Superkicks in Hyperbolic Encounters of Binary Black Holes

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Generic inspirals and mergers of binary black holes produce beamed emission of gravitational radiation that can lead to a gravitational recoil or *kick* of the final black hole. The kick velocity depends on the mass ratio and spins of the binary as well as on the dynamics of the binary configuration. Studies have focused so far on the most astrophysically relevant configuration of quasicircular inspirals, for which kicks as large as $\sim 3300\ \mathrm{km\,s^{-1}}$ have been found. We present the first study of gravitational recoil in *hyperbolic* encounters. Contrary to quasicircular configurations, in which the beamed radiation tends to average during the inspiral, radiation from hyperbolic encounters is plunge dominated, resulting in an enhancement of preferential beaming. As a consequence, it is possible in highly relativistic scatterings to achieve kick velocities as large as $10\,000\ \mathrm{km\,s^{-1}}$.

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Introduction.—Numerical relativity estimates of the kick inflicted on the final (BH) from generic inspirals and mergers of (BBH) have triggered tremendous excitement in astrophysics. This is mainly due to the fact that most galaxies host a (SMBH) at their centers [1,2]. As galaxies merge, a kick to the final BH could have profound implications in subsequent mergers, affecting the growth of SMBHs via mergers as well as the population of galaxies containing SMBHs. In addition, there have been several suggestions of direct observational signatures of putative BH recoils [3–8], with one study [9] presenting evidence for the first candidate of a recoiling SMBH.

BH kick velocities depend on the mass ratio and spins of the merging BHs as well as on the initial configuration and subsequent dynamics of the binary. Studies published to date have concerned the most astrophysically relevant configuration, that of quasicircular inspirals [10–17].

Motivated by our previous study [18] of the final spin of BHs from scattering mergers of BBHs, we present the first extension of kicks to hyperbolic encounters. There are crucial differences between hyperbolic and quasicircular configurations that affect the kick to the final BH. For quasicircular orbits, the emitted radiation is asymmetrically beamed, and carries linear momentum with it. But it tends to average out during the inspiral [19], producing a modest wobbling and drift of the center of mass of the binary. The final kick arises because as the binary approaches the plunge, the *averaging* loses its effectiveness, leading to a gradual recoil build-up. Both numerical simulations and post-Newtonian studies [19,20] have confirmed the gradual kick accumulation during the inspiral, and, in addition, the studies have shown that most of the recoil is generated during the plunge. In some instances, during the plunge and ring-down there is also a period of *antikick* before reaching the final kick value [21,22].

The main motivation for the present study was to consider plunge-dominated configurations to investigate whether kicks comparable to those for quasicircular inspirals can be found. We were surprised to find that kicks as large as $10\,000\ \mathrm{km\,s^{-1}}$ are produced for spin configurations equivalent to those studied in quasicircular inspirals. Two qualitative features of hyperbolic encounters contribute to these larger kicks. Not only are hyperbolic encounters plunge dominated, but the nature of the plunge is such, i.e., highly relativistic, that it enhances the beaming of radiated linear momentum [23].

Computational methodology.—We use the same computational infrastructure and methodology as in previous studies [10,12,18], namely, a BSSN code with moving puncture gauge conditions. The hyperbolic encounters are initiated with Bowen-York initial data [24]. The data consist of two equal-mass BHs with masses $m = M/2$ located along the x axis: $\mathrm{BH}_{\pm}$ is located at $x = \pm 5M$ and has linear momentum $\vec{P}_{\pm} = \pm(P\cos\theta, P\sin\theta, 0)$ with θ the angle in the orbital plane with respect to the x axis. The total initial orbital angular momentum is then $\vec{L}/M^2 = 10(P/M)\sin\theta\hat{z}$. The spins of each BH are in the orbital plane: $\mathrm{BH}_{\pm}$ has spin $\vec{S}_{\pm} = \pm(S\cos\phi_{\pm}, S\sin\phi_{\pm}, 0)$, with $\phi_{\pm}$ the angle in the orbital plane with respect to the $\pm x$-axis. The parameter space of our encounters is quite large: $\{P, \theta, S, \phi_{\pm}\}$. However, from exploratory runs we have gained a good understanding of the parameter space and isolated those parameters that can be kept fixed without seriously compromising the goals of the study.

