## Surface Dead Layer for Quasiparticles Near a Mott Transition

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Electron quasiparticles are progressively weakened by correlations upon approaching a continuos Mott metal-insulator transition in a bulk solid. We show that corresponding to the bulk weakening, a dead layer forms below the surface of the solid, where quasiparticles are exponentially suppressed. The surface dead layer depth is a bulk property and diverges when the Mott transition is approached. We describe this phenomenon in a Hubbard model within a self-consistent Gutzwiller approximation. The photoemission data of Rodolakis *et al.* [Phys. Rev. Lett. **102**, 066805 (2009)] in  $V_2O_3$  appear to be in accord with this physical picture.

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The Mott transition [1] where a lattice of atoms or molecules abandons the metallic state and turns insulating due to electron-electron repulsion has a very intuitive physical explanation. Electron motion in the lattice is caused by kinetic energy and is favored by electron-ion energy because the same electron can feel in this way the attraction of more than one nucleus. It is opposed by Coulomb repulsion, higher for itinerant electrons due to the higher chance of collision during motion. When the first two terms (which form the band energy) prevail, the system is a band metal; otherwise the electrons localize, and we have an insulator. Despite the conceptual simplicity, the properties of Mott insulators and especially of the strongly correlated metallic state close to a Mott transition remain quite difficult to capture both theoretically and experimentally. Theoretically, the reason is that the Mott transition is a collective phenomenon, which escapes single-particle or mean field theories such as Hartree-Fock or density-functional theory-local-density approximations. Experimentally, complications such as magnetism, lattice distortions, etc., often conspire to mask the nature of metal-insulator transitions.

Fresh progress on this problem has come in the past two decades with dynamical mean field theory (DMFT) [2], which in the standard Hubbard model showed that, as the electron-electron repulsion parameter U increases, the initial band metal evolves first to a strongly correlated metal well before the Mott transition. In the strongly correlated metal the electron spectral function undergoes a profound change exhibiting well formed, localized Mott-Hubbard bands coexisting with delocalized, propagating quasiparticles—the latter narrowly centered in energy near the Fermi level. Only successively do the quasiparticles disappear as the Mott transition takes place when U is increased to reach  $U = U_c$ . This intriguing prediction—simultaneous metallic and insulating features, though on well separated energy scales—has stimulated a consider-

able experimental effort to reveal coexisting quasiparticles and Mott-Hubbard bands in strongly correlated metals [3-12]. A large amount of work has been done on  $V_2O_3$ , the prototype compound where a Mott transition was first discovered [13] and studied theoretically [14,15]. At the metal-insulator transition of  $(V_{1-r}Cr_r)_2O_3$ , early photoemission experiments [16–19] failed to reveal the sharp quasiparticle peak predicted by DMFT. The electronic spectrum was simply dominated by the lower Mott-Hubbard band with barely a hint of metallic weight at the Fermi energy. A similar puzzle was actually reported much earlier in *f*-electron materials [20], and soon ascribed to large surface effects in the presence of strong correlations [21], the same conclusion reached by more recent photoemission experiments [3,4,6,7,11,12,22]. In V2O3, using higher kinetic energy photoelectrons, whose escape depth is larger, a prominent quasiparticle peak coexisting with incoherent Mott-Hubbard bands was eventually observed [5,10,23]. Quasiparticle suppression in surface-sensitive probes was attributed [23] to surface-modified Hamiltonian parameters, the reduced atomic coordination pushing the surface closer to the Mott transition than the underlying bulk. Larger electronic correlations at the surface have been discussed by several authors through ad hoc formulations of DMFT [24–26]. There is general agreement on intrinsically different quasiparticle properties near a surface, even if all Hamiltonian parameters were to remain identically the same up to the outermost atomic layer [24].

This conclusion, although not unexpected, raises a more fundamental question. A metal does not possess any intrinsic length scale at long distances other than the Fermi wavelength. Thus an imperfection like a surface can only induce at large depth a power-law decaying disturbance such as that associated with Friedel's oscillations. Since one does not expect Luttinger's theorem to break down, even in a strongly correlated metal these oscillations should be controlled by the same Fermi wavelength as in

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the absence of interaction, irrespective of the proximity of the Mott transition. However, a strongly correlated metal does possess an intrinsic energy scale, the parametric distance of the Hamiltonian from the Mott transition, where that distance could be associated with a length scale. The surface as a perturbation should alter the quasiparticle properties within a depth corresponding to that length, a bulk property increasing near the Mott transition, unlike the Fermi wavelength that remains constant. In this respect, it is not a priori clear whether the recovery of bulk quasiparticles' spectral properties with increasing depth should be strictly power law, compatible with the common view of a metal as an inherently critical state of matter, or whether it should be exponential, as one would expect by regarding the Mott transition as any other critical phenomena where power laws emerge only at criticality. We find here in the simple half-filled Hubbard model that the quasiparticle spectral weight below the surface is actually recovered exponentially inside the bulk with a length scale that depends only on the bulk properties and diverges approaching the continuous Mott transition.

