Using the Beam-Echo Effect for Generation of Short-Wavelength Radiation

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We propose to use the mechanism of an echo effect previously observed in hadron accelerators for up-frequency conversion of density modulation in an electron beam. We show that, for generation of high harmonics, this method is much more efficient in comparison with the currently used approach. A one-dimensional model of the effect is developed which allows us to optimize the amplitude of the modulation for a given harmonic number.

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The development of high-power short-wavelength free electron lasers (FELs) over the last years holds a promise of providing researches with a light source capable to generate coherent ultra short pulses of radiation in the spectral range from infrared to hard x rays. The two main approaches that are currently pursued in the design of the next generation light sources are the self-amplified spontaneous emission (SASE) FELs [1,2], and the high-gain harmonic generation (HGHG) scheme [3,4]. One of the main advantages of the HGHG over the SASE FEL is that, by using up-frequency conversion of the initial seed signal, HGHG allows us to produce not only transversely, but also temporally coherent pulses. In contrast, the SASE radiation starts from initial shot noise in the beam, with the resulting radiation having an excellent spatial coherence, but a rather poor temporal one.

Unfortunately, the standard HGHG suffers an essential drawback in that a single stage frequency conversion allows only a limited frequency multiplication factor [4]. This leads to multistage approach for x-ray production seeded at an ultraviolet wavelength [5], with a significant complication in the overall design. Some improvements in the cascading efficiency can be achieved by modification of the original idea, as demonstrated in a recent Letter [6]. However, generation of harmonic numbers in the range of 10 to 20 still requires a large energy modulation of the beam and deteriorates the beam properties as a lasing medium. In addition, the laser power for the seed light scales as a square of the modulation amplitude [7] and becomes expensive with an increased modulation amplitude.

The goal of this Letter is to point out a new physical mechanism that has several important advantages over the classical approach to the frequency cascading. This mechanism is intimately linked to the *echo* effect in circular accelerators [8], hence the name. In the echo experiments [9], it was demonstrated that modulating the beam energy with the frequency ω_1 and, after some delay, with the frequency ω_2 leads, after a "sleep" time, to an echo signal at the frequency $m\omega_1 - k\omega_2$, where m and k are integer numbers [10]. The experiments [9] were carried out using the modulation frequencies in the range from megahertz to hundreds of megahertz. As we will show below, the echo effect can also be implemented at much higher laser frequencies. Before describing a setup for such an experiment, we will quickly review a traditional way to modulate the beam current using a laser and an undulator tuned to the laser frequency.

The setup for such a device is shown in Fig. 1(a). An electron bunch with an average energy E_0 interacts with a laser beam of frequency ω in a short undulator (called a modulator) with the resonant frequency tuned to ω. Typically, the bunch length is much larger than the laser wavelength, and one can locally consider a longitudinally uniform beam, neglecting variation of the beam current over the distance of several laser wavelength. We assume an initial Gaussian beam energy distribution with the variance σ_E and use the variable $p = (E - E_0)/\sigma_E$ for the dimensionless energy deviation of a particle. The initial distribution function of the beam is $f(p) = N_0(2\pi)^{-1/2}e^{-p^2/2}$, where N_0 is the number of particles per unit length of the beam.

After passage through the undulator, the beam energy is modulated with the amplitude ΔE so that the final dimensionless energy deviation p' is related to the initial one p by the equation $p' = p + A\sin(qz)$, where $A = \Delta E/\sigma_E$, $q = \omega/c$, and z is the longitudinal coordinate in the beam. The distribution function after the interaction with the laser becomes $f(\zeta, p) = N_0(2\pi)^{-1/2}\exp[-(p - A\sin\zeta)^2/2]$ where we now use the dimensionless variable $\zeta = qz$.

