Dynamically Induced Zeeman Effect in Massless QED

Efrain J. Ferrer and Vivian de la Incera

Department of Physics, University of Texas at El Paso, El Paso, Texas 79968, USA

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It is shown that in nonperturbative massless QED an anomalous magnetic moment is dynamically induced by an applied magnetic field. The induced magnetic moment produces a Zeeman splitting for electrons in Landau levels higher than $l = 0$. The expressions for the nonperturbative Lande g factor and Bohr magneton are obtained. Possible applications of this effect are outlined.

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The theory of the electron magnetic moment has historically played an important role in the development of QED. As is known, the electron intrinsic magnetic moment $\vec{\mu}$ is related to the spin vector $\vec{s}$ by $\vec{\mu} = g\mu_B\vec{s}$, where $\mu_B = e\hbar/2mc$ is the Bohr magneton, and g is the Lande g factor. One of the great triumphs of the Dirac relativistic theory for the electron was the prediction of the value $g = 2$. Nevertheless, experimental measurements of the g factor showed a deviation from this prediction. The solution of the apparent contradiction came only after Schwinger calculated the first-order radiative correction to $\vec{\mu}$, due to the electron-photon interactions [1]. Schwinger's results led to an anomalous magnetic moment with a correction to the g factor of order ($\frac{g-2}{2} = \frac{\alpha}{2\pi}$), α being the fine-structure constant. Subsequently higher-order radiative corrections to g have given rise to a series in powers of α/π [2] that is in excellent agreement with the experiment.

Now, in the case of massless QED, one cannot follow Schwinger's approach to obtain the anomalous magnetic moment. The reason is that an anomalous magnetic moment would break the chiral symmetry of the massless theory, but this symmetry is protected against perturbative corrections. However, the chiral symmetry can be broken dynamically via nonperturbative effects. In fact, such a dynamical symmetry breaking has been shown to occur if the massless electrons interact with the photons in the presence of a constant magnetic field. This mechanism is known in the literature [3–7] as the magnetic catalysis of chiral symmetry breaking (MCχSB). The phenomenon of MCχSB consists of the formation of a chiral condensate due to the dimensional reduction in the dynamics of the fermions in the lowest Landau level (LLL). This dimensional reduction makes the nonperturbative fermion-antifermion interaction effectively stronger, hence favoring fermion-antifermion pairing even at weak coupling.

All the previous studies of MCχSB in QED [4,5] focused on the generation of a fermion dynamical mass. None of them, however, considered the possibility of a dynamically generated magnetic moment. In the present Letter we are going to show that, along with the dynamical mass, the chiral condensate necessarily produces a dynamical magnetic moment. Physically it is easy to understand the origin of the new dynamical quantity. The chiral condensate carries nonzero magnetic moment, since the particles forming the condensate have opposite spins and opposite charges. Therefore, chiral condensation will inexorably provide the quasiparticles with both a dynamical mass and a dynamical magnetic moment. Symmetry arguments can help us also to better understand this phenomenon. A magnetic moment term does not break any additional symmetry that has not already been broken by a mass term. Hence, once MCχSB occurs, there is no reason why only one of these parameters should be different from zero. We will show below that a very important consequence of the dynamically generated magnetic moment is a splitting in the electron energy spectrum that can be interpreted as a nonperturbative Zeeman effect.

To explore the dynamical generation of a magnetic moment in massless QED, we can start from the Schwinger-Dyson (SD) equation for the fermion self-energy in the presence of a constant magnetic field along the Z direction ($F_{12} = H$). We will work in the quenched-ladder approximation where

$$\Sigma(x, x') = ie^2\gamma^\mu G(x, x')\gamma^\nu D_{\mu\nu}(x - x'). \qquad (1)$$

Here, $\Sigma(x, x')$ is the electron self-energy operator, $D_{\mu\nu}(x - x')$ is the bare photon propagator, and $G(x, x')$ is the full fermion propagator depending on the dynamically induced quantities and the magnetic field.

