Driven Quantum Coarsening

Camille Aron,[1] Giulio Biroli,[2] and Leticia F. Cugliandolo[1]
[1]Université Pierre et Marie Curie - Paris VI, LPTHE UMR 7589, 4 Place Jussieu, 75252 Paris Cedex 05, France
[2]Institut de Physique Théorique, CEA, IPhT, F-91191 Gif-sur-Yvette, CNRS, URA 2306, France
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We study the driven dynamics of quantum coarsening. We analyze models of M-component rotors coupled to two electronic reservoirs at different chemical potential that generate a current threading through the system. In the large M limit, we derive the dynamical phase diagram as a function of temperature, strength of quantum fluctuations, voltage, and coupling to the leads. We show that the slow relaxation in the ordering phase is universal. On large time and length scales, the dynamics are analogous to stochastic classical ones, even for the quantum system driven out of equilibrium at zero temperature. We argue that our results apply to generic driven quantum coarsening.

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Phase transitions are central to condensed matter and statistical physics. Initially, emphasis was put on classical and quantum *equilibrium* phase transitions. Later, attention moved to *nonequilibrium* phase transitions in which quantum fluctuations can be neglected. These are realized when a system is forced in a nonequilibrium steady state (by a shear rate, an external current, etc.) [1,2] or when it just fails to relax (e.g., after a quench) and displays aging phenomena [3,4]. The study of steady states in small quantum systems driven out of equilibrium [5] has been recently boosted by their relevance for nanodevices. In contrast, the effect of a drive on a *macroscopic* system close to a quantum phase transition is a rather unexplored subject. Some works have focused on nonlinear transport properties close to an (equilibrium) quantum phase transition [6–8]. Others have studied the effect of a drive on the critical properties [9–11]. However, a global understanding of phase transitions in the parameter space T (temperature), V (drive), Γ (strength of quantum fluctuations), and the rôle played by the environment, is still lacking. Furthermore, experiments in $2d$ electronic systems [12,13] show interesting features in the *relaxation* toward the quantum nonequilibrium steady state (QNESS) but these have not been addressed theoretically yet (except for [14]).

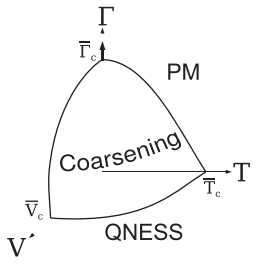
A number of intriguing questions arise in the context of driven quantum phase transitions, some of which are: How long does it take to reach the QNESS after one of the parameters T, V, Γ is changed? Do the systems always relax to the QNESS or, as for classical systems, do deep quenches in the T, V, Γ phase diagram lead to aging phenomena and glassy dynamics? What are the properties of the latter "doubly nonequilibrium" dynamics? Are quantum quenches, obtained by changing V and Γ at $T = 0$, different from their classical counterpart?

The aim of this Letter is to answer these questions for a class of analytically tractable models, systems of M-component quantum rotors that encompass an infinite range spin glass and its $3d$ pure counterpart that models coarsening. Models of quantum rotors are nontrivial but still relatively simple and provide a coarse-grained description of Bose-Hubbard models and double layer antiferromagnets [15]. The out of equilibrium drive is provided by two electron reservoirs that induce a current flowing through the system. In the simplest setting , each rotor is coupled to two independent reservoirs. We analyze the out of equilibrium dynamics in the large M limit. We find a phase transition (Fig. 1) between a QNESS ($V \neq 0$) and an ordered phase, we study its critical properties, and we discuss the effect of the environment.