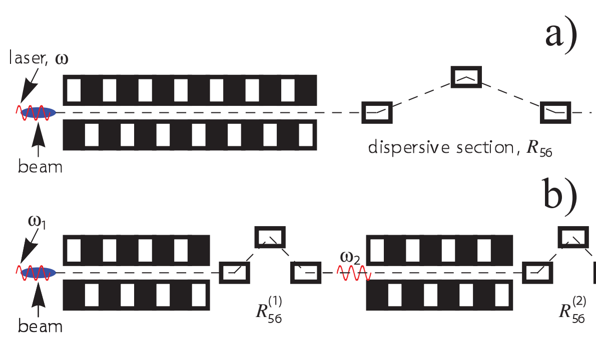

FIG. 1 (color online). A traditional system (a) consists of an undulator and a dispersive section. The proposed new scheme (b) includes two consecutive modulators.

Sending then the beam through a dispersive system with the dispersive strength R_{56} converts the longitudinal z into z', $z' = z + R_{56} p \sigma_E / E_0$, and makes the distribution function

$$f(\zeta, p) = \frac{N_0}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}[p - A\sin(\zeta - Bp)]^2\right], \quad (1)$$

where $B = R_{56} q \sigma_E / E_0$ (for notational clarity, we dropped primes in the arguments of f). Integration of f over p gives the distribution of the beam density N as a function of the coordinate ζ,

$$N(\zeta) = \int_{-\infty}^{\infty} dp f(\zeta, p). \quad (2)$$

Noting that this density is a periodic function of ζ, one can expand it into Fourier series

$$\frac{N(\zeta)}{N_0} = 1 + \sum_{k=1}^{\infty} b_k \cos(k\zeta + \psi_k), \quad (3)$$

where the coefficient b_k is the amplitude of the harmonic k. Calculations with the function (1) give an analytical expression for b_k (see, e.g., [4] and references therein)

$$|b_k| = 2e^{-(1/2)B^2k^2} |J_k(ABk)|, \quad (4)$$

where J_k is the Bessel function of order k.

It follows from this formula that if $A \lesssim 1$, then b_k decays exponentially when k increases. For a numerical illustration, let us take $A = 1$; then the modulation in the forth harmonic is approximately equal to 1%, $|b_4| = 0.01$, and $|b_5| = 1.9 \times 10^{-3}$. To obtain 10% modulation in 10th harmonic ($|b_{10}| = 0.1$), A is required to be not less then 5.9, which would result in a sixfold increase in the energy spread of the beam after the passage through the modulator.

To overcome the low efficacy of the standard approach to modulate the beam, we propose to use a modulation analogous to the experiments [9], albeit extended into the region of optical frequencies. The setup is depicted in Fig. 1(b). After passing through the system described above, the beam is sent through one more energy modulator and an additional dispersive section. To distinguish between the first and the second modulators and dispersive sections, we will now use indices 1 and 2, respectively (so that the distribution function (1) is now written with A and B replaced by A_1 and B_1). In general, the frequency ω_2 of the second laser beam can differ from the frequency of the first laser ω_1.

The final distribution function at the exit from the second dispersion section can be easily found by applying consecutively two more transformations to (1), similar to the derivation outlined above. The first of these two transformations corresponding to the modulation of the beam energy with dimensionless amplitude A_2 is $p' = p + A_2 \sin(q_2 z + \phi)$, where ϕ is a phase of the second laser beam, and the second one corresponding to the passage through the second dispersive element is $z' = z + pR_{56}^{(2)} \sigma_E / E_0$. The resulting final distribution function is (we again drop primes in the arguments of f):

$$f(\zeta, p) = \frac{N}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\{p - A_2 \sin(K\zeta - KB_2 p + \phi) - A_1 \sin[\zeta - (B_1 + B_2)p + A_2 B_1 \sin(K\zeta - KB_2 p + \phi)]\}^2\right], \quad (5)$$

where we now use the notation $\zeta = q_1 z$, $B_1 = R_{56}^{(1)} q_1 \sigma_E / E_0$, $B_2 = R_{56}^{(2)} q_1 \sigma_E / E_0$, and $K = q_2 / q_1$.