For most of the runs, we have kept the spin magnitudes at $S/M^2 = 0.2$ ($a/m = 0.8$), with the exception of those runs used to investigate the dependence of the kick on S. We kept fixed also the impact angle at $\theta = 153.4°$. We considered some other angles but found that this is the angle for which we obtained the largest kicks. Finally, for most cases we kept the spin direction of BH_- located at $x = -5M$ fixed at $\phi_- = 0°$ or $45°$. Once other parameters were fixed from among a small set of values, the parameter we varied in general was the linear momentum magnitude P.

Kicks are computed from a surface integral [25,26] involving the Weyl curvature tensor Ψ_4. Although strictly speaking this kick formula must be evaluated in the limit $r \to \infty$, we applied it at extraction radii $r/M = \{40, 50, 75, 85, 100\}$. The resultant kicks were fitted to both $V(r) = V_\infty^{(1)} + K_0/r$ and $V(r) = V_\infty^{(2)} + K_1/r + K_2/r^2$. The extrapolated $r \to \infty$ kicks and their errors were estimated from $V_\infty = (V_\infty^{(1)} + V_\infty^{(2)})/2$ and $\delta V_\infty = |V_\infty^{(1)} - V_\infty^{(2)}|$, respectively. Unless explicitly noted, all the reported kick velocities and energy radiated were obtained with resolution $h = M/0.8$ on the mesh used for the kick computation. Every run had 10 levels of factor-of-2 refinement, with outer boundaries at $\sim 320M$. We discuss below the convergence and errors of kick estimates as a function of the grid spacing and the extraction radius.

Results.—Table I gives the components of the recoil along the initial orbital angular momentum $V_\|$ and in the initial orbital plane $V_\perp$ in km s^{-1} (and the total recoil V) for the cases with $\phi_- = 0°$. Notice that the largest recoil in this case happens when $\phi_+ = 45°$. Another important observation is that, although the dominant component of the kick is $V_\|$, as we increase ϕ_+ a substantial component of the kick is also generated in the orbital plane (see $V_\perp$). The rightmost column in Table I gives the $r \to \infty$ extrapolated radiated energy as a percentage of the total initial energy of the binary. We see that large angular momentum increases the energy radiated, up to very substantial values, as large as 13%. For even larger angular momenta, we expect a falloff of the radiated energy.

For the case $P/M = 0.2665$, $\phi_+ = 45°$ in Table I, we have carried out simulations to investigate the dependence of the kick with the initial BH spins (a/m). The results are displayed in Fig. 1. We find that, to first order, the kick is proportional to a/M_h, as found in quasicircular orbits [12]. However, as the initial spin of the BHs grows, we found hints of the quadratic spin dependence also obtained in quasicircular orbits [27].

The configuration we have found to yield the largest kick has the spins antialigned $\phi_+ = \phi_- = \phi$, as in the case of quasicircular orbits [14–16], and initial linear momentum $P/M = 0.3075$, corresponding to initial angular momentum $L/M^2 = 1.275$. The kicks are essentially along the direction of the initial orbital angular momentum. Figure 2 shows the kick velocity $V_\|$ as a function of $\vartheta = \theta - \phi$, where ϑ measures the angle between the initial spin and linear momentum vectors. The solid line represents the fit $V_\| = -10\,239\cos(\vartheta + 86°)$ km s^{-1}. This is the same angular dependence found in quasicircular orbits [16]. Furthermore, as with circular inspirals, the maximum kick is obtained for $\vartheta \approx 90°$ although in these hyperbolic encounters the kick is significantly larger, close to 10 000 km s^{-1}.

