Proposed Bell Experiment with Genuine Energy-Time Entanglement

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Franson's Bell experiment with energy-time entanglement [Phys. Rev. Lett. **62**, 2205 (1989)] does not rule out all local hidden variable models. This defect can be exploited to compromise the security of Bell inequality-based quantum cryptography. We introduce a novel Bell experiment using genuine energy-time entanglement, based on a novel interferometer, which rules out all local hidden variable models. The scheme is feasible with actual technology.

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Two particles exhibit "energy-time entanglement" when they are emitted at the same time in an energy-conserving process and the essential uncertainty in the time of emission makes undistinguishable two alternative paths that the particles can take. Franson [1] proposed an experiment to demonstrate the violation of local realism [2] using energy-time entanglement, based on a formal violation of the Bell Clauser-Horne-Shimony-Holt (CHSH) inequality [3]. However, Aerts *et al.* [4] showed that, even in the ideal case of perfect preparation and perfect detection efficiency, there is a local hidden variable (LHV) model that simulates the results predicted by quantum mechanics for the experiment proposed by Franson [1]. This model proves that "the Franson experiment does not and cannot violate local realism" and that "[t]he reported violations of local realism from Franson experiments [5] have to be reexamined" [4].

Despite this fundamental deficiency, and despite that this defect can be exploited to create a Trojan horse attack in Bell inequality-based quantum cryptography [6], Franson-type experiments have been extensively used for Bell tests and Bell inequality-based quantum cryptography [7], have become standard in quantum optics [8,9], and an extended belief is that "the results of experiments with the Franson experiment violate Bell's inequalities" [9]. This is particularly surprising, given that recent research has emphasized the fundamental role of a (loophole-free) violation of the Bell inequalities in proving the device-independent security of key distribution protocols [10], and in detecting entanglement [11].

Polarization entanglement can be transformed into energy-time entanglement [12]. However, to our knowledge, there is no single experiment showing a violation of the Bell-CHSH inequality using genuine energy-time entanglement (or "time-bin entanglement" [13]) that cannot be simulated by a LHV model. By "genuine" we mean not obtained by transforming a previous form of entanglement, but created because the essential uncertainty in the time of emission makes two alternative paths undistinguishable.

Because of the above reasons, a single experiment using energy-time entanglement able to rule out all possible LHV models is of particular interest. The aim of this Letter is to describe such an experiment by means of a novel interferometric scheme. The main purpose of the new scheme is not to compete with existing interferometers used for quantum communication in terms of practical usability, but to fix a fundamental defect common to all of them.

We will first describe the Franson Bell-CHSH experiment. Then, we will introduce a LHV model reproducing any conceivable violation of the Bell-CHSH inequality. The model underlines why a Franson-type experiment does not and cannot be used to violate local realism. Then, we will introduce a new two-photon energy-time Bell-CHSH experiment that avoids these problems and can be used for a conclusive Bell test.

The Franson Bell-CHSH experiment.—The setup of a Franson Bell-CHSH experiment is in Fig. 1. The source emits two photons, photon 1 to the left and photon 2 to the right. Each of them is fed into an unbalanced interferometer. BS_i are beam splitters and M_i are perfect mirrors. There are two distant observers, Alice on the left and Bob on the right. Alice randomly chooses the phase of the phase shifter ϕ_A between A_0 and A_1, and records the counts in each of her detectors (labeled $a = +1$ and $a = -1$), the

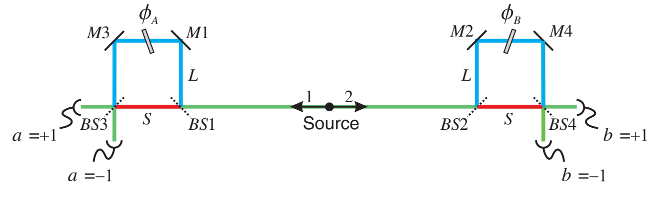

FIG. 1 (color online). Generic setup of the Franson Bell experiment.

