Large-Scale Dynamo Action Driven by Velocity Shear and Rotating Convection

David W. Hughes*
Department of Applied Mathematics, University of Leeds, Leeds LS2 9JT, United Kingdom

Michael R. E. Proctor†
Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, United Kingdom
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By incorporating a large-scale shear flow into turbulent rotating convection, we show that a sufficiently strong shear can promote dynamo action in flows that are otherwise nondynamos. Our results are consistent with a dynamo driven either by the shear-current effect or by a fluctuating α effect interacting with the shear, but not with either a classical α^2 or $\alpha\omega$ dynamo.

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Magnetic fields are observed in virtually all cosmical bodies, from planets to stars and accretion discs; in many cases their presence can be categorically attributed to dynamo action. The most pressing problem in astrophysical dynamo theory is to explain the generation of *large-scale* magnetic fields, i.e., fields with significant energy on scales large compared with those of the driving flow. The Sun, with its global magnetic field manifested through surface activity, represents the most well-known example of a large-scale dynamo.

Astrophysical dynamos are often studied within the framework of mean field electrodynamics, in which the evolution of a mean (large-scale) magnetic field is described in terms of transport coefficients determined from averaged small-scale properties of the flow and field. The generation of magnetic field can then be ascribed to the α effect, which relates the mean electromotive force (emf) to the mean magnetic field. The α effect is nonzero only in flows that lack reflectional symmetry [1]; consequently helical flows are prime candidates for large-scale dynamo action. Indeed, in certain limiting cases the relation between α and helicity can be made explicit [1,2]; however, and importantly, there is no theory relating these two quantities when the magnetic Reynolds number $\mathrm{Rm} \gg 1$ and the Strouhal number St is of order unity, the case of astrophysical relevance. Numerical simulations reveal that the relationship between α and helicity is indeed far from straightforward [3].

The most natural system for investigating astrophysical dynamo action is that of rotating thermal convection [4,5]. Recent studies of convection in a domain of large horizontal extent—namely one that encompasses many convective cells—have demonstrated that although there is both significant helicity and healthy dynamo action (provided that Rm is sufficiently large), there is no evidence of any significant large-scale magnetic field [6,7]. Indeed, attempts to measure the α effect directly reveal a strongly fluctuating quantity with a very small mean. The similarity of the spectra of the magnetic fields generated by rotating and nonrotating convection—for which the flows are not helical—provides further evidence that the dynamo is controlled by small-scale processes (such as stretching and cancellation; see [8]) and not by mean field processes (such as a lack of reflectional symmetry).

The failure of rotating turbulent convection to act as a large-scale dynamo suggests that the notion that helical flows will necessarily lead to large-scale field generation is too simplistic. However, most astrophysical bodies possess a strong large-scale shear flow (differential rotation) and, indeed, most mean field astrophysical dynamo models incorporate this feature. It is therefore of interest to examine the additional effects arising from incorporating such a shear into the rotating convection model: we find that dynamo action is promoted and that there can be a significant large-scale field component, in contrast to the basic model.

One can envisage four possible beneficial effects of the shear on the mean field dynamo process: (i) that the large spatial scale of the shear leads to an enhanced α through greater spatial correlation of the small-scale motions [7,9]; (ii) that even though the mean α remains small there may nonetheless be an effective $\alpha\omega$ dynamo when the shear is significant; (iii) that the anisotropy induced by the shear may lead to a significant shear-current effect [10–12] (see also [13], for a related effect); (iv) that the shear may interact with temporal fluctuations in α to produce an effective mean field dynamo [14,15]. Here we explore these various possibilities by introducing large-scale velocity shear into the model of [6,7].

As in [6,7] we consider a plane Boussinesq convective layer ($0 < x,\ y < \lambda,\ 0 < z < 1$) with rotation about the vertical axis. In order to investigate turbulent dynamo action it is necessary to consider sufficiently large values of λ. Here, as in [6,7], we take $\lambda = 5$; this is large enough to provide a reasonable scale separation between the size of the domain and the size of a convective cell, with $O(100)$ convective cells in the domain, but small enough to allow us to undertake a number of computational runs. The basic model is extended by the inclusion of a horizontal flow of the form

$$\boldsymbol{U}_0 = U_0 \cos\frac{2\pi y}{\lambda}\hat{\boldsymbol{x}}, \qquad (1)$$

accomplished by replacing $\boldsymbol{u}$ with $\boldsymbol{u} + \boldsymbol{U}_0$ in the governing equations. It should be noted that although a flow with a large-scale component [i.e., with the same spatial dependence as the "target flow" (1)] does indeed occur, it may have a very different amplitude; the hydrodynamic state that ensues depends on interactions between the shear flow and convection and, possibly, on instabilities of the shear flow itself. Importantly, the scale of variation of this shear flow is much greater than all scales of the convection; this is essential if the results are to be explained within the mean field framework. Tobias *et al.* [16] have presented results in a related geometry but with a very different shear flow, one with no horizontal structure but with a strong vertical variation.