To simplify analysis, we first consider the limit of small energy modulations, A_1, $A_2 \ll 1$. At the same time, we assume that the product $A_2 B_1$ may not be small. We then expand the distribution function (5) keeping only linear terms in A_1 and A_2. It is easy to see that the result of such an expansion gives three terms: the zero order term $N_0 (2\pi)^{-1/2} e^{-(p^2/2)}$, the term $N_0 (2\pi)^{-1/2} e^{-(p^2/2)} p A_2 \times \sin(K\zeta - KB_2 p + \phi)$ linear in A_2, and finally, the term, which we denote f_3,

$$f_3(\zeta, p) = \frac{N_0}{\sqrt{2\pi}} e^{-(p^2/2)} p A_1 \sin[\zeta - (B_1 + B_2)p + A_2 B_1 \sin(K\zeta - KB_2 p + \phi)]. \quad (6)$$

This last term is responsible for the echo effect, so we will focus on this term only. Using the mathematical identity $\sin[\phi_1 + x\sin(\phi_2)] = \sum_{k=-\infty}^{\infty} J_k(x) \sin(\phi_1 + k\phi_2)$ and integrating Eq. (6) over p gives

$$\int_{-\infty}^{\infty} f_3(\zeta, p) dp = \sum_{k=-\infty}^{\infty} c_k \cos(\zeta + kK\zeta + k\phi), \quad (7)$$

with

$$c_k = -A_1 [B_1 + B_2(kK + 1)] J_k(B_1 A_2) e^{-(1/2)[B_1 + B_2(kK+1)]^2}. \quad (8)$$

Each term in sum (7) corresponds to modulation with the wave number q_{echo} equal to

$$q_{\text{echo}} = kq_2 + q_1, \quad (9)$$

and we see that large values of k give up-frequency conversion of the wave number of the second laser q_2. Note that k takes both positive and negative values; a negative q_{echo} means a modulation with a wavelength $2\pi/|q_{\text{echo}}|$.

For a given B_1, one can maximize the absolute value of $|c_k|$ by choosing

$$B_2 = -\frac{B_1 \pm 1}{kK + 1}, \quad (10)$$

which gives

$$|c_k| = \frac{A_1}{\sqrt{e}} |J_k(A_2 B_1)|, \quad (11)$$

where $e = 2.71$. Note that since k takes both positive and negative values, Eq. (10) allows for solutions with $R_{56}^{(1)}$ and $R_{56}^{(2)}$ having either the same or opposite signs. For a given amplitude of the modulation A_1, one can now further maximize the value of the Bessel function in this expression by optimizing the value of B_1, that is by properly choosing $R_{56}^{(1)}$. For $k > 4$, with an accuracy better than a few percent, the maximum of the Bessel function J_k is approximately equal to $0.67/k^{1/3}$, and it is attained for the value of its argument equal to $k + 0.81k^{1/3}$. This gives the maximum value of $|c_k|$

$$|c_k| \approx \frac{0.41A_1}{|k|^{1/3}}, \tag{12}$$

for

$$|B_1| \approx \frac{|k|}{A_2}(1 + 0.81|k|^{-2/3}). \tag{13}$$

Since we assume that $A_2 \ll 1$, it follows from the above equation that $|B_1|$ should be much larger than unity for the validity of our approximation. Note that although our definitions of $A_{1,2}$ and $B_{1,2}$ involve the energy spread in the beam σ_E, the optimal settings for the dimensional factors $R_{56}^{(1)}$ and $R_{56}^{(2)}$, as follows from Eqs. (13) and (11), do not depend on σ_E. They do depend however on the amplitude of the energy modulation $A_2\sigma_E$ in the second modulator.

Equation (12) demonstrates a remarkable feature of the echo modulation—a very slow decay $\propto |k|^{-1/3}$ with the harmonic number k. This kind of dependence is in sharp contrast with the exponential decrease of the bunching factor with k demonstrated by Eq. (4) for $A_1 \lesssim 1$.

One can show that in the general case of arbitrary values of parameters A_1 and A_2, the wave number for the echo signal is $q_{\rm echo} = kq_2 + mq_1$, where k and m are arbitrary (positive or negative) integer numbers.