The model we focus on is an infinite-range quantum disordered system made of N M-component rotors interacting via random Gaussian distributed couplings, J_{ij}, with zero mean and variance J^2/N. Its Hamiltonian is

$$H_S = \frac{\Gamma}{2\hbar^2 M}\sum_{i=1}^{N}\mathbf{L}_i^2 - \sum_{i<j}J_{ij}\mathbf{n}_i\mathbf{n}_j, \quad \mathbf{n}_i^2 = M \ \ \forall\, i. \qquad (1)$$

n_i^μ are the M components of the i-th rotor, and $L_i^{\mu\nu} = n_i^\mu p_i^\nu - n_i^\nu p_i^\mu$, with $p_i^\mu = -i\hbar\partial/\partial n_i^\mu$, are the $M(M-1)$ components of the i-th generalized angular momentum operator with $\mathbf{L}_i^2 = \sum_{\mu<\nu}(L_i^{\mu\nu})^2$ [15,16]. Γ controls the strength of quantum fluctuations; as $\Gamma \to 0$, the model



$$\Gamma_c(V=0) \sim \overline{T}_c - T$$
$$\Gamma_c(T=0) \sim \text{NA}$$
$$T_c(\Gamma=0) \sim \text{NA}$$
$$T_c(V=0) \sim \left(\overline{\Gamma}_c - \Gamma\right)^{1/2}$$
$$V_c(T=0) \sim \left(\Gamma_c - \Gamma\right)^{1/2}$$
$$V_c(\Gamma=0) \sim \left(\overline{T}_c - T\right)^{1/2}$$

FIG. 1. Nonequilibrium phase diagram for reservoirs with a much larger variation scale at the Fermi level than all other energy scales ($\hbar\omega_F \gg J$). Close to the critical point $\bar{V}_c = V_c(T = \Gamma = 0)$, the critical lines are nonanalytical (NA). Close to $\bar{T}_c$ and $\bar{\Gamma}_c$, they are power laws with exponents given in the right. The arrow above $\bar{\Gamma}_c$ indicates that the critical surface is pulled up by increasing the coupling to the leads.

approaches the classical Heisenberg fully connected spin-glass. In the large M limit, it is equivalent to the quantum fully connected $p = 2$ spherical spin glass [17,18]. The classical mapping to ferromagnetic coarsening in the $O(\mathcal{N})$ model with $\mathcal{N} \to \infty$ [4] holds, as we shall show, for the quantum model as well.

The system is coupled to two independent and noninteracting "left" (L) and "right" (R) electronic reservoirs in equilibrium at different chemical potential $\mu_L = \mu$ and $\mu_R = \mu + eV$ and the same temperature T. R and L reservoirs act as source and drain, respectively. The details of their Hamiltonian are not important since only the electronic Green functions matter in the small rotor-bath coupling we concentrate on. We focus on free electrons with the same symmetric density of states, $\rho_L(\epsilon) = \rho_R(\epsilon) = \rho(\epsilon)$, centered at $\epsilon = 0$, and with typical variation scale $\hbar\omega_F$ at the Fermi level.

Each rotor is coupled nonlinearly to its (double) electron bath. For example, for $M = 3$ rotors, we take $H_{SB} = \frac{1}{\mathcal{M}}\sum_{i\alpha ll'\gamma\gamma'}\hbar\omega_{\gamma\gamma'}n_i^{\alpha}[c^{\dagger}_{liL\gamma}\frac{\sigma^{\alpha}_{ll'}}{2}c_{l'iR\gamma} + L \leftrightarrow R]$ where $c^{\dagger}_{liR(L)\gamma}$, $c_{liR(L)\gamma}$ are the fermionic operators of the $R(L)$ reservoirs, $\hbar\omega_{\gamma\gamma'}$ is the electron-bath couplings that we choose to be constant, $\hbar\omega_{\gamma\gamma'} = \hbar\omega_c$, σ^{α} are the Pauli matrices ($\alpha = 1, ..., M$), and $\gamma = 1, ..., \mathcal{M}$ is the fermion label inside the reservoirs.

System and reservoirs are uncoupled at time $t < 0$ and evolve with $H = H_S + H_B + H_{SB}$ at $t > 0$. The density matrix at $t = 0$, $\varrho = \varrho_S \otimes \varrho_L \otimes \varrho_R$, provides the initial condition. ϱ_S, ϱ_L, and ϱ_R correspond to equilibrium of the system at temperature $T_0 \gg 1$, and the L and R reservoirs at temperature T and chemical potential μ and $\mu + eV$, respectively. For simplicity, ϱ_S is taken to be the identity; this choice is equivalent to any other one uncorrelated with disorder [19].