To investigate the dependence on the initial orbital angular momentum, we selected the case $\phi_+ = \phi_- = \phi = 45°$ or $\vartheta = 108°$ in Fig. 2, which yields a kick magnitude of 9589 km s^{-1} and 15% energy radiated, and carried out simulations varying the initial momentum of the BHs. Figure 3 shows the magnitude of the kick velocity V (top panel) and the energy radiated E_{rad} in percent of the initial energy (bottom panel) as a function of L/M^2. Note

TABLE I. Configuration parameters P/M, ϕ_+, L/M^2 for $\phi_- = 0°$ and $S/M^2 = 0.2$ as well as final BH kick velocity (km s^{-1}) and percent of energy radiated.

P/M	ϕ_+	L/M^2	$V_\|$	$V_\perp$	V	E_{rad}(%)
0.2379	0	1.064	5155	0	5155	4.9
0.2665	0	1.192	6561	0	6561	7.6
0.2739	0	1.225	6505	0	6505	8.4
0.2851	0	1.275	5424	0	5424	10.1
0.2952	0	1.320	3140	0	3140	11.8
0.2379	45	1.064	5483	709	5529	5.2
0.2665	45	1.192	7614	932	7671	8.0
0.2739	45	1.225	7830	930	7885	8.9
0.2851	45	1.275	7227	778	7269	10.4
0.2952	45	1.320	5026	384	5041	12.2
0.2379	90	1.064	4291	1074	4423	5.5
0.2665	90	1.192	6485	1519	6661	8.7
0.2739	90	1.225	6740	1528	6911	9.6
0.2851	90	1.275	6001	1197	6119	11.5
0.2952	90	1.320	3791	410	3813	13.1

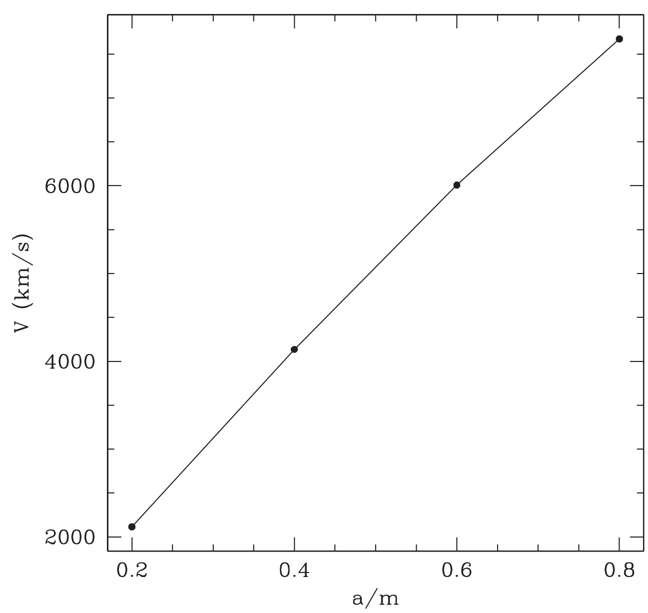

FIG. 1. Kick velocity as a function of the initial BH spin (a/m) for $P/M = 0.2665$, $\phi_+ = 45°$