In this initial study we focus on the regime in which convection is fairly vigorous but in which there is no dynamo action in the absence of shear; specifically we set the Rayleigh number Ra = 150 000, the Taylor number Ta = 500 000, the Prandtl number = 1, and the magnetic Prandtl number = 5; this leads to a Reynolds number Re ≈ 60 and a magnetic Reynolds number Rm ≈ 300. A useful *a priori* measure of the imposed shear is given by the shear parameter S, defined by

$$S = U_0(\ell/u_{\rm rms}L), \qquad (2)$$

where $u_{\rm rms}$ is the rms velocity in the absence of shear, L is the scale of the shear, and ℓ is the horizontal scale of the convection cells in the absence of shear. For the parameters used here, $S \approx U_0/300$. One can also define an *effective* value of S, $S_{\rm eff}$ say, analogous to (2) but involving the shear flow that emerges dynamically in the sheared convective state.

We have investigated flow and dynamo properties for the range $0 \leq S \lesssim 7$. A weak seed magnetic field of zero mean is introduced into an established, stationary, purely hydrodynamic state of sheared convection. Figure 1 shows the evolution of the magnetic energy versus time for a range of values of S, and Fig. 2 the kinematic growth rate γ as a function of S. We see immediately that dynamo action ensues for sufficiently large values of S, although the dependence of γ on S is not straightforward. Following the onset of dynamo action (with the critical value of S in the interval $1/3 < S < 1/2$) γ is linearly related to S, the strongest dependence possible [17]. For larger S though this simple relationship no longer holds. This can be explained, at least partially, by inspection of the purely hydrodynamic states (see Fig. 3). For the two largest values of S considered ($S = 5, 20/3$), a coherent vortex forms and the proportion of energy in the "target mode" becomes much smaller, leading to a reduction in $S_{\rm eff}$. Note also that for $S \gtrsim 1$ the amplitude of the saturated magnetic energy is fairly insensitive to the value of S.

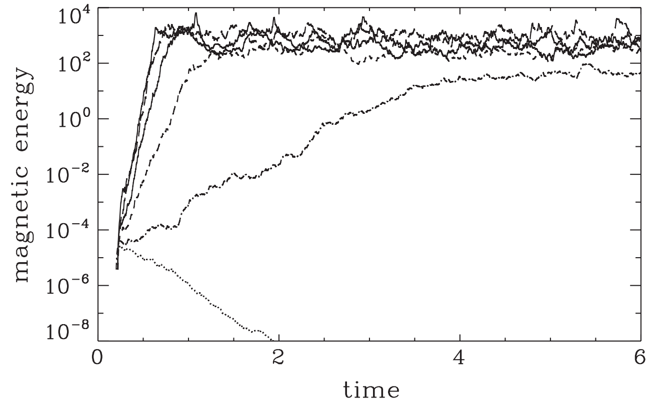

FIG. 1. Magnetic energy evolution for a range of S. In terms of increasing linear growth rate, $S = 1/3$ (not a dynamo), $2/3$, $5/3$, 5, $20/3$, $10/3$.

We have examined the spatial structure of the dynamo-generated magnetic fields, in both the kinematic and dynamic regimes, for evidence of large-scale dynamo action. Figure 4 shows the spectra of the horizontal fields for $S = 5/3$ and, for comparison, the spectra of the (small-scale) dynamo fields for Ra = 1 000 000 in the absence of shear. Note particularly that for the case of $S = 5/3$ there is roughly equal energy in all scales comparable with and greater than that of the driving convective flow, in contrast to the case of no shear, for which the spectrum is peaked at the scale of the convection. (Note that in Fig. 4 only the shapes of the spectra, and not their amplitudes, are significant, since the two runs are at different values of Ra and S.) In both cases the shapes of the spectra in the kinematic and dynamic regimes are similar, indicating that the structure of the field in the final nonlinear state is determined, to a large extent, from kinematic considerations.