detection times, and the phase settings at $t_D - t_I$, where t_D is the detection time and t_I is the time the photon takes to reach the detector from the location of the phase shifter ϕ_A. Similarly, Bob chooses ϕ_B between B_0 and B_1, and records the counts in each of his detectors (labeled $b = +1$ and $b = -1$), the detection times, and the phase settings. The setup must satisfy four requirements: (I) To have two-photon interference, the emission of the two photons must be simultaneous, the moment of emission unpredictable, and both interferometers identical. If the detections of the two photons are coincident, there is no information about whether both photons took the short paths S or both took the long paths L. A simultaneous random emission is achieved in actual experiments by two methods, both based on spontaneous parametric down conversion. In energy-time experiments, a nonlinear crystal is pumped continuously by a monochromatic laser so the moment of emission is unpredictable in a temporal window equal to the coherence time of the pump laser. In time-bin experiments, a nonlinear crystal is pumped by pulses previously passing through an unbalanced interferometer, so it is the uncertainty of which pulse, the earlier or the later, has caused the emission what provokes the uncertainty in the emission time. In both cases, the simultaneity of the emission is guaranteed by the conservation of energy. (II) To prevent single-photon interference, the difference between paths L and S, i.e., twice the distance between $BS1$ and $M1$, $\Delta\mathcal{L} = 2d(BS1, M1)$ (See Fig. 1), must satisfy $\Delta\mathcal{L} > ct_{\rm coh}$, where c is the speed of light and $t_{\rm coh}$ is the coherence time of the photons. (III) To make distinguishable those events where one photon takes S and the other takes L, $\Delta\mathcal{L}$ must satisfy $\Delta\mathcal{L} > c\Delta t_{\rm coinc}$, where $\Delta t_{\rm coinc}$ is the duration of the coincidence window. (IV) To prevent that the local phase setting at one side can affect the outcome at the other side, the local phase settings must randomly switch (ϕ_A between A_0 and A_1, and ϕ_B between B_0 and B_1) with a frequency of the order c/D, where $D = d(\text{Source}, BS1)$.

The observers record all their data locally and then compare them. If the detectors are perfect they find that

$$P(A_i = +1) = P(A_i = -1) = \tfrac{1}{2}, \tag{1a}$$

$$P(B_j = +1) = P(B_j = -1) = \tfrac{1}{2}, \tag{1b}$$

for $i, j \in \{0, 1\}$. $P(A_0 = +1)$ is the probability of detecting a photon in the detector $a = +1$ if the setting of ϕ_A was A_0. They also find 25% of two-photon events in which photon 1 is detected a time $\Delta\mathcal{L}/c$ before photon 2, and 25% of events in which photon 1 is detected $\Delta\mathcal{L}/c$ after photon 2. The observers reject this 50% of events and keep the 50% that are coincident. For these selected events, quantum mechanics predicts that

$$P(A_i = a, B_j = b) = \tfrac{1}{4}[1 + ab\cos(\phi_{A_i} + \phi_{B_j})], \tag{2}$$

where $a, b \in \{-1, +1\}$ and ϕ_{A_i} (ϕ_{B_j}) is the phase setting corresponding to A_i (B_j).

The Bell-CHSH inequality is

$$-2 \leq \beta_{\rm CHSH} \leq 2, \tag{3}$$

where

$$\beta_{\rm CHSH} = \langle A_0B_0\rangle + \langle A_0B_1\rangle + \langle A_1B_0\rangle - \langle A_1B_1\rangle. \tag{4}$$

According to quantum mechanics, the maximal violation of the Bell-CHSH inequality is $\beta_{\rm CHSH} = 2\sqrt{2}$ [14], and is obtained, e.g., with $\phi_{A_0} = 0$, $\phi_{A_1} = \frac{\pi}{2}$, $\phi_{B_0} = -\frac{\pi}{4}$, $\phi_{B_1} = \frac{\pi}{4}$.

