion

$$[z^2 \partial_z^2 - (d - 1 - 2J) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2] \Phi_J = 0, \tag{15}$$

where the fifth dimensional mass is rescaled according to $(\mu R)^2 \to (\mu R)^2 - J(d - J)$. One can then construct an effective action in terms of high spin modes $\Phi(x, z)_{\mu_1 \cdots \mu_J}$ with only the physical degrees of freedom [15].

Upon the substitution $z \to \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} \Phi_J(\zeta)$ in (15) we recover for $d = 4$ the QCD light-front wave equation (11) in the conformal limit

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} \right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}, \tag{16}$$

where the fifth-dimensional mass is not a free parameter but scales according to $(\mu R)^2 = -(2 - J)^2 + L^2$. In the hard-wall model there is a total decoupling of the total spin J. For $L^2 \geq 0$ the LF Hamiltonian is positive definite $\langle \phi_J | H_{\rm LF} | \phi_J \rangle \geq 0$ and we find the stability bound $(\mu R)^2 \geq -(2 - J)^2$. For $J = 0$ the stability condition gives the bound $(\mu R)^2 \geq -4$. The quantum-mechanical stability conditions discussed here are thus equivalent to the Breitenlohner-Freedman stability bound in AdS [18]. The scaling dimensions are $\Delta = 2 + L$ independent of J in agreement with the twist scaling dimension of a two-parton bound state in QCD.

We have shown that the use of the invariant coordinate ζ in light-front QCD which is related to the fundamental constituent structure, allows the separation of the dynamics of quark and gluon binding from the kinematics of constituent spin and internal orbital angular momentum. The result is a single-variable LF Schrödinger equation which determines the spectrum and LFWFs of hadrons for general spin and orbital angular momentum. This LF wave equation serves as a semiclassical first approximation to QCD and is equivalent to the equations of motion which describe the propagation of spin-J modes on AdS. Remarkably, the AdS equations correspond to the kinetic energy terms of the partons inside a hadron, whereas the interaction terms build confinement and correspond to the truncation of AdS space. As in this approximation there are no quantum corrections, there are no anomalous dimensions. This may explain the experimental success of power-law scaling in hard exclusive reactions where there are no indications of the effects of anomalous dimensions. In the hard-wall model there is total orbital decoupling from hadronic spin J and thus the LF excitation spectrum of hadrons depends only on orbital and principal quantum numbers. In this model the mass dependence has the linear form: $\mathcal{M} \sim 2n + L$. In the soft-wall model the usual Regge behavior is found $\mathcal{M}^2 \sim n + L$, where the slope in L and n is identical. Both models predict the same multiplicity of states for mesons and baryons as observed experimentally [19]. As in the Schrödinger equation, the semiclassical approximation to light-front QCD described in this Letter does not account for particle creation and absorption, and thus it is expected to break down at short distances where hard gluon exchange and quantum corrections become important. However, one can systematically improve the holographic approximation by diagonalizing the QCD light-front Hamiltonian on the AdS-QCD basis.

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