We have also directly determined the α effect, by imposing a uniform horizontal magnetic field and measuring the induced emf. Since this procedure has an unambiguous interpretation only in the absence of small-scale dynamo action [18], we have considered the value $U_0 = 100$ ($S \approx 1/3$), which is strong enough to influence the flow but is not

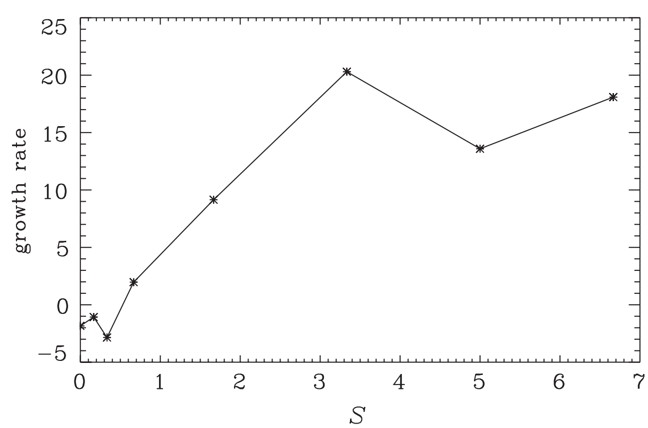

FIG. 2. Growth rates of the magnetic field versus S.

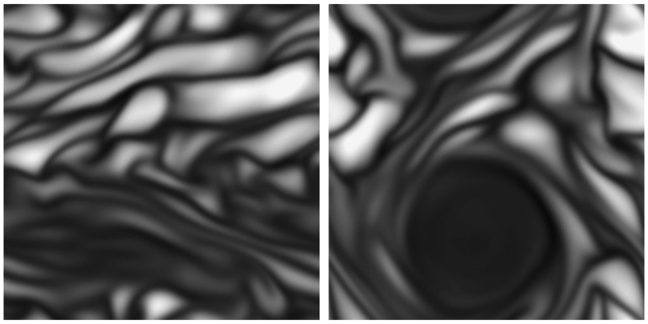

FIG. 3. Snapshots of horizontal slices of the temperature profile near the upper boundary for the sheared convective basic state; $S = 5/3$ (left) and $S = 5$ (right).

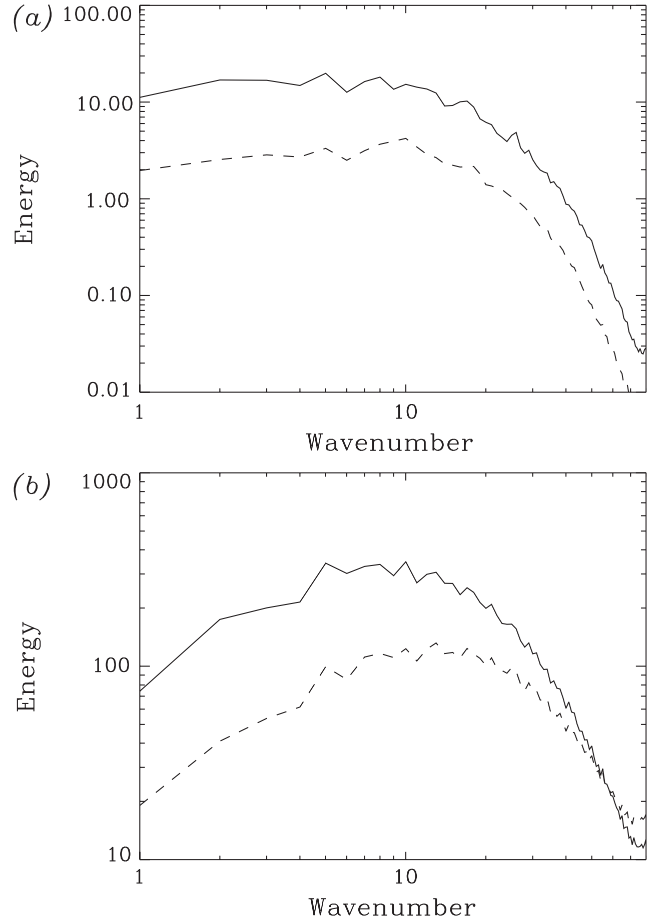

FIG. 4. Horizontal power spectra for the magnetic field in both the kinematic (dashed) and dynamic (solid) regimes. In (a) $S = 5/3$, Ra = 150 000; in (b) $S = 0$, Ra = 1 000 000; in both cases Ta = 500 000. The spectra were computed over the interior region of the domain ($0.06 < z < 0.94$). The arbitrary amplitudes of the kinematic spectra have been scaled so as to be on the same plot.