We analyze the $t > 0$ dynamics by using the Schwinger-Keldysh formalism yielding a functional-integral representation of the Heisenberg evolution [19–21]. Each field carries a $\pm$ index associated to the forward and backward evolution. The action corresponding to Eq. (1) is

$$S_S = \sum_{a=\pm} a \int dt\left[\frac{\hbar^2}{2\Gamma}\sum_i(\dot{\mathbf{n}}_{ia})^2 + \sum_{i<j}J_{ij}\mathbf{n}_{ia}\mathbf{n}_{ja}\right].$$

The path integral runs over paths such that $\mathbf{n}_{ia}^2(t) = M$, $\forall\ i, a, t$. One may lift this constraint by introducing auxiliary fields $\lambda_{ia}(t)$ and adding $S_\lambda = \sum_{a=\pm}\frac{a}{2}\times \int dt\sum_i\lambda_{ia}(t)(\mathbf{n}_{ia}^2(t) - M)$ to the action. After expanding the system-leads interaction up to second order in $g \equiv \omega_c/\omega_F$, integrating out the fermionic fields, and taking the large M limit, we obtain a Feynman-Vernon-like action for the rotors. The detailed computation [21] confirms that several system-reservoir coupling that preserve the $O(M)$ symmetry and the addition of different LL and RR couplings do not modify our results qualitatively. In short, we obtain

$$S_{SB} = -\frac{1}{2}\sum_{ab=\pm}\int dtdt'\Sigma^B_{ab}(t,t')\sum_i\mathbf{n}_{ia}(t)\mathbf{n}_{ib}(t'),$$

$$\Sigma^B_{ab}(t,t') = -iab\hbar\omega_c^2[G^R_{ab}(t,t')G^L_{ba}(t',t) + L \leftrightarrow R].$$

The electronic Green functions are $G_{ab}(t,t') \equiv -i\langle\mathcal{T}\psi_a(t)\psi_b^{\dagger}(t')\rangle$ with $\psi_a(t)$, $\psi_a^{\dagger}(t)$ the fermionic fields and $\mathcal{T}$ the time-ordering operator on the closed contour. It is convenient to use retarded, $G^B_R = (G^B_{+-} - G^B_{++})/\hbar$, advanced, $G^B_A = (G^B_{-+} - G^B_{++})/\hbar$, and Keldysh, $G^B_K = i(G^B_{++} + G^B_{--})/2$, Green functions. The Σ's transform in a similar way. For identical reservoirs at temperature T and chemical potential μ ($V = 0$), the self-energy components verify the usual fluctuation-dissipation relation of a standard *bosonic* bath $\Sigma^B_K(\omega) = \hbar\coth(\beta\hbar\omega/2)\Im\Sigma^B_R(\omega)$, with $\beta = 1/T$ and $k_B = 1$.

Collecting all contributions, the total action, $S = S_S + S_\lambda + S_{SB}$, is $O(MN)$. Since the zero-source generating functional equals one, one can simply compute its average over quenched randomness [19] and use a saddle point evaluation that becomes exact in the large M and N limits. At the saddle point, λ is a spatially homogenous function with time dependence determined by the condition $\langle\mathbf{n}_{ia}^2(t)\rangle = M$ with the average taken over S [21].

The macroscopic dynamic order parameters are the symmetric two-time correlation and instantaneous linear response that in the operator formalism are defined as $MC(t,t_w) \equiv \langle\{\mathbf{n}_i(t), \mathbf{n}_i(t_w)\}/2\rangle$ and $MR(t,t_w) \equiv \delta\langle\mathbf{n}_i(t)\rangle/\delta\mathbf{h}_i(t_w)|_{h=0} = -1/\hbar\langle[\mathbf{n}_i(t), \mathbf{n}_i(t_w)]\rangle\theta(t - t_w)$. The field $\mathbf{h}_i$ couples linearly to the i-th rotor, and the last identity is the Kubo formula valid in linear response. The exact Schwinger-Dyson equations then read

$$\mathcal{D}(t)R(t,t_w) = \delta(t - t_w) + \int_{t_w}^{t} dt''\Sigma_R(t,t'')R(t'',t_w),$$