From a practical point of view, a particularly interesting case is represented by selection of equal frequencies for both modulators, $q_1 = q_2$. This case can be realized with a single laser system by splitting the laser light and sending it to both undulators. Remarkably, this case allows an analytical solution for arbitrary values of parameters A_1 and A_2. The echo wave number in this case is equal to an integer numbers of q_1, and hence the dimensionless beam density can again be expanded in Fourier series (3). Omitting a lengthy derivation, we present here the final result for the amplitudes of the modulation in this case:

$$b_k = 2\left|\sum_{m=-\infty}^{\infty} e^{im\phi} J_{-m-k}\{A_1[(m+k)B_1 + kB_2]\} \times J_m(kA_2B_2)e^{-(1/2)[(m+k)B_1+kB_2]^2}\right|. \tag{14}$$

Equation (14) is too complicated for analytical maximization of b_k. However, if A_1 and A_2 are not very large, one can use the results of the small amplitude analysis Eq. (10) and (13) as a first approximation, with the following numerical optimization of Eq. (14).

We will demonstrate this approach for the case $A_1 = A_2 = 1$, that is when the energy modulation in both modulators is equal to the original energy spread in the beam. We will first try to maximize the amplitude of the 10th harmonic using Eqs. (10) and (13) for B_1 and B_2. Assuming that they are of the same sign [which means a negative $k = -11$ in Eq. (9)], we find $B_1 = 12.8$ and $B_2 = 1.18$. Equation (12) predicts the amplitude $b_{10} = 0.18$. This prediction is then corrected by scanning the values of B_1 and B_2 in the vicinity of the above values. The plot of b_{10} as a function of B_1 obtained this way with the help of Eq. (14) is shown in Fig. 2 for several different values of B_2. As it follows from this plot, the maximal value of b_{10} is actually attained when $B_1 = 12.1$ and $B_2 = 1.3$, and is equal to 0.16, in reasonably good agreement with the small A approximation. Note also a weaker echo effect in the region from 5 to 7 of values of B_1. It is important to emphasize here that, as an additional numerical analysis shows, the maximum value of b_{10} is insensitive to the value of the phase ϕ.

Figure 3 shows the evolution of the phase space of the beam as it travels through the system, for parameters discussed above. These pictures demonstrate a simple physical mechanism behind the echo effect. A large value of $R_{56}^{(1)}$ in the first modulator leads to "shredding" of the beam phase space in the longitudinal direction and generation of multiple "beamlets" in the phase space. Each beamlet is imaged as a stripe in the top right picture of Fig. 3. It has an almost uniform density distribution in the z direction and an energy spread much smaller than that of the original beam. The role of the second modulator consists in a simultaneous compression of all beamlets with a relatively modest value of $R_{56}^{(2)}$.

As an example of practical parameters for a possible application of the proposed scheme, we estimated the required strengths of the dispersion elements using the beam and the seed laser parameters of the

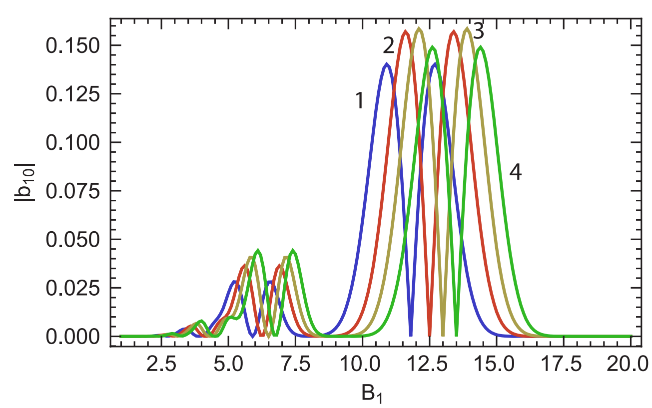

FIG. 2 (color online). The bunching factor for $k = 10$ as a function of parameter B_1 for four different values of B_2: $1 - B_2 = 1.18$, $2 - B_2 = 1.25$, $3 - B_2 = 1.3$, $4 - B_2 = 1.35$.