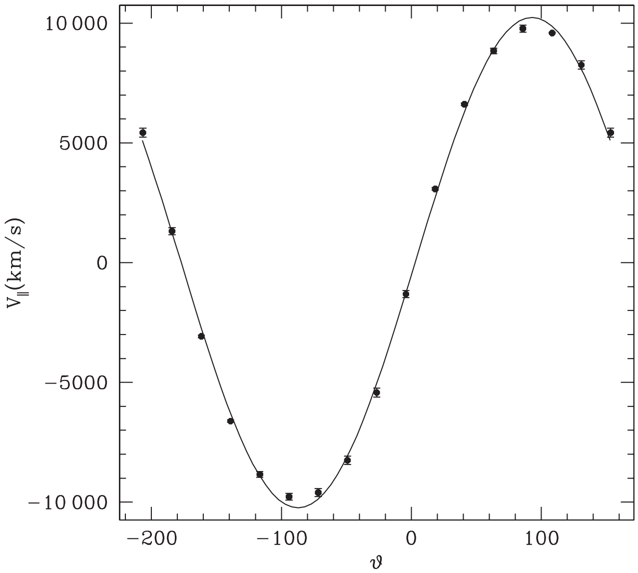

FIG. 2. Kick velocity $V_\parallel$ as a function of $\vartheta = \theta - \phi$ for $\phi_+ = \phi_- = \phi$ and $P/M = 0.3075$ ($L/M^2 = 1.275$). The solid line is the fit $V_\parallel = -10\,256\ \cos(\vartheta + 86°)\ \mathrm{km\,s^{-1}}$.

that the maximum of V does not occur for maximum $E_{\rm rad}$. The case with the largest initial angular momentum, $L/M^2 = 1.375$, has an interesting feature as displayed in the top panel of Fig. 4. There is a pronounced antikick before the recoil reaches its final value. The reason for this antikick could be due to the fact that the plunge is not as pronounced, appearing more *circularlike* or with a *hung-up*. That is, there is a decrease in the rate at which the binary comes together, as one can see in the bottom panel of Fig. 4 during the time interval 20–40M. Thus, the flux vectors responsible for the kick have more time to undergo the phase shift needed for the appearance of an antikick [22]. In Fig. 4 we have included for comparison the case $L/M^2 = 1.225$ (dash lines) which is devoid of antikick and hung-up.

In addition to the errors from extracting the kick at a finite radius, the values of the kicks are affected by numerical finite differencing resolution. To investigate these errors, we selected the case $P/M = 0.2665$, $\phi_+ = 45°$ from Table I and carried out simulations with resolutions $h = \{M/0.4, M/0.5, M/0.63, M/0.8\}$ on the extraction level. Figure 5 shows the data (points) and the corresponding fitting (lines) to $V(r) = V_\infty^{(2)} + K_1/r + K_2/r^2$. We find that at a given extraction radius, the error in the kick scales as h^3, and the kick itself grows with resolution. Based on this h^3 behavior, we estimate errors of $\lesssim 5\%$ in the extrapolated, $r \rightarrow \infty$, values of the kicks when computed using a resolution of $h = M/0.8$. We expect similar accuracy for all the other kicks presented in this work.

Conclusions.—We have carried out a study of the gravitational recoil of the final BH in the merger of hyperbolic BBH encounters. We have found that in general the kick velocities for in-plane initial BH spins are significantly larger than those from the corresponding quasicircular mergers. Our results suggest that kicks as large as $10\,000\ \mathrm{km\,s^{-1}}$ are possible. We have also found that the dependence of the kick on the initial magnitude of the BH's spins is similar to the quasicircular case. An analytic multipolar analysis of encounters for similar configurations can be found in Ref. [23]. A recent study by O'Leary, Kocsis, and Loeb [28] has found that in dense population environ-

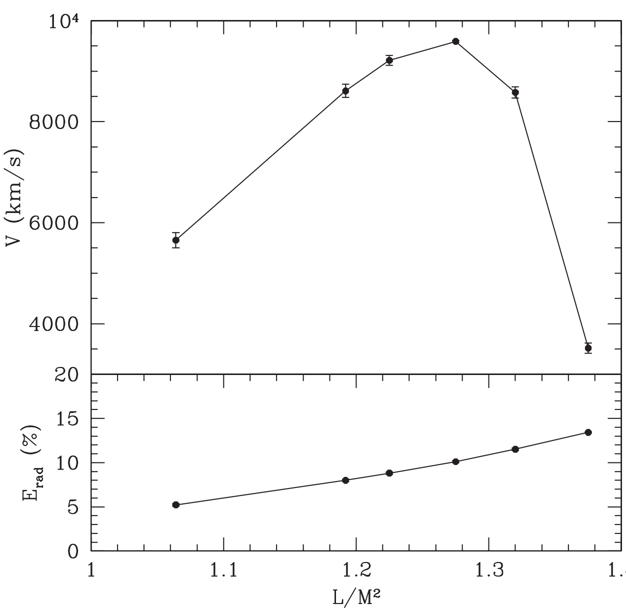

FIG. 3. Magnitude of the kick velocity (top panel) and percent of energy radiated (bottom panel) as a function of L/M^2 for $\phi_+ = \phi_- = 45°$.