$$\mathcal{D}(t)C(t,t_w) = \int_0^t dt''\Sigma_R(t,t'')C(t'',t_w) + \int_0^{t_w} dt''\Sigma_K(t,t'')R(t_w,t''), \quad (2)$$

with $\mathcal{D}(t) = \hbar^2\Gamma^{-1}\partial_t^2 + \lambda(t)$, the retarded and Keldysh self-energies given by $\Sigma_R = \Sigma^B_R + J^2R$ and $\Sigma_K = \Sigma^B_K + J^2C$, and $\lambda(t) = -\hbar^2\Gamma^{-1}\partial^2_{t^2}C(t,t_w \to t^-) + \int_0^t dt''[\Sigma_R(t,t'')C(t,t'') + \Sigma_K(t,t'')R(t,t'')]$.

In the QNESS, the dynamics are stationary but do not verify the fluctuation-dissipation theorem (FDT) for $V \neq 0$. The Lagrange multiplier approaches a constant, $\lambda(t) \to \lambda_\infty$. The linear response satisfies a closed equation that once Fourier transformed, reads $R(\omega) = -[\hbar^2\Gamma^{-1}\omega^2 - \lambda_\infty + \Sigma^B_R(\omega) + J^2R(\omega)]^{-1}$ the physical solution of which satisfies $R(\omega \to \infty) = 0$. The correlation is given by $C(\omega) = \Sigma_K(\omega)|R(\omega)|^2$, and the spherical constraint, $C(t,t) = 1$, implies

$$\int_0^\infty \frac{d\omega}{2\pi}\frac{\Sigma^B_K(\omega)}{\Im\Sigma^B_R(\omega)}\Im R(\omega) = 1/2. \quad (3)$$

The phase transition occurs when $R(\omega = 0) = \int dtR(t)$

ceases to be real, indicating that the stationary condition necessary to Fourier transform is no longer valid. Concomitantly, the derivatives of $R(\omega)$ in $\omega = 0$ diverge, and hence the real-time response function shows a power law decay. This happens when $\lambda_\infty = \lambda_c = 2J + \Sigma_R^B(\omega = 0)$. Inserting λ_c in Eq. (3), we obtain the equation for the critical manifold in the T, V, Γ space (for a given g and $\hbar\omega_F$). We shall derive the critical manifold for different reservoirs in full detail in [21]; we summarize here some of the salient features.

We first consider $g \to 0$ *after* the longtime limit such that the asymptotic regime has been established and we take $\hbar\omega_F$ much larger than any other energy scale ($\hbar\omega_F \to \infty$). For $\Gamma = V = 0$, we recover the classical critical temperature, $\bar{T}_c = J$ [16,17]. At $V = T = 0$, we obtain $\bar{\Gamma}_c = (3\pi/4)^2 J$, as for the $p = 2$ quantum spherical model in equilibrium [16] and its dynamics coupled to an equilibrium oscillator bath [18]. Finally, the critical point $\bar{V}_c$ on the $\Gamma = T = 0$ line is determined by

$$\int_\mu^{\mu + e\bar{V}_c} d\epsilon \left[\frac{\rho_L(\epsilon)}{2J} - \dot{\rho}_L(\epsilon) \right] \rho_R(\epsilon) = \rho_L(\mu)\rho_R(\mu).$$

In the large variation scale limit $\hbar\omega_F \gg J$, we find $e\bar{V}_c = 2J$ for the $\rho_L = \rho_R = \rho$ symmetric case, with $\mu = 0$ and differentiable at the origin. The form of the critical lines are shown in Fig. 1.

As for finite $\hbar\omega_F$, we find that $\bar{V}_c$ varies (contrary to $\bar{T}_c$ and $\bar{\Gamma}_c$) upon decreasing $\hbar\omega_F/J$, the critical line $V_c(T, \Gamma = 0)$ is reentrant, and, for a single band, eV is bounded when the R reservoir is filled.