To transform to momentum space in the presence of a magnetic field we can use the so-called Ritus' method, originally developed for fermions in [8] and later extended to vector fields in [9]. In Ritus' approach, the transformation to momentum space is carried out using the eigenfunctions $E^l_p(x)$ of the asymptotic states of the charged fermions in a uniform magnetic field

$$E^l_p(x) = E^+_p(x)\Delta(+) + E^-_p(x)\Delta(-), \qquad (2)$$

where $\Delta(\pm) = (I \pm i\gamma^1\gamma^2)/2$ are up ($+$) and down ($-$) spin projectors; $E^{+/-}_p(x) = N(l/l-1)\exp(p_0x^0 + p_2x^2 + p_3x^3)D_{(l/l-1)}(\rho)$, with $D_l(\rho)$ the parabolic cylinder functions of argument $\rho = \sqrt{2|eH|}(x_1 - p_2/|eH|)$, and $N(l) = (4\pi|eH|)^{1/4}/\sqrt{l!}$ a normalization constant. The index $l = 0, 1, 2, \ldots$ denotes the Landau levels (LL). The $E_p^l(x)$ functions (2) play the role in the magnetized medium of the usual plane-wave (Fourier) functions e^{ipx} at zero field. They satisfy the field-dependent eigenvalue equation

$$(\Pi \cdot \gamma)E_p^l(x) = E_p^l(x)(\gamma \cdot \bar{p}), \tag{3}$$

with generalized momenta $\Pi_\mu = i\partial_\mu - eA_\mu$ and $\bar{p} = (p_0, 0, -\text{sgn}(eH)\sqrt{2|eH|l}, p_3)$.

In momentum space the fermion self-energy is given by

$$\Sigma(p, p') = \int dxdy \bar{E}_p^l(x)\Sigma(x, y)E_p^l(y) = (2\pi)^4\hat{\delta}^{(4)}(p - p')\Pi(l)\tilde{\Sigma}^l(\bar{p}) \tag{4}$$

since the E_p^l are precisely linear combinations of the eigenfunctions of the fermion self-energy in the presence of a magnetic field [8]. In (4) $\bar{E}_p^l \equiv \gamma^0(E_p^l)^\dagger\gamma^0$, and we used that $\int d^4x \bar{E}_p^l(x)E_{p'}^{l'}(x) = (2\pi)^4\hat{\delta}^{(4)}(p - p')\Pi(l)$ with $\hat{\delta}^{(4)}(p - p') = \delta^{ll'}\delta(p_0 - p_0')\delta(p_2 - p_2')\delta(p_3 - p_3')$ and $\Pi(l) = \Delta(+)\delta^{l0} + I(1 - \delta^{l0})$ [10].

As proven in [11], in the presence of a magnetic field H, the general structure of $\tilde{\Sigma}^l(\bar{p})$ consistent with the Ward-Takahashi identity in the ladder approximation is

$$\tilde{\Sigma}^l(\bar{p}) = Z_\parallel^l(\bar{p}, F)\gamma \cdot \bar{p}_\parallel + Z_\perp^l(\bar{p}, F)\gamma \cdot \bar{p}_\perp + M^l(\bar{p}, F)I + \tfrac{1}{2}T^l(\bar{p}, F)\bar{F}^{\mu\nu}\sigma_{\mu\nu}. \tag{5}$$

Here, $\bar{F}^{\mu\nu} = F^{\mu\nu}/|H|$, $\bar{p}_\mu^\perp = (0, 0, -\text{sgn}(eH)\sqrt{2|eH|l}, 0)$, and $\bar{p}_\mu^\parallel = (p_0, 0, 0, p_3)$. The coefficients M^l, Z^l, and T^l depend on the field strength F, LL l and momentum $\bar{p}$. M^l is the dynamical mass already considered in previous works on MCχSB [3–7]. T^l corresponds to the dynamically induced magnetic moment and should be found, along with M^l, from the SD equations. The operator $\tilde{\Sigma}^l(\bar{p})$ can be conveniently written, with the help of the projectors $\Lambda_\parallel^\pm = \frac{1}{2}(1 \pm \frac{\gamma^\parallel \cdot \bar{p}_\parallel}{|\bar{p}_\parallel|})$, $\Lambda_\perp^\pm = \frac{1}{2}(1 \pm i\gamma^2)$, as

$$\tilde{\Sigma}^l(\bar{p}) = Z_\parallel^l(\Lambda_\parallel^+ - \Lambda_\parallel^-)|\bar{p}_\parallel| + iZ_\perp^l(\Lambda_\perp^- - \Lambda_\perp^+)|\bar{p}_\perp| + (M^l + T^l)\Delta(+) + (M^l - T^l)\Delta(-). \tag{6}$$

