Circumstantial Evidence for a Soft Nuclear Symmetry Energy at Suprasaturation Densities

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Within an isospin- and momentum-dependent hadronic transport model, it is shown that the recent FOPI data on the π^-/π^+ ratio in central heavy-ion collisions at SIS/GSI energies [Willy Reisdorf *et al.*, Nucl. Phys. A **781**, 459 (2007)] provide circumstantial evidence suggesting a rather soft nuclear symmetry energy $E_{\rm sym}(\rho)$ at $\rho \geq 2\rho_0$ compared to the Akmal-Pandharipande-Ravenhall prediction. Some astrophysical implications and the need for further experimental confirmations are discussed.

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The density dependence of the nuclear symmetry energy $E_{\rm sym}(\rho)$ is critical for understanding not only the structure of rare isotopes [1] and heavy-ion reactions [2–6], but also many interesting issues in astrophysics [6–9]. To determine the $E_{\rm sym}(\rho)$ and thus the equation of state (EOS) of neutron-rich nuclear matter has been a long-standing goal of both nuclear physics and astrophysics. While both fields have some promising tools for probing the $E_{\rm sym}(\rho)$ over a broad density range, they all have some limitations. Thus, only by combining carefully complementary information from both fields will we have ultimately a good understanding about the $E_{\rm sym}(\rho)$. While significant progress has been made over the last few years in constraining the $E_{\rm sym}(\rho)$ at subsaturation densities using terrestrial nuclear laboratory data [6], still very little is known about the $E_{\rm sym}(\rho)$ at suprasaturation densities.

In fact, the high density behavior of the $E_{\rm sym}(\rho)$ has long been regarded as the most uncertain property of dense neutron-rich nuclear matter [10,11]. Presently, at suprasaturation densities, even the trend of the $E_{\rm sym}(\rho)$, i.e., whether it increases continuously or decreases at some point with the increasing density, is still controversial. While many microscopic and/or phenomenological many-body theories using various interactions, such as the Relativistic Mean-Field [12] and Brueckner-Hartree-Fock approaches [13], predict that the $E_{\rm sym}(\rho)$ increases continuously at all densities, an approximately equal number of other models including the Variational Many-Body theory [14–16] and the Dirac-Brueckner-Hartree-Fock [17] predict that the $E_{\rm sym}(\rho)$ first increases to a maximum and then may start decreasing at certain supra-saturation densities depending on the interactions used [18]. Additionally, the nonrelativistic Hartree-Fock (HF) approach using many Skyrme [1,19–21], Gogny [22], Myers-Swiatecki [23], and the density-dependent M3Y interactions [24,25] or the nonrelativistic Thomas-Fermi approach using the Seyler-Blanchard interaction [26] also lead to decreasing $E_{\rm sym}(\rho)$ starting at some suprasaturation densities. Thus, currently the theoretical predictions on the $E_{\rm sym}(\rho)$ at suprasaturation densities are extremely diverse. Therefore, to make further progress in determining the $E_{\rm sym}(\rho)$ at suprasaturation densities, what is most critically needed is some guidance from experiments. In this Letter, we report circumstantial evidence suggesting a rather soft nuclear symmetry energy $E_{\rm sym}(\rho)$ at $\rho \geq 2\rho_0$ compared to the Akmal-Pandharipande-Ravenhall (APR) prediction [27] based on a transport model (IBUU04 [28]) analysis of the recent FOPI data on the π^-/π^+ ratio from central heavy-ion collisions at SIS/GSI energies [29].

The isospin and momentum-dependent mean-field potential (MDI) used in the IBUU04 reads [30]

$$U(\rho,\delta,\mathbf{p},\tau) = A_u(x)\frac{\rho_{\tau'}}{\rho_0} + A_l(x)\frac{\rho_\tau}{\rho_0} + B\left(\frac{\rho}{\rho_0}\right)^{\sigma}(1-x\delta^2) - 8x\tau\frac{B}{\sigma+1}\frac{\rho^{\sigma-1}}{\rho_0^{\sigma}}\delta\rho_{\tau'} + \sum_{t=\tau,\tau'}\frac{2C_{\tau,t}}{\rho_0}\int d^3\mathbf{p}'\frac{f_t(\mathbf{r},\mathbf{p}')}{1+(\mathbf{p}-\mathbf{p}')^2/\Lambda^2} \quad (1)$$