When the coupling to the electronic reservoirs, g, is finite, the critical line in the $\Gamma = 0$ plane remains unaltered, but the critical surface on the Γ direction is pulled "upwards" enlarging the low temperature phase for increasing values of g. This is similar to what was found for quantum oscillator Ohmic baths and is due to a spin-localization-like effect [18,22]. We now turn to the dynamics. Our numerical and analytical analysis of Eqs. (2) show that after a quench in the low-T, weak-Γ and weak-V phase, the dynamics do not reach a QNESS [21]. There is a separation of two-time scales typical of aging phenomena [4]. First, a stationary regime for short time differences $t - t_w$ with respect to the waiting-time after the quench, t_w, in which the symmetric correlation approaches a plateau asymptotically in the time-difference. Later, an aging regime in which C depends on the two times explicitly. This behavior is shown in Fig. 2(a). The plateau value $q_{\rm EA}$, so-called Edwards-Anderson parameter, measures the fraction of frozen rotor fluctuations on time scales much smaller than t_w. The stationary decay depends on all control parameters. $q_{\rm EA}$ approaches one at $T = \Gamma = V = 0$ and zero on the critical manifold as in a second order transition. In the aging regime, the correlation normalized by $q_{\rm EA}$ is identical to the classical one [17]:

$$C(t, t_w)/q_{\rm EA} \simeq 2\sqrt{2}(t_w/t)^{3/4}(1 + t_w/t)^{-3/2} \qquad (4)$$

for $0 < t_w/t < 1$. We shall prove this result and unveil the connection with coarsening anticipated previously by exploiting the quadradic form of the full action in the $\mathbf{n}$ fields. Indeed, under the Keldysh rotation $(\mathbf{n}^+, \mathbf{n}^-) \to (i\hat{\mathbf{n}}, \mathbf{n})$ with $i\hat{\mathbf{n}} \equiv i(\mathbf{n}^+ - \mathbf{n}^-)/\hbar$ and $\mathbf{n} \equiv (\mathbf{n}^+ + \mathbf{n}^-)/2$, the action is identical to the Martin-Siggia-Rose one for a *classical* Langevin process in a harmonic potential $\sum_{ij}(J_{ij} - \lambda(t)\delta_{ij})\mathbf{n}_i \cdot \mathbf{n}_j$. The noise statistics is, however, peculiar: because of the quantum origin of the environment, it has memory, depends on $\hbar$, and satisfies the quantum FDT in the $V = 0$ case. The Langevin equations are rendered independent—apart from a residual coupling through the Lagrange multiplier—once expressed in the diagonal basis of the interaction matrix J_{ij} with eigenvalues J_μ. The analysis then follows the same route as in [17], see [21]. One finds quite naturally that the longtime dynamics corresponds to a Bose-Einstein-like condensation process of the M N-dimensional "vectors" $\mathbf{n}_i$ on the direction of the edge eigenvector. The relaxation is controlled by the decay of $\rho(J_\mu)$ close to its edge. For Gaussian-independent identically distributed couplings, $\rho(J_\mu) \propto (2J - J_\mu)^{1/2}$ coincides with the distribution of the modulus of the Laplacian eigenvalues, $dk^2 k^{2(d/2-1)} = dk^2 k^{2\times 1/2}$ in $d = 3$. For this reason, all models with a square root singularity of the distribution of "masses" J_μ, as the ferromagnetic rotor model in $d = 3$ and the completely connected spin-glass rotor model, are characterized by the same longtime dynamics. Now let us show that the *aging* dynamics are indeed equivalent to their classical counterpart. In the ordered phase, taking the long t_w *and* $t - t_w$ limits with t/t_w fixed (low-frequency aging regime), the second-time derivatives in the effective Langevin equations can be neglected. Furthermore, only the low-frequency ($\omega \ll 1/\beta\hbar$) behavior of the kernels plays a rôle in this regime. In this $\omega \to 0$ limit, $\Sigma_K^B(\omega) \to {\rm ct} \in \Re$, and $\Sigma_R(\omega)$ is linear. Therefore, the noise kernels approach a classical Ohmic white-noise limit with "temperature"