LHV models for the Franson experiment.—A LHV theory for the Franson experiment must describe how each of the photons makes two decisions. The $+1/-1$ decision: the decision of a detection to occur at detector $+1$ or at detector -1, and the S/L decision: the decision of a detection to occur at time $t_D = t$ or a time $t_D = t + \frac{\Delta\mathcal{L}}{c}$. Both decisions may be made as late as the detection time t_D, and may be based on events in the backward light cones of the detections. In a Franson-type setup both decisions may be based on the corresponding local phase setting at $t_D - t_I$. For a conclusive Bell test, there is no problem if photons make the $+1/-1$ decision based on the local phase setting. The problem is that the 50% postselection procedure should be independent on the phase settings, otherwise the Bell-CHSH inequality (3) is not valid. In the Franson experiment the phase setting at $t_D - t_I$ can causally affect the decision of a detection of the corresponding photon to occur at time $t_D = t$ or a time $t_D = t + \frac{\Delta\mathcal{L}}{c}$. If the S/L decision can depend on the phase settings, then, after the 50% postselection procedure, one can formally obtain not only the violations predicted by quantum mechanics, as proven in [4], but any value of $\beta_{\rm CHSH}$, even those forbidden by quantum mechanics. This is proven by constructing a family of explicit LHV models.

Consider the 64 sets of local instructions in Tables I and II. For instance, if the pair of photons follows the first set of

TABLE I. 32 sets of instructions (out of 64) of the LHV model (the other 32 are in Table II). Each row represents 4 sets of local instructions (first 4 entries) and their corresponding contributions for the calculation of $\beta_{\rm CHSH}$ after applying the postselection procedure of the Franson experiment (last 4 entries). For each row, two sets (corresponding to $\pm$ signs) are explicitly written, while the other two can be obtained by changing all signs.

A_0	A_1	B_0	B_1	$\langle A_0B_0\rangle$	$\langle A_0B_1\rangle$	$\langle A_1B_0\rangle$	$\langle A_1B_1\rangle$
$S+$	$S+$	$S+$	$L\pm$	$+1$	rejected	$+1$	rejected
$L+$	$L+$	$L+$	$S\pm$	$+1$	rejected	$+1$	rejected
$S+$	$S-$	$L\pm$	$S+$	rejected	$+1$	rejected	-1
$L+$	$L-$	$S\pm$	$L+$	rejected	$+1$	rejected	-1
$S+$	$L\pm$	$S+$	$S+$	$+1$	$+1$	rejected	rejected
$L+$	$S\pm$	$L+$	$L+$	$+1$	$+1$	rejected	rejected
$L\pm$	$S+$	$S+$	$S-$	rejected	rejected	$+1$	-1
$S\pm$	$L+$	$L+$	$L-$	rejected	rejected	$+1$	-1

TABLE II. 32 sets of instructions of the LHV model.

A_0	A_1	B_0	B_1	$\langle A_0B_0\rangle$	$\langle A_0B_1\rangle$	$\langle A_1B_0\rangle$	$\langle A_1B_1\rangle$
$S+$	$S+$	$S-$	$L\pm$	-1	rejected	-1	rejected
$L+$	$L+$	$L-$	$S\pm$	-1	rejected	-1	rejected
$S+$	$S-$	$L\pm$	$S-$	rejected	-1	rejected	$+1$
$L+$	$L-$	$S\pm$	$L-$	rejected	-1	rejected	$+1$
$S-$	$L\pm$	$S+$	$S+$	-1	-1	rejected	rejected
$L-$	$S\pm$	$L+$	$L+$	-1	-1	rejected	rejected
$L\pm$	$S-$	$S+$	$S-$	rejected	rejected	-1	$+1$
$S\pm$	$L-$	$L+$	$L-$	rejected	rejected	-1	$+1$