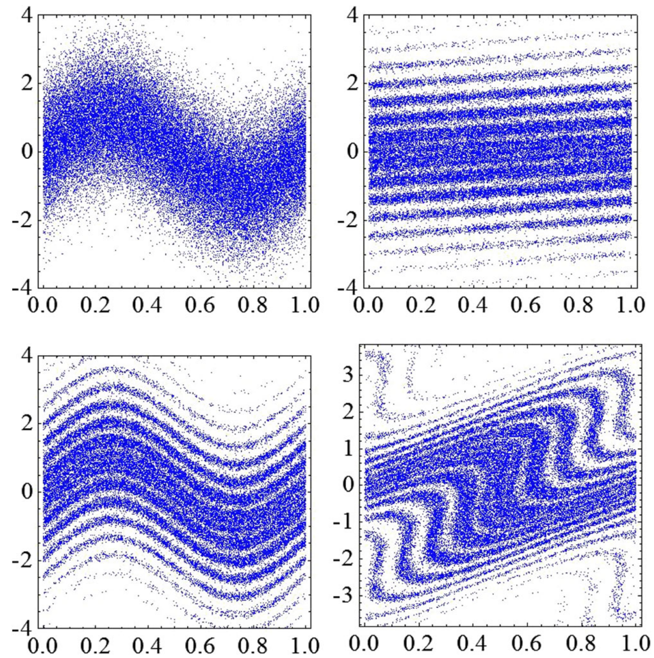

FIG. 3 (color online). The phase space of the beam after the first undulator (top left), the first dispersive element (top right), the second undulator (bottom left), and the second dispersive element (bottom right). Horizontal axes in the plots are ζ, and the vertical axes are p.

ELETTRA@FERMI project in Trieste [11]. The relevant parameters are: the beam energy $E_0 = 1.2$ GeV, the beam energy spread $\sigma_E = 150$ keV, and the laser wavelength 0.24 μm. For the example of an optimized 10th harmonic setup discussed above with $A_1 = A_2 = 1$, one finds that the maximum amplitude of the modulation 0.16 requires $R_{56}^{(1)} = 4$ mm and $R_{56}^{(2)} = 0.37$ mm.

In another example, increasing 3 times the modulation in the first undulator, $A_1 = 3$ (and leaving $A_2 = 1$), allows one to reach the relative amplitude of the 24th harmonic (corresponding to the wavelength of 10 nm) of 0.2. The required dispersive elements for this case are: $R_{56}^{(1)} =$ 8.2 mm and $R_{56}^{(2)} = 0.35$ mm.

To illustrate a possible performance of the echo modulated beam in the previous example (with $A_1 = 3$ and $A_2 =$ 1), we simulated FEL radiation of such a beam at the wavelength of 10 nm using the upgraded code Genesis [12]. In this simulation, the first modulator was chosen to be 135 cm long with the undulator period length of 15 cm. The input laser beam had a waist of 310 microns and the peak power of 64 MW. The second modulator was 45 cm long and had just 3 undulator periods, with the laser parameters being the same as in the first modulator.

The evolution of the radiation power in the undulator is shown in Fig. 4. The peak power of the 24th harmonic radiation exceeds 1.6 GW and it saturates after 5 undulator sections (the total magnet length is 12.5 m).

There are several practical physical effects that are left behind our simplified one-dimensional analysis of the echo

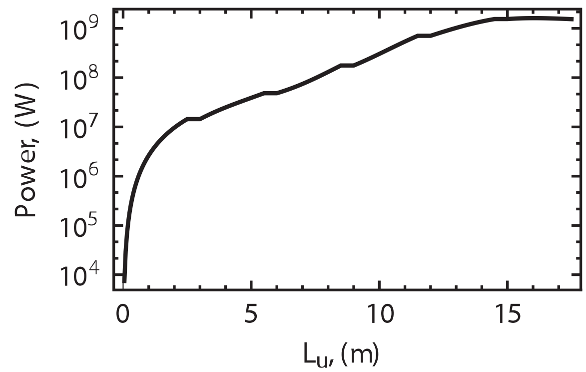

FIG. 4. FEL power versus the undulator length L_u.

effect. They include coupling between transverse and longitudinal degrees of freedom, coherent and incoherent radiation of the bunched beam in dispersion elements, and additional energy spread introduced by the radiation, radial inhomogeneity of the laser beam in the undulator, and its effect on the amplitude of the echo effect. These and other effects should be taken in the account in the design and optimization of the echo experiment setup. It is likely that they will determine the ultimate shortest wavelength of modulation achievable with the proposed approach.

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