Using the E_p^l transformation, the full fermion propagator in momentum space is given by

$$G^l(p - p') = \int dxdy \bar{E}_p^l(x)G(x, y)E_{p'}^{l'}(y) = (2\pi)^4\hat{\delta}^{(4)}(p - p')\Pi(l)\tilde{G}^l(\bar{p}), \tag{7}$$

where

$$\begin{aligned}\tilde{G}^l(\bar{p}) &= \frac{1}{\gamma \cdot \bar{p} - \tilde{\Sigma}^l(\bar{p})} \\ &= \frac{N^l(T, V_\parallel)}{D^l(T)}\Delta(+)\Lambda_\parallel^+ + \frac{N^l(T, -V_\parallel)}{D^l(-T)}\Delta(+)\Lambda_\parallel^- \\ &\quad + \frac{N^l(-T, V_\parallel)}{D^l(-T)}\Delta(-)\Lambda_\parallel^+ + \frac{N^l(-T, -V_\parallel)}{D^l(T)}\Delta(-)\Lambda_\parallel^- \\ &\quad - iV_\perp^l(\Lambda_\perp^+ - \Lambda_\perp^-)\Big[\frac{\Lambda_\parallel^+\Delta(+) + \Lambda_\parallel^-\Delta(-)}{D^l(T)} \\ &\quad + \frac{\Lambda_\parallel^-\Delta(+) + \Lambda_\parallel^+\Delta(-)}{D^l(-T)}\Big]\end{aligned} \tag{8}$$

with coefficients

$$\begin{aligned}N^l(T, V_\parallel) &\equiv M^l - T^l - V_\parallel^l, \\ D^l(T) &\equiv (M^l)^2 - (V_\parallel^l + T^l)^2 + (V_\perp^l)^2, \\ V_\parallel^l &\equiv (1 - Z_\parallel^l)|\bar{p}_\parallel|, \\ V_\perp^l &\equiv (1 - Z_\perp^l)|\bar{p}_\perp| = (1 - Z_\perp^l)\sqrt{2|eH|l}.\end{aligned} \tag{9}$$

Transforming Eq. (1) to momentum space with the help of the E_p^l functions, taking the photon propagator in the Feynman gauge, $D_{\mu\nu}(x - x') = \int \frac{d^4q}{(2\pi)^4}\frac{e^{iq\cdot(x-x')}}{q^2 - i\varepsilon}g_{\mu\nu}$, and carrying out derivations and approximations similar to those done in [5], we obtain that the SD equation for arbitrary Landau level l is given by

$$\begin{aligned}\tilde{\Sigma}^l(\bar{p})\Pi(l) = ie^2(2eH)\Pi(l)\int \frac{d^4\hat{q}}{(2\pi)^4}\frac{e^{-\hat{q}_\perp^2}}{\hat{q}^2} &\times [\gamma_\mu^\parallel \tilde{G}^l(\overline{p - q})\gamma_\mu^\parallel \\ &+ \Delta(+)\gamma_\mu^\perp \tilde{G}^{l+1}(\overline{p - q})\gamma_\mu^\perp\Delta(+) \\ &+ \Delta(-)\gamma_\mu^\perp \tilde{G}^{l-1}(\overline{p - q})\gamma_\mu^\perp\Delta(-)],\end{aligned} \tag{10}$$

where $\overline{p - q} \equiv (p_0 - q_0, 0, -\text{sgn}(eH)\sqrt{2|eH|n}, p_3 - q_3)$ for $n = l - 1, l, l + 1$ and the normalized quantities are defined as $\hat{Q}_\mu = Q_\mu/\sqrt{2|eH}$. Since the equation for a given Landau level l involves dynamical parameters that depend on l, $l - 1$, and $l + 1$, the SD equations for all the LL's actually form a system of infinite coupled equations. Fortunately, in the infrared region, the leading contribution to each equation will come from the propagators with the lower LL's, since the magnetic field appearing in the denominator of the fermion propagator for $l \neq 0$ acts as a suppressing factor. Using this approximation, one can find a consistent solution at each level. On the other hand, the solutions for any M^l and T^l can be ultimately expressed in terms of the LLL solution, indicating that the physical origin of all the dynamical quantities is actually due to the infrared dynamics taking place at the LLL. For the LLL ($l = 0$) case, the leading contribution to the right-hand side (rhs) of (10) comes from the $\tilde{G}^0(\overline{p - q})$ term, and we find

$$(M^0+T^0)+Z^0_{\parallel}(\Lambda^+_{\parallel}-\Lambda^-_{\parallel})|\bar{p}_{\parallel}| = ie^2(2|eH|)\cdot\int\frac{d^4q}{(2\pi)^4}\frac{e^{-\hat{q}^2_{\perp}}}{\hat{q}^2}\frac{(M^0+T^0)}{(\bar{p}_{\parallel}-\overline{q}_{\parallel})^2-(M^0+T^0)^2}. \quad (11)$$