where ρ_n and ρ_p denote the neutron ($\tau = 1/2$) and proton ($\tau = -1/2$) density, respectively, and $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ is the isospin asymmetry of the nuclear medium. All parameters in the above equation can be found in Refs. [28]. The variable x is introduced to mimic different forms of the $E_{\rm sym}(\rho)$ predicted by various many-body theories without changing any property of the symmetric nuclear matter and the $E_{\rm sym}(\rho)$ at normal density ρ_0. The $E_{\rm sym}(\rho)$ with x values of 1, 0.5, 0, and −1 are shown in Fig. 1. By setting $x = 1$, one recovers the HF prediction using the original Gogny force [30]. As it is well known, the latter predicts a soft $E_{\rm sym}(\rho)$ decreasing with increasing density at suprasaturation densities. For comparisons, shown also are the $E_{\rm sym}(\rho) = 12.5\rho/\rho_0 + 12.7(\rho/\rho_0)^{2/3}$ used in the IQMD (Isospin-Dependent Quantum Molecular Dynamics) [31] and the well-known APR prediction [27]. The latter has been widely used in calibrating other model calculations. In terms of reproducing the experimental

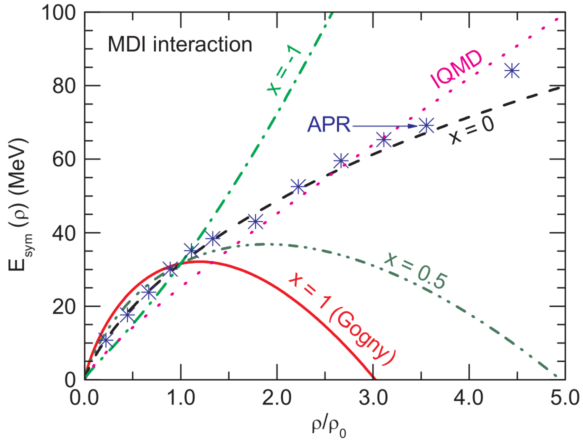

FIG. 1 (color online). Density dependence of nuclear symmetry energy predicted by APR, used in IQMD and the present work (MDI).

data, the IBUU04 model has had modest successes so far [6]. While the NSCL/MSU isospin diffusion data [32] allowed us to limit the $E_{sym}(\rho)$ at subsaturation densities to be between that with $x = 0$ and $x = -1$ [33,34], the same model parameter sets underpredict [35] significantly the double neutron/proton ratio of Ref. [36]. Nevertheless, it is interesting to mention that the above limited range of the $E_{sym}(\rho)$ for $\rho \leq \rho_0$ is consistent with that extracted very recently from analyses using the ImQMD (Improved QMD) model which can reproduce both the isospin diffusion data and the double neutron/proton ratio simultaneously [37]. It is also worth noting that the APR prediction for the $E_{sym}(\rho)$ at subsaturation densities lies right between that with $x = 0$ and $x = -1$.