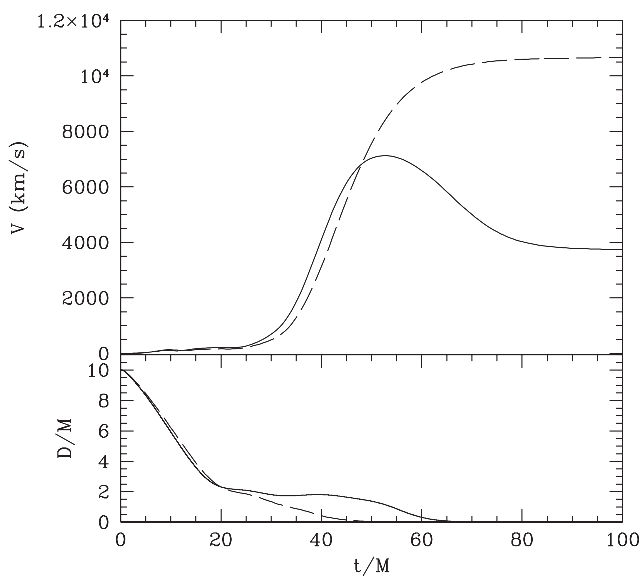

FIG. 4. Kick velocity accumulation (top panel) and binary separation (bottom panel) as a function of time for $\phi_+ = \phi_+ = 45°$ with $L/M^2 = 1.375$ (solid line) and $L/M^2 = 1.225$ (long dash).

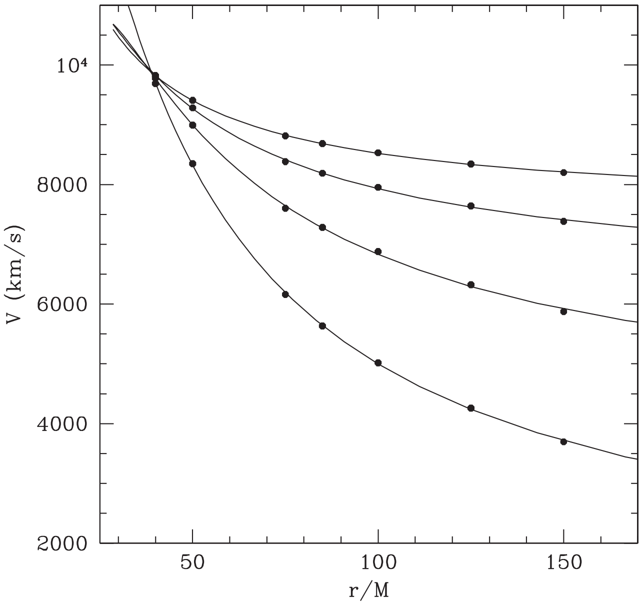

FIG. 5. Magnitude of the kick velocity as a function of the extraction radius for ($P/M = 0.2665$, $\phi_+ = 45°$). From bottom to top are, respectively, resolutions of $h = \{M/0.4, M/0.5, M/0.63, M/0.8\}$ at the extraction level. Lines are the fit $V(r) = V_\infty^{(2)} + (K_1/r + K_2/r^2)$.

ments, there is a non-negligible probability for close flybys or hyperbolic encounters [29]. Most of the cases they considered are those in which after the first passage, the BHs release enough energy to become bound with large initial eccentricity. The hyperbolic encounters we considered are the extreme case: relativistic scatterings with velocities of $\sim 0.4c$ at $t \to -\infty$ and immediate merger. This study represents an example in general relativity of phenomena dominated by the nonlinear regime in the theory, a study accessible only with the tools provided by numerical relativity (see Ref. [30] for another example).

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