Equation (11) implies that $Z^0_{\parallel}=0$, while for the combination M^0+T^0 it gives, in the infrared limit ($p_{\parallel}\sim 0$),

$$1 = ie^2(4|eH|)\int\frac{d^4\hat{q}}{(2\pi)^4}\frac{e^{-\hat{q}^2_{\perp}}}{\hat{q}^2}\frac{1}{(M^0+T^0)^2-q^2_{\parallel}}. \quad (12)$$

If M^0+T^0 is replaced in (12) by the dynamical mass $m_{\rm dyn}$ of Refs. [4,5], Eq. (12) turns identical to the gap equation found there. Hence, the solution of (12) is formally the same as the one found in [4,5], but with the combination M^0+T^0 now playing the role previously played only by the dynamical mass. Hence,

$$M^0+T^0 \simeq \sqrt{2|eH|}e^{-\sqrt{\pi/\alpha}}. \quad (13)$$

As in [4,5], this solution is obtained considering that M^0+T^0 does not depend on the momentum, an assumption consistent within the ladder approximation [12]. As proved in [13], when the polarization effect was included in the gap equation through the improved-ladder approximation, the solution for $m_{\rm dyn}$ was of the same form as (13), but with the replacement $\sqrt{\pi/\alpha}\to\pi/\alpha\log(\pi/\alpha)$ in the exponent. Since the inclusion of the magnetic moment in the LLL SD equation merely implies the replacement $m_{\rm dyn}\to M^0+T^0$, it is expected that a similar effect will occur in the solution (13). However, this effect will not qualitatively change the nature of our findings.

Since in the LLL propagator $G^0(p-p')$ the dynamical mass M^0 and magnetic moment T^0 always enter through the combination M^0+T^0, the solution of the LLL SD equation (13) can only determine the sum of these dynamical parameters. This indicates that at the LLL, the effect of a magnetic moment is irrelevant, it just redefines the rest energy due to the replacement $m_{\rm dyn}\to M^0+T^0$. This is physically natural, since the electrons in the LLL can only have one spin projection, so for them there is no spin degeneracy and hence, no possible energy splitting due to the magnetic moment. $E^0=M^0+T^0$ represents then a dynamically induced rest energy. This can be easily seen considering the Dirac equation for the electrons in the LLL with dynamically induced parameters,

$$[\bar{p}^{\parallel}\cdot\tilde{\gamma}_{\parallel}-E^0]\psi_{\rm LLL}=0, \quad (14)$$

where $\psi_{\rm LLL}$ is the spin-up two-component wave function. Equation (14) coincides with the free $(1+1)$-Thirring model [14], with corresponding gamma matrices $\tilde{\gamma}_0=\sigma_1$, $\tilde{\gamma}_3=-i\sigma_2$, where σ_i are the Pauli matrices. The dispersion relation of the electrons in the LLL obtained from (14), $p_0^2=p_3^2+(E^0)^2$, is in agreement with the above discussion. As we will see below, the interesting effect associated to T comes from the higher LL's.

For electrons in the first LL ($l=1$), the leading contribution to the rhs of (10) in the infrared limit ($p_{\parallel}\sim 0$) comes from the term containing $\tilde{G}^0(\overline{p-q})$. Then,

$$Z^{(1)}_{\perp}\gamma_2(2|eH|)+(M^1+T^1)\Delta(+)+(M^1-T^1)\Delta(-) = ie^2(4|eH|)\Delta(-)\int\frac{d^4\hat{q}}{(2\pi)^4}\frac{e^{-\hat{q}^2_{\perp}}}{\hat{q}^2}\frac{E^0}{(E^0)^2-q^2_{\parallel}}. \quad (15)$$

From (15) we obtain the solutions

$$M^1=-T^1=\tfrac{1}{2}E^0=\sqrt{|eH|/2}e^{-\sqrt{\pi/\alpha}}, \quad Z^1_{\perp}=0. \quad (16)$$

This result corroborates the relevance of the LLL dynamics (both M^1 and T^1 are determined by E^0) in the generation of the dynamical mass and magnetic moment for electrons in the first LL. Given that the magnitude of the magnetic moment for the electrons in the first LL is determined by the dynamically generated rest-energy of the electrons in the LLL, any modification of the theory producing an increase in E^0 will, in turn, drive an increase in the magnitude of T^1. From the experience with the MCχSB phenomenon, such modifications could be, for example, lowering the space dimension [15], introducing scalar-fermion interactions [6,12], or considering a nonzero bare mass [16].