Among the most sensitive probes of the $E_{sym}(\rho)$ at suprasaturation densities proposed in the literature [6], the π^-/π^+ ratio in heavy-ion collisions is particularly promising. Qualitatively, the advantage of using the π^-/π^+ ratio is evident within both the $\Delta(1232)$ resonance model [38] and the statistical model [39] for pion production. Assuming only first chance inelastic nucleon-nucleon collisions produce pions and neglecting their reabsorptions, the Δ resonance model predicts a primordial π^-/π^+ ratio of $(\pi^-/\pi^+)_{res} \equiv (5N^2 + NZ)/(5Z^2 + NZ) \approx (N/Z)^2_{dense}$, where the N and Z are neutron and proton numbers in the participant region of the reaction. The π^-/π^+ ratio is thus a direct measure of the isospin asymmetry $(N/Z)_{dense}$ of the dense matter formed. The latter is determined by the $E_{sym}(\rho)$ through the dynamical isospin fractionation [40], namely, the high (low) density region is more neutron-rich (poor) with a lower $E_{sym}(\rho)$ at suprasaturation densities. Since effects of the $E_{sym}(\rho)$ are obtained mainly through the corresponding nuclear mean-field which dominates the dynamics of heavy-ion reactions at relatively low energies, based on the resonance model one thus expects the π^-/π^+ probe to be most effective at beam energies near the pion production threshold $E^{\pi}_{th} \approx$ 300 MeV. On the other hand, assuming pions have gone through multiple production-reabsorption cycles and reached thermal-chemical equilibrium, the statistical model predicts that $\pi^-/\pi^+ \propto \exp[2(\mu_n - \mu_p)/T] = \exp[8\delta E_{sym}(\rho)/T]$, where T is the temperature. Thus, in this model the π^-/π^+ ratio measures directly the $E_{sym}(\rho)$ at the pion freeze-out. Meanwhile, at energies much higher than the E^{π}_{th} where pions are abundant, the reaction dynamics is dominated by scatterings among all hadrons instead of the nuclear mean-field [41]. Therefore, one expects that the π^-/π^+ probe becomes less effective at very high energies where other observables, such as, the neutron-proton differential flow [42,43], may be more useful for probing the high density $E_{sym}(\rho)$. More quantitatively and realistically compared to the above two idealized models, several hadronic transport models have shown consistently that the π^-/π^+ ratio is indeed sensitive to the $E_{sym}(\rho)$ [43–45] especially near the E^{π}_{th}. Moreover, by varying separately the $E_{sym}(\rho)$ at sub and suprasaturation densities in IBUU04 simulations we found that the π^-/π^+ ratio in collisions near the E^{π}_{th} is much more sensitive to the variation of the $E_{sym}(\rho)$ at suprasaturation rather than subsaturation densities.

Recently, Reisdorf *et al.* studied systematically the π^-/π^+ ratio in $^{40}Ca + ^{40}Ca$, $^{96}Ru + ^{96}Ru$, $^{96}Zr + ^{96}Zr$, and $^{197}Au + ^{197}Au$ reactions using the FOPI detector at SIS/GSI [29]. Their π^-/π^+ data are the most extensive and accurate ones available in the literature, thus providing us the best opportunity so far to extract the $E_{sym}(\rho)$ at suprasaturation densities.

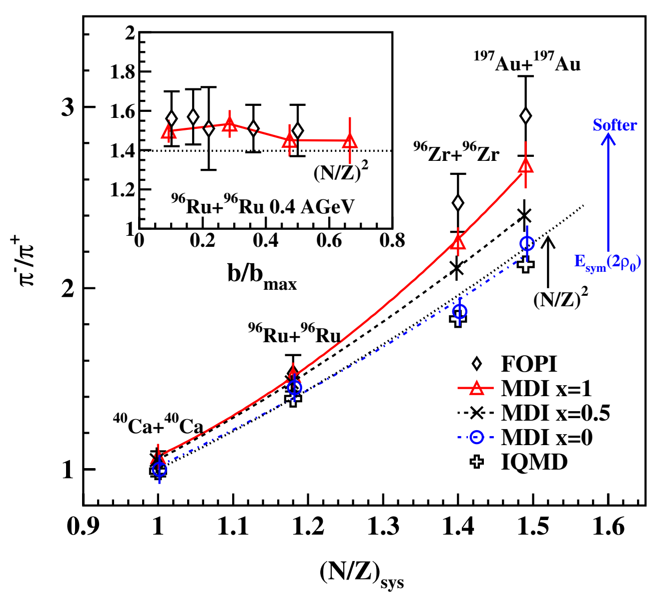

FIG. 2 (color online). The π^-/π^+ ratio as a function of the neutron/proton ratio of the reaction system at 0.4A GeV with the reduced impact parameter of $b/b_{max} \leq 0.15$. The inset is the impact parameter dependence of the π^-/π^+ ratio for the $^{96}Ru + ^{96}Ru$ reaction at 0.4A GeV.