To address the generic surface features of a strongly correlated metal, we study the simplest Hamiltonian exhibiting a Mott transition, namely, the Hubbard model at half filling:

$$H = -t \sum_{\langle \mathbf{R}\mathbf{R}' \rangle \sigma} c^{\dagger}_{\mathbf{R}\sigma} c_{\mathbf{R}'\sigma} + \text{H.c.} + \sum_{\mathbf{R}} U_{\mathbf{R}} n_{\mathbf{R}\uparrow} n_{\mathbf{R}\downarrow}, \quad (1)$$

where  $\langle {\bf R} {\bf R}' \rangle$  are nearest neighbor sites,  $c^{\dagger}_{{\bf R}\sigma}$  creates an electron at site **R** with spin  $\sigma$ , and  $n_{\mathbf{R}\sigma} = c_{\mathbf{R}\sigma}^{\dagger} c_{\mathbf{R}\sigma}$ . Conventionally, the Mott transition of the half-filled Hubbard model is studied restricting to the paramagnetic sector of the Hilbert space [2,14,15] so as to avoid spurious effects due to magnetism. We assume a cubic lattice of spacing a with periodic boundary conditions in x and ydirections and open boundary conditions in the z direction, in an N-layer slab geometry with two surfaces at z = 0 and z = Na. The Hubbard electron-electron interaction parameter  $U_{\mathbf{R}}$  is U everywhere except at the top surface layer (z = 0), where it takes a generally higher value  $U_s > U$ . In this way we can compare effects at the ideal lower surface (z = Na), where  $U_{Na} = U$ , with the more correlated upper surface (z = 0). DMFT [2] offers an ideal tool to attack this model in the paramagnetic sector, assuming a local selfenergy that depends on the layer index z [24–26]. However, a full DMFT calculation of this sort is numerically feasible only for a small number of layers, e.g., N = 20 as in Ref. [27], making the critical regime near the Mott transition hard to access. As a useful approximate alternative, one can resort to the so-called linearized DMFT [24,28] to treat moderately larger sizes. We decided to adopt a different method altogether, the Gutzwiller variational approximation [29]. Despite its limitations (static mean field character, inability to describe the insulating phase), it is known to provide a good description of quasiparticle properties close to the Mott transition [2] with very little size limitations and great simplicity and flexibility (it may treat intersite interactions, any kind of lattice, etc.). We study (1) by means of a Gutzwiller-type variational wave function

$$|\Psi\rangle = \prod_{\mathbf{R}} \mathcal{P}_{\mathbf{R}} |\Psi_0\rangle, \qquad (2)$$

where  $|\Psi_0\rangle$  is a paramagnetic Slater determinant. The operator  $\mathcal{P}_{\mathbf{R}}$  has the general expression

$$\mathcal{P}_{\mathbf{R}} = \sum_{n=0}^{2} \lambda_{n}(z) |n, \mathbf{R}\rangle \langle n, \mathbf{R}|, \qquad (3)$$

where  $|n, \mathbf{R}\rangle\langle n, \mathbf{R}|$  is the projector at site  $\mathbf{R} = (x, y, z)$  onto configurations with *n* electrons, and  $\lambda_n(z)$  are layer-dependent variational parameters. We calculate average values on  $|\Psi\rangle$  using the so-called Gutzwiller approximation [30,31] (for details see, e.g., Ref. [29], whose notations we use hereafter), and require that

$$\langle \Psi_0 | \mathcal{P}_{\mathbf{R}}^2 | \Psi_0 \rangle = 1, \quad \langle \Psi_0 | \mathcal{P}_{\mathbf{R}}^2 n_{\mathbf{R}\sigma} | \Psi_0 \rangle = \langle \Psi_0 | n_{\mathbf{R}\sigma} | \Psi_0 \rangle.$$
(4)