quite strong enough to induce dynamo action. Figure 5(a) shows the time history of the longitudinal α effect (i.e., α_{11} calculated from $\mathcal{E}_x = \alpha_{11} B_{0x}$), obtained from a spatial average over half the domain [6], for $S = 0$. As discussed in detail in [7], even though α is the result of a spatial average over many convective cells, it remains highly fluctuating in time, with only a small mean. Thus, as shown in Fig. 5(b), a further long temporal average is needed in order to pin down α, with the resulting value being small in comparison with the rms velocity; from Fig. 5(b) it can be seen that the long-time average value of α is given by $\bar{\alpha} \approx 0.05$, whereas $u_{\rm rms} \approx 60$. For $S = 1/3$, when the influence of the shear on the flow is by no means negligible, there is essentially no difference in the behavior of α from that when $S = 0$; as can be seen in Figs. 5(c) and 5(d) it is again characterized by large fluctuations and the same small mean. We also tried to determine the emf in the case of $S = 5/3$ (when there is small-scale dynamo action), both with and without a weak imposed field; these gave almost the same result, being characterized by substantial temporal fluctuations, and thus rendering meaningless the extraction of a mean emf (and hence α) for the former case.

Having thus shown that the introduction of a large-scale shear flow does indeed promote vigorous large-scale dynamo action, we now return to the four possibilities discussed earlier. As indicated above, α is essentially unchanged by the shear flow, so we can rule out possibility (i). For a conventional $\alpha\omega$ dynamo model we expect the growth rate to vary either as $S^{1/2}$ (for a disturbance of fixed wave number) or $S^{2/3}$ (if the optimal wave number is permitted in the system). Our calculations show that once dynamo action sets in then, for a range of S, the growth rate varies linearly with S; thus possibility (ii) is also inconsistent with the results. Both remaining possibilities would seem to allow the growth rate to be linearly proportional to S [15,19]. However, distinguishing between the two is far from straightforward. Although the physical mechanisms appear quite distinct, they are both manifested as nondiffusive contributions to the turbulent diffusivity tensor β_{ijk}.

We have demonstrated conclusively that the flow resulting from the interaction of a large-scale shear flow and turbulent rotating convection can lead to large-scale dynamo action, i.e., the generation of magnetic fields with a significant component of energy on scales large compared with that of the convective cells. This may be significant in understanding the generation of large-scale fields in astrophysical bodies. It is important to note that this large-scale field generation can be explained within the kinematic framework. As such, our work is related to that of Yousef *et al.* [20,21], who consider kinematic dynamo action in sheared, forced turbulence and obtain a similar relation between the growth rate and S. The mechanism we have described is very different from that discussed in [22], in which the evolution of a large-scale magnetic field component is attributed to a boundary flux of magnetic helicity and is entirely a nonlinear effect. In this Letter we have concentrated on the regime in which the convection, although fairly vigorous, does not induce dynamo action of itself. Thus the magnetic Reynolds numbers involved are fairly modest. Obviously it is also of interest to inves-