Shown in Fig. 2 are the calculated π^-/π^+ ratios in comparison with the FOPI data at $0.4A$ GeV with the reduced impact parameter $b_0 \equiv b/b_{max} \leq 0.15$ as a function of the neutron/proton ratio of the reaction system. The inset shows the π^-/π^+ ratio as a function of b_0 for the ^{96}Ru + ^{96}Ru reaction at $0.4A$ GeV. It is seen that both the data and the calculations exhibit very weak b_0 dependence for the π^-/π^+ ratio, even for midcentral reactions where we found that the multiplicities of both π^- and π^+ vary appreciably with the b_0. For the symmetric ^{40}Ca + ^{40}Ca and the slightly asymmetric ^{96}Ru + ^{96}Ru reactions, calculations using both $x = 1$ and $x = 0$ can well reproduce the FOPI data. Interestingly, for the more neutron-rich reactions of ^{96}Zr + ^{96}Zr and ^{197}Au + ^{197}Au calculations with $x = 0$, 0.5 and 1 are clearly separated from each other. The FOPI data favor clearly the calculation with $x = 1$. As shown in Fig. 1, with $x = 1$ the $E_{sym}(\rho)$ at $\rho \geq 2\rho_0$ reached in the reaction is very small, leading to a rather high N/Z in the participant region and thus the larger π^-/π^+ ratio observed. For comparisons, the IQMD result from Ref. [29] is also shown. As seen in Fig. 1, the $E_{sym}(\rho)$ used in the IQMD, the MDI $E_{sym}(\rho)$ with $x = 0$ and the APR prediction are all very close to each other for $\rho_0 < \rho \leq 3\rho_0$. Thus, not surprisingly, the π^-/π^+ ratios from the IQMD and the IBUU04 with $x = 0$ are very close, too. Moreover, they both grow approximately according to the scaling $\pi^-/\pi^+ \approx (N/Z)^2$ predicted by the $\Delta(1232)$ resonance model but fall far below the FOPI data. Our calculations with varying values of x indicate that a strong symmetry potential is at work at this energy as one expects. While not perfectly reproduced by our calculations even with $x = 1$, the FOPI data suggest unambiguously that the $E_{sym}(\rho)$ is rather soft at supra-saturation densities compared to the APR prediction.

Since the most central Au + Au collision is most sensitive to the $E_{sym}(\rho)$ among the reactions considered here, we now turn to the excitation functions of the pion yield and the π^-/π^+ ratio in these reactions. Figure 3 displays the excitation function of the pion multiplicity per participant M_π/A_{part}. In order to compare with the data directly, the total pion multiplicity is obtained from the charged pions only by using $1.5(M_{\pi^-} + M_{\pi^+})$ and the number of participants is calculated from $0.9A_{sys}$ where A_{sys} is the total mass of the colliding system as done in the data analysis [29]. It is seen that the results of the calculations are in reasonably good agreement with the available data. We notice here that, unlike the π^-/π^+ ratio which has the advantage of reducing significantly not only systematic errors but also effects of isoscalar nuclear potentials, the pion yield also depends appreciably on the EOS of symmetric nuclear matter [29,38]. The inset illustrates the time evolution of the central density with the maximum value varying from about $2.2\rho_0$ to $3.5\rho_0$ for the beam energy from 0.25 to $1.2A$ GeV.

Shown in Fig. 4 are the excitation functions of the π^-/π^+ ratio calculated with the IBUU04 and the IQMD in comparison with the FOPI data. First of all, the decreasing trend of the π^-/π^+ ratio with the increasing beam energy is well reproduced by all calculations. Most interestingly, IBUU04 calculations with $x = 1$ can best describe the FOPI data over the whole energy range. Moreover, the π^-/π^+ ratio is more sensitive to the $E_{sym}(\rho)$ at lower beam energies as expected.

Putting together all available information, we infer that the $E_{sym}(\rho)$ reaches a maximum somewhere between ρ_0 and $2\rho_0$ before it starts decreasing at higher densities. This indicates the importance of the development at NSCL/MSU [46] and RIKEN [47] of Time Projection Chambers