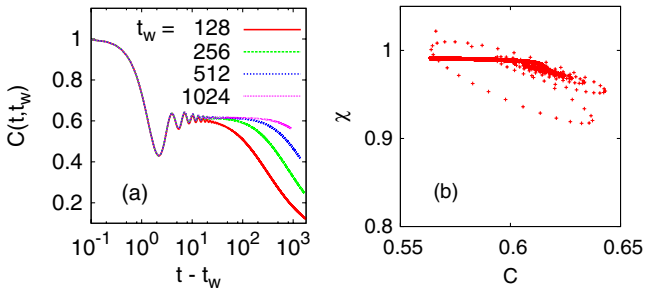


FIG. 2 (color online). Dynamics in the driven coarsening regime: numerical solution to Eqs. (2) after a quench to $T = 0.2J$, $V = 0.2J$, $\Gamma = 1$, $g = 1$ with $\hbar\omega_F = 10J$. (a) The symmetric correlation $C(t, t_w)$. (b) The integrated linear response, $\chi(t, t_w) = \int_{t_w}^t dt' R(t, t')$ against C, for $t_w = 1024$ and using t as a parameter. The curved part corresponds to the stationary and oscillatory regime with $(t - t_w)/t_w \to 0$ while the straight line is for times in the monotonic aging decay of C.

$$T^* = \lim_{\omega\to 0} \Sigma_K^B(\omega)/[2\partial_\omega \Im\Sigma_R^B(\omega)]. \quad (5)$$

At $V = 0$, one gets $T^* = T$. Instead, at $T = 0$ and $eV \ll \hbar\omega_F$, one has $T^* = eV/2$: the voltage plays the rôle of a *bath* temperature. This fact has already been reported, and it is at the root of the derivation of the stochastic Gilbert equation for a spin under bias [23]. Having argued that the longtime dynamics is governed by a *classical* Langevin equation at temperature T^*, we have proven that correlation scales as in Eq. (5) for $t/t_w = O(1)$ [17], a result with two interesting consequences. In the case of (large M) quantum $3d$ coarsening, the classical-quantum mapping extends to space-time correlations [21] and proves the existence of a growing coherence length $\xi(t_w) \propto t_w^{1/2}$ over which the rotors are oriented in the same direction and provides a real-space interpretation of aging. Moreover, in the same longtime regime, the linear response also scales as in the classical limit. Therefore, the quantum fluctuation-dissipation relation between integrated linear-response, χ, and symmetric correlation approaches the classical one, $\chi \sim \mathrm{ct} + (q_{\mathrm{EA}} - C)/T_{\mathrm{eff}}$, with an *infinite* effective temperature [24], $T_{\mathrm{eff}} \to \infty$, as shown in Fig. 2(b) (see also [19,25]). In short, the asymptotic aging coarsening is universal, in the sense that the scaling functions do not depend on T, Γ, V and, hence, are identical to the classical undriven ones ($\Gamma = V = 0$) [26]. This result mirrors the one obtained in [9] for steady state dynamics.

The environment plays a dual rôle: its quantum character basically determines the phase diagram but the coarsening process at long times and large length-scales only ''feels'' a classical white bath at temperature T^*. The two-time dependent decoherence phenomenon [absence of oscillations, validity of a classical FDT when $t/t_w = O(1)$, etc.] is intimately related to the development of a nonzero (actually infinite) effective temperature, T_{eff}, of the system as defined from the deviation from the (quantum) FDT [24]. T_{eff} should be distinguished from T^* as the former is generated not only by the environment but by the system interactions as well ($T_{\mathrm{eff}} > 0$ even at $T^* = 0$ [19,25]). Moreover, we found an extension of the irrelevance of T in classical ferromagnetic coarsening ($T = 0$ ''fixed-point'' scenario): after a suitable normalization of the observables that takes into account all microscopic fluctuations (e.g., q_{EA}), the scaling functions are independent of all parameters including V and Γ. Although we proved this result through a mapping to a Langevin equation that applies to quadratic models only, we expect it to hold in all instances with the same type of ordered phase, say ferromagnetic, and a longtime aging dynamics dominated by the slow motion of large domains. Thus, a large class of coarsening systems (classical, quantum, pure, and disordered) should be characterized by the same scaling functions. It could be worth studying carefully systems evolving by barrier crossing, a rapid process in which not only the low-frequency behavior of the bath may be relevant.

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