Because of particle-hole symmetry,  $\langle \Psi_0 | n_{\mathbf{R}\sigma} | \Psi_0 \rangle = 1/2$ , from which it follows that Eq. (4) is satisfied if  $\lambda_2(z) = \lambda_0(z)$ ,  $\lambda_1(z)^2 = 2 - \lambda_0(z)^2$ . The average value of (1) is then [29,32]

$$E = \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$
  
=  $\sum_{\mathbf{R}} \frac{U_{\mathbf{R}}}{4} \lambda_0(z)^2 - t \sum_{\langle \mathbf{R} \mathbf{R}' \rangle \sigma} R(z) R(z') \langle \Psi_0 | c_{\mathbf{R}\sigma}^{\dagger} c_{\mathbf{R}'}$  (5)

where  $R(z) = \lambda_0(\mathbf{R})\sqrt{2 - \lambda_0(\mathbf{R})^2}$  plays the role of a wave function renormalization factor. Its square is the actual quasiparticle weight,  $Z(z) = R^2(z)$ , since Fermi liquid theory renormalizes  $c_{\mathbf{R}\sigma}^{\dagger}$  to  $R(z)c_{\mathbf{R}\sigma}^{\dagger}$  [note also that the Hubbard bands, carrying the remaining weight (1-Z(z)), do not appear explicitly]. One can invert this equation to express  $\lambda_0(z)$  as a function of R(z), which become the actual variational parameters together with the Slater determinant  $|\Psi_0\rangle$ . In order to minimize E in Eq. (5) we assume that the Slater determinant  $|\Psi_0\rangle$  is built with single-particle wave functions that, because of the slab geometry, have the general expression  $\phi_{\epsilon \mathbf{k}_{\parallel}}(\mathbf{R}) =$  $\sqrt{1/A}e^{i\mathbf{k}_{\parallel}\cdot\mathbf{R}}\phi_{\epsilon\mathbf{k}_{\parallel}}(z)$ , where A is the number of sites per layer and  $\mathbf{k}_{\parallel}$  the momentum in the x-y plane. The stationary value of E with respect to variation of  $\phi_{\epsilon \mathbf{k}_{\parallel}}(z)$  and R(z) corresponds to the coupled equations

$$\epsilon \phi_{\epsilon \mathbf{k}_{\parallel}}(z) = R(z)^{2} \epsilon_{\mathbf{k}_{\parallel}} \phi_{\epsilon \mathbf{k}_{\parallel}}(z) - tR(z) \\ \times \sum_{p=\pm} R(z+pa) \phi_{\epsilon \mathbf{k}_{\parallel}}(z+pa), \quad (6)$$

$$R(z) = \frac{4\sqrt{1 - R(z)^2}}{U(z)A} \sum_{\epsilon \mathbf{k}_{\parallel}}^{\text{occupied}} \left[ -2R(z)\epsilon_{\mathbf{k}_{\parallel}}\phi_{\epsilon \mathbf{k}_{\parallel}}(z)^2 + t\phi_{\epsilon \mathbf{k}_{\parallel}}(z)\sum_{p=\pm}R(z + pa)\phi_{\epsilon \mathbf{k}_{\parallel}}(z + pa) \right], \quad (7)$$

where  $\epsilon_{\mathbf{k}_{\parallel}} = -2t(\cos k_x a + \cos k_y a)$  and the sum in Eq. (7) runs over all pairs of  $(\epsilon, \mathbf{k}_{\parallel})$  that are occupied in the Slater determinant  $|\Psi_0\rangle$ . The first equation has the form of a Schrödinger equation that the single-particle wave functions  $\phi_{\epsilon \mathbf{k}_{\parallel}}(z)$  must satisfy, depending parametrically on R(z). The second equation has been intentionally cast in the form of a map  $R_{i+1}(z) = F[R_i(z), R_i(z+a), R_i(z-a)]$ a)] whose fixed point we have verified to coincide with the actual solution of (7) in the parameter region of interest. Equations (6) and (7) can be solved iteratively as follows. First solve the Schrödinger equation at fixed  $R_i(z)$ , next find the new  $R_{j+1}(z)$  using the old  $R_j(z)$  and the newly determined wave functions  $\phi_{\epsilon \mathbf{k}_{\parallel}}(z)$ . With the new  $R_{j+1}(z)$ , repeat the above steps and iterate until convergence. Because of the large number of variational parameters, this iterative scheme is much more efficient than-while fully equivalent to—a direct minimization of E, Eq. (5).

In Fig. 1 we plot  $Z(z) = R^2(z)$ , experimentally the total spectral weight carried by quasiparticles, calculated as a function of z (in units of the lattice spacing a) for  $U_s = 20t$ , for two different bulk values 15t and 15.98t of U below the critical Mott-transition value  $U_c = 16t$ .