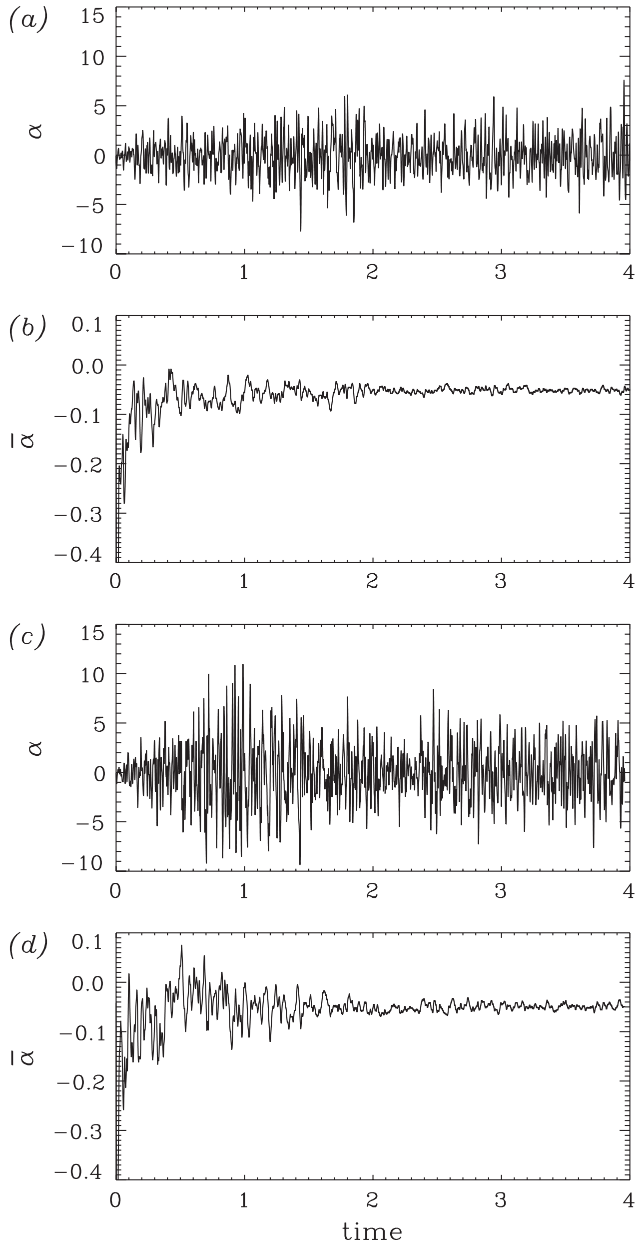

FIG. 5. (a) Longitudinal α effect versus time for $S = 0$; (b) $\bar{\alpha}$, the cumulative temporal average of α, for $S = 0$; (c) α for $S = 1/3$; (d) $\bar{\alpha}$ for $S = 1/3$.

tigate the role of shear on the small-scale dynamo action that sets in at higher Rm and for which the underlying mechanism is related to local stretching and folding properties of the flow, characterized, for example, by Lyapunov exponents and cancellation exponents [23], rather than helicity. Our results on this will be presented in a future paper.

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*d.w.hughes@leeds.ac.uk
†mrep@cam.ac.uk

[1] H. K. Moffatt, *Magnetic Field Generation in Electrically Conducting Fluids* (Cambridge University Press, Cambridge, U.K., 1978).
[2] F. Krause and K.-H. Rädler, *Mean-Field Magnetohydrodynamics and Dynamo Theory* (Pergamon, New York, 1980).
[3] A. Courvoisier, D. W. Hughes, and S. M. Tobias, Phys. Rev. Lett. **96**, 034503 (2006).
[4] S. Childress and A. M. Soward, Phys. Rev. Lett. **29**, 837 (1972).
[5] A. M. Soward, Phil. Trans. R. Soc. A **275**, 611 (1974).
[6] F. Cattaneo and D. W. Hughes, J. Fluid Mech. **553**, 401 (2006).
[7] D. W. Hughes and F. Cattaneo, J. Fluid Mech. **594**, 445 (2008).
[8] S. Childress and A. D. Gilbert, *Stretch, Twist, Fold: The Fast Dynamo* (Springer-Verlag, Berlin, 1995).
[9] A. Courvoisier, D. W. Hughes, and S. M. Tobias, J. Fluid Mech. (to be published).
[10] I. Rogachevskii and N. Kleeorin, Phys. Rev. E **68**, 036301 (2003).
[11] N. Kleeorin and I. Rogachevskii, Planet. Space Sci. **55**, 2315 (2007).
[12] A. Brandenburg, K.-H. Rädler, M. Rheinhardt, and P. J. Käpylä, Astrophys. J. **676**, 740 (2008).
[13] K.-H. Rädler, Monatsber. Dtsch. Akad. Wiss. Berlin **11**, 272 (1969).
[14] N. A. Silant'ev, Astron. Astrophys. **364**, 339 (2000).
[15] M. R. E. Proctor, Mon. Not. R. Astron. Soc. **382**, L39 (2007).
[16] S. M. Tobias, F. Cattaneo, and N. H. Brummell, Astrophys. J. **685**, 596 (2008).
[17] G. E. Backus, Ann. Phys. (N.Y.) **4**, 372 (1958).
[18] F. Cattaneo and D. W. Hughes, arXiv:0805.2138 [Mon. Mot. R. Astron. Soc. (to be published)].
[19] I. Rogachevskii and N. Kleeorin, Phys. Rev. E **75**, 046305 (2007).
[20] T. A. Yousef, T. Heinemann, A. A. Schekochihin, N. Kleeorin, I. Rogachevskii, A. B. Isakov, S. C. Cowley, and J. C. McWilliams, Phys. Rev. Lett. **100**, 184501 (2008).
[21] T. A. Yousef, T. Heinemann, F. Rincon, A. A. Schekochihin, N. Kleeorin, I. Rogachevskii, S. C. Cowley, and J. C. McWilliams, Astron. Nachr. **329**, 737 (2008).
[22] P. J. Käpylä, M. J. Korpi, and A. Brandenburg, Astron. Astrophys. **491**, 353 (2008).
[23] Y. Du and E. Ott, J. Fluid Mech. **257**, 265 (1993).