Let us find now the dispersion relations for electrons in higher LL's, taking into account the dynamically induced quantities. Starting from the modified electron equation in the presence of the magnetic field

$$[\bar{p}\cdot\gamma-M^lI-iT^l\gamma^1\gamma^2]\psi_l=0, \quad (17)$$

the dispersion relations are found from

$$\det[\bar{p}\cdot\gamma-M^lI-iT^l\gamma^1\gamma^2]=[(M^l)^2-(\bar{p}_{\parallel}-T^l)^2+\overline{p}^2_{\perp}]\times[(M^l)^2-(\bar{p}_{\parallel}+T^l)^2+\bar{p}^2_{\perp}]=0. \quad (18)$$

yielding

$$p_0^2=p_3^2+[\sqrt{(M^l)^2+2eHl}\pm T^l]^2, \quad (19)$$

and thus showing that the induced magnetic moment breaks the energy degeneracy between the spin states in the same LL.

In particular for $l=1$, plugging (16) into (19), taking into account that $\hat{M}^1$, $\hat{T}^1\ll 1$, and Taylor expanding the term in parenthesis, the dispersion relations can be expressed as

$$p_0^2\simeq p_3^2+2eH+(M^1)^2+(T^1)^2\pm 2T^1\sqrt{2eH}, \quad (20)$$

thereby producing an energy splitting

$$\Delta E=|2T^1|=2\sqrt{|eH|/2}e^{-\sqrt{\pi/\alpha}}. \quad (21)$$

Expression (21) can be conveniently written in the well-known form of the Zeeman energy splitting for the two spin projections

$$\Delta E=\tilde{g}\tilde{\mu}_BH, \quad (22)$$

where $\tilde{g}$ and $\tilde{\mu}_B$ are the nonperturbative Lande g factor and Bohr magneton given, respectively, by

$$\tilde{g} = 2e^{-2\sqrt{\pi/\alpha}}, \qquad \tilde{\mu}_B = \frac{e}{2M^1}. \tag{23}$$

Notice that the Lande g factor depends nonperturbatively on the coupling constant α, and that the Bohr magneton is given in terms of the dynamically induced electron mass.

We want to call attention to possible applications of the dynamically induced Zeeman effect obtained in this Letter. One area of potential interest is condensed matter, since recent experiments [17] have shown that the 2-dimensional crystalline form of carbon, known as graphene, has charge carriers that behave as massless Dirac electrons. In particular, a phenomenon where the dynamically induced Zeeman effect can bring some new light is the lifting of the fourfold degeneracy of the $l = 0$ LL, and twofold degeneracy of the $l = 1$ LL in the recently found quantum Hall states corresponding to filling factors $\nu = 0, \pm 1, \pm 4$ under strong magnetic fields [18]. Notice that dispersion relations similar to (19) were found within certain region of the parameter space in a two-dimensional modeling of Dirac quasiparticles in graphene with magnetically catalyzed masses and other order parameters connected to quantum Hall ferromagnetism [19].

Another domain where the finding we are reporting can be of interest is color superconductivity. An important aspect of color superconductivity is its magnetic properties [20–23]. In spin-zero color superconductivity, although the color condensate has nonzero electric charge, there is a linear combination of the photon and a gluon that remains massless, hence giving rise, in both the 2SC and CFL phases, to a long-range remnant "rotated-electromagnetic" field [20]. To understand this, notice that the quarks participating in the pairing are neutral or have equal and opposite "rotated" $\tilde{Q}$ charge. That is, the condensate is always $\tilde{Q}$ neutral. In the case that the pair is formed by $\tilde{Q}$-charged quarks of opposite sign, although the condensate is $\tilde{Q}$ neutral, an applied magnetic field can interact with the quarks forming the pair [22]. Hence, with respect to the "rotated-electromagnetism" the color-superconducting pair resembles the chiral condensate under a conventional electromagnetic field. It should be expected, then, that a nonperturbative Zeeman effect can also be induced in a color superconductor under an applied magnetic field. Since, on the other hand, the Meissner instabilities that appear in some density regions of the color superconductor can be removed by the induction of a magnetic field [23], it will be interesting to investigate what could be the role in this process of a dynamically induced magnetic moment.