FIG. 1 (color online). The quasiparticle weight  $Z(z) = R^2(z)$  as a function of the coordinate *z* perpendicular to the surface (in units of the lattice spacing) for a 100-layer slab. The interaction parameter at z = 0 is  $U_s = 20t$ , while the bulk *U* is 15.98*t* in the upper panel and 15*t* in the lower one (while  $U_c = 16$ ). The insets show the behavior of *Z* close to the two surfaces, the highest curve corresponding to the bulklike surface, the other to  $U_s = 20t$ .

Coming from the bulk, the quasiparticle weight Z(z) decreases monotonically on approaching both surfaces, where it attains much smaller values than in bulk. As expected, the more correlated surface has a smaller quasiparticle weight, Z(0) < Z(N). Note, however, that so long as the slab interior (the "bulk") remains metallic, the surface quasiparticle weight never vanishes no matter how large  $U_s$  [24]. Mathematically, this follows from Eq. (7), which is not satisfied by choosing R(0) = 0 while  $R(z > 0) \neq 0$ . Physically, some metallic character can always tunnel from the interior to the surface, so long as the bulk is metallic. The quasiparticle weight approaches the surface with upward curvature when U is closest to  $U_c$ , upper panel in Fig. 1, whereas the behavior is linear well below  $U_c$ , as found earlier within linearized DMFT [24]. We note that an upward curvature is in better accord with photoemission spectra of Rodolakis *et al.* on  $V_2O_3$  [33]. The curvature becomes more manifest if the number of surface layers where  $U_s > U$  is increased, as shown in Fig. 2. Next, we analyze the dependence of R(z) at large distance  $1 \ll z \ll N/2$  below the surface. As Fig. 3 shows, we find no trace of a power law, and R is best fit by an exponential  $R(z) = R_{\text{bulk}} + (R_{\text{surf}} - R_{\text{bulk}})e^{-z/\lambda}$ , where  $R_{\text{bulk}}$  is the bulk value (a function of U only) and  $R_{\text{surf}} < R_{\text{bulk}}$ .  $R_{\text{surf}}$  now depends on both U and on  $U_s$ , and vanishes only when  $R_{\text{bulk}}$  does at  $U > U_c$ . A detailed study by varying U and  $U_s$  shows that the surface "dead layer" thickness  $\lambda$  depends only on bulk properties and diverges at the Mott transition as  $\lambda \propto (U_c - U)^{-\nu}$ . Numerically we find  $\nu = 0.53 \pm 0.3 \simeq 0.5$ , a typical mean field exponent [27]. The same conclusion can actually be drawn by analyzing Eqs. (6) and (7) deep inside the bulk. We note that the precise behavior at the outermost surface layers would in a real system depend on details, such as lack of electronhole symmetry and/or surface dipoles, not included in our model. However, we believe that the exponential behavior and its divergence at a continuous Mott transition should be generic and universal, and thus independent of these and other details. In conclusion, we have shown in a simple approximation the existence in the Hubbard model of strongly correlated metals of a "dead layer" below the



FIG. 2 (color online). Quasiparticle weight dependence on the distance z from the surface for two different bulk U values and for two cases: one where only the first layer has  $U_s = 20t > U$  (upper curve in each panel), the other where five surface layers have  $U_s = 20t$ .



FIG. 3 (color online). Log scale plot of  $R_{\text{bulk}} - R(z)$  versus z for U = 15.99,  $U_s = 20t$  and for different thicknesses of the slab N = 60, 100, 200, 400.

crystal surface. Within this layer-whose depth is a bulk property and not a surface property of the metal-the quasiparticle weight decays exponentially on approaching the surface. The dead layer thickness  $\lambda$  inversely depends on the distance in parameter space to the bulk continuous Mott transition, where it diverges critically. The physical significance of  $\lambda$  is that of a correlation length of the bulk metallic state, where the quasiparticle weight acts as an order parameter, critically vanishing at a continuous Mott transition. Like other features of the Hubbard model, this result should, we believe, carry over to real systems with an ideal Mott transition, not obscured by, e.g., symmetry breaking phenomena like magnetic order, the critical region not preempted by strong first order jumps, like that in the  $\alpha$ - $\gamma$  transition of Ce. It could therefore apply to high temperature  $V_2O_3$  near the paramagnetic metal-insulator weakly first order line, notwithstanding complications including orbital degeneracy, Hund's rules, or lattice couplings [34]. One may thus expect a surface dead layer in the metal phase of  $V_2O_3$ , with thickness increasing (although not diverging because of the first order transition) on approaching the Mott-transition line. The associated Letter by Rodolakis et al. [33] reports photoemission evidence which lends some support to this picture. It is also interesting to note that an anomalously thick subsurface dead layer has long been observed in mixed valent YbInCu<sub>4</sub> [35], with a depth not smaller that 60 Å [36].

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