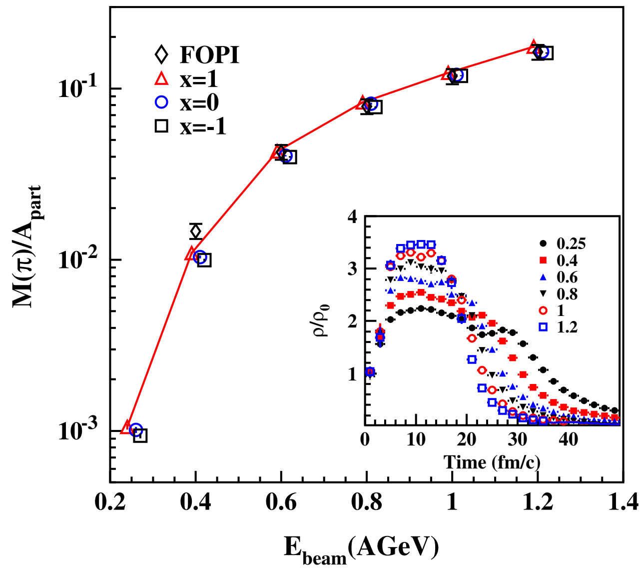

FIG. 3 (color online). Excitation function of the pion multiplicity per participant M_π/A_{part} in the most central Au + Au collisions. The inset shows the time evolution of the central density.

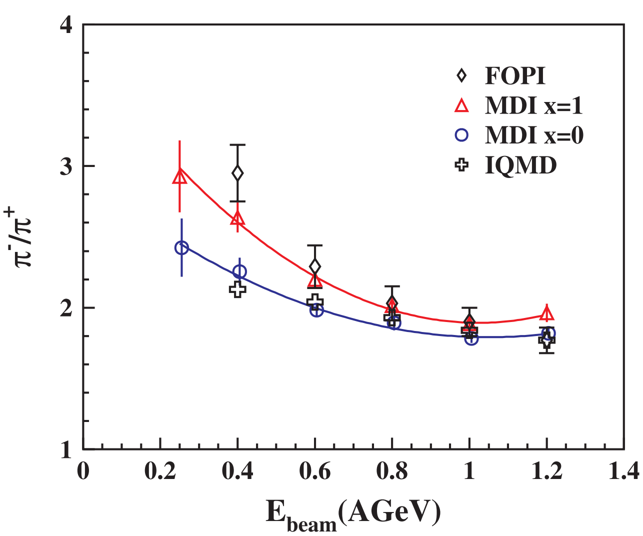

FIG. 4 (color online). Excitation function of the π^-/π^+ ratio in the most central Au + Au collisions.

(TPC) to measure π^-/π^+ ratios in heavy-ion reactions induced by radioactive beams from about 120 to 400A MeV. These measurements around the $E_{\rm th}^{\pi}$ can help constrain the $E_{\rm sym}(\rho)$ at densities from about ρ_0 to $2\rho_0$. Moreover, the neutron-proton differential flow measurements planned at SIS/GSI [48] and possibly also at the future FAIR/GSI will not only test our findings here but may also enable the exploration of the $E_{\rm sym}(\rho)$ at even higher densities. Generally speaking, there are limitations on how much one can learn reliably about the high density $E_{\rm sym}(\rho)$ from high energy heavy-ion collisions. For instance, to reach higher baryon densities by using more energetic beams, more baryon and meson resonances are excited. Moreover, with the increase of the beam energy, the collision becomes more transparent [49]. Therefore, the hot and dense system formed is further away from the pure nucleonic matter in thermodynamical equilibrium. It thus becomes more difficult to model and relate properties of this system with the $E_{\rm sym}(\rho)$ of nucleonic matter. In this regard, neutron stars are unique laboratories for probing the $E_{\rm sym}(\rho)$ of cold high density nucleonic matter. On the other hand, astrophysical studies of neutron stars have their own limitations and challenges. It is thus important to do cross checking between the two fields. If our conclusion on the $E_{\rm sym}(\rho)$ is confirmed by more experimental and theoretical studies, it not only posts a serious challenge to some nuclear many-body theories but also has important implications on several critical issues in nuclear astrophysics, such as, the cooling of proto-neutron stars, the possible formation of polarons due to the isospin separation instability [10,18], the possible formation of quark droplets [50] and hyperons [26] in the core of neutron stars.

In summary, circumstantial evidence for a rather soft nuclear symmetry energy $E_{\rm sym}(\rho)$ at $\rho \geq 2\rho_0$ compared to the APR prediction was obtained from analyzing the recent FOPI data on the π^-/π^+ ratio using the IBUU04 transport model. Further experimental and theoretical confirmations of our findings are important.

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