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[1] J. Schwinger, Phys. Rev. **73**, 416 (1948).
[2] T. Kinoshita and W. B. Lindquist, Phys. Rev. D **42**, 636 (1990).
[3] V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Phys. Rev. Lett. **73**, 3499 (1994); K. G. Klimenko, Z. Phys. C **54**, 323 (1992).
[4] V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Phys. Rev. D **52**, 4747 (1995); Nucl. Phys. **B462**, 249 (1996); C. N. Leung, Y. J. Ng, and A. W. Ackley, Phys. Rev. D **54**, 4181 (1996).
[5] D.-S Lee, C. N. Leung, and Y. J. Ng, Phys. Rev. D **55**, 6504 (1997).
[6] E. J. Ferrer and V. de la Incera, Phys. Lett. B **481**, 287 (2000).
[7] E. Rojas, A. Ayala, A. Bashir, and A. Raya Phys. Rev. D **77**, 093004 (2008).
[8] V. I. Ritus, Ann. Phys. (N.Y.) **69**, 555 (1972); Zh. Eksp. Teor. Fiz. **75**, 1560 (1978).[Sov. Phys. JETP **48**, 788 (1978)].
[9] E. Elizalde, E. J. Ferrer, and V. de la Incera, Ann. Phys. (N.Y.) **295**, 33 (2002); Phys. Rev. D **70**, 043012 (2004).
[10] C. N. Leung and S.-Y. Wang Nucl. Phys. **B747**, 266 (2006).
[11] E. J. Ferrer and V. de la Incera, Phys. Rev. D **58**, 065008 (1998).
[12] E. Elizalde, E. J. Ferrer, and V. de la Incera, Phys. Rev. D **68**, 096004 (2003).
[13] V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Nucl. Phys. **B563**, 361 (1999).
[14] W. E. Thirring, Ann. Phys. (N.Y.) **3**, 91 (1958).
[15] V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Phys. Rev. D **52**, 4718 (1995).
[16] V. P. Gusynin and A. V. Smilga, Phys. Lett. B **450**, 267 (1999); S. Y. Wang, Phys. Rev. D **77**, 025031 (2008); K. G. Klimenko and V. Ch. Zhukovsky, Phys. Lett. B **665**, 352 (2008).
[17] K. S. Novoselov *et al.*, Science **306**, 666 (2004); Nature (London) **438**, 197 (2005); Y. Zhang *et al.*, Nature (London) **438**, 201 (2005).
[18] Y. Zhang *et al.*, Phys. Rev. Lett. **96**, 136806 (2006).
[19] V. Gorbar, V. P. Gusynin, and V. A. Miransky, Low Temp. Phys. **34**, 790 (2008).
[20] M. Alford, K. Rajagopal, and F. Wilczek, Nucl. Phys. **B537**, 443 (1999).
[21] K. Iida and G. Baym, Phys. Rev. D **66**, 014015 (2002); I. Giannakis and H-C Ren, Nucl. Phys. **B669**, 462 (2003); E. J. Ferrer and V. de la Incera, Phys. Rev. Lett. **97**, 122301 (2006); Phys. Rev. D **76**, 045011 (2007); J. L. Noronha and I. A. Shovkovy, Phys. Rev. D **76**, 105030 (2007); K. Fukushima and H. J. Warringa, Phys. Rev. Lett. **100**, 032007 (2008).
[22] E. J. Ferrer, V. de la Incera, and C. Manuel, Phys. Rev. Lett. **95**, 152002 (2005); Nucl. Phys. **B747**, 88 (2006); Proc. Sci., JHW2005 (2006) 022; J. Phys. A **39**, 6349 (2006); E. J. Ferrer and V. de la Incera, J. Phys. A **40**, 6913 (2007).
[23] E. J. Ferrer and V. de la Incera, Phys. Rev. D **76**, 114012 (2007); AIP Conf. Proc. **947**, 401 (2007); D. T. Son and M. A. Stephanov, Phys. Rev. D **77**, 014021 (2008).