local instructions in Table I, $(A_0 =)S+$, $(A_1 =)S+$, $(B_0 =)S-$, $(B_1 =)L+$, then, if the setting of ϕ_A is A_0 or A_1, photon 1 will be detected by the detector $a = +1$ at time t (corresponding to the path S), and if the setting of ϕ_B is B_0, photon 2 will be detected by $b = -1$ at time t, but if the setting of ϕ_B is B_1, photon 2 will be detected by $b = +1$ at time $t + \frac{\Delta\mathcal{L}}{c}$ (corresponding to the path L). If each of the 32 sets of instructions in Table I occurs with probability $p/32$, and each of the 32 sets of instructions in Table II with probability $(1-p)/32$, then it is easy to see that, for any value of $0 \leq p \leq 1$, the model gives 25% of SL events, 25% of LS events, 50% of SS or LL events, and satisfies (1a) and (1b). If $p = 0$, the model gives $\beta_{\rm CHSH} = -4$. If $p = 1$, the model gives $\beta_{\rm CHSH} = 4$. If $0 < p < 1$, the model gives any value between $-4 < \beta_{\rm CHSH} < 4$. Specifically, a maximal quantum violation $\beta_{\rm CHSH} = 2\sqrt{2}$, satisfying (2), is obtained when $p = (2+\sqrt{2})/4$.

The reason why this LHV model is possible is that the 50% postselection procedure in Franson's experiment allows the subensemble of selected events to depend on the phase settings. For instance, the first 8 sets of instructions in Table I are rejected only when $\phi_B = B_1$. The main aim of this Letter is to introduce a similar experiment which does not have this problem.

There is a previously proposed solution consisting on replacing the beam splitters BS_1 and BS_2 in Fig. 1 by switchers synchronized with the source [13]. However, these active switchers are replaced in actual experiments by passive beam splitters [7,13] that force a Franson-type postselection with the same problem described above.

One way to avoid the problem is to make an extra assumption, namely, that the decision of being detected at time $t_D = t$ or a time $t_D = t + \frac{\Delta\mathcal{L}}{c}$ is actually made at the first beam splitter, before having information of the local phase settings [4,15]. This assumption is similar to the fair sampling assumption, namely, that the probability of rejection does not depend on the measurement settings. As we have seen, there are local models that do not satisfy this assumption. The experiment we propose does not require this extra assumption.

Proposed energy-time entanglement Bell experiment.— The setup of the new Bell experiment is illustrated in Fig. 2. The source emits two photons, photon 1 to the left

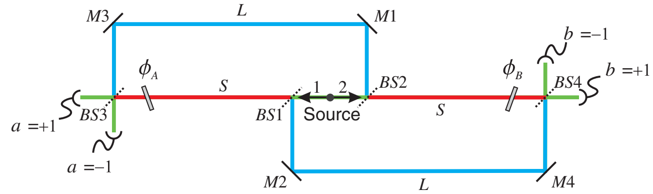

FIG. 2 (color online). The generic setup of the proposed energy-time (and time-bin) Bell experiment.

and photon 2 to the right. The S path of photon 1 (photon 2) ends on the detectors a on the left (b on the right). The difference with Fig. 1 is that now the L path of photon 1 (photon 2) ends on the detectors b (a). In this setup, the two photons end in different sides only when both are detected in coincidence. If one photon takes S and the other photon takes L, both will end on detectors of the same side. An interferometer with this last property is described in [16].

The data that the observers must record are the same as in Franson's experiment. The setup must satisfy the following requirements: (I′) To have two-photon interference, the emission of the two photons must be simultaneous, the moment of emission unpredictable, and both arms of the setup identical. The phase stabilization of the entire setup of Fig. 2 is more difficult than in Franson's experiment. (II′) Single-photon interference is not possible in the setup of Fig. 2. (III′) To temporally distinguish two photons arriving at the same detector at times t and $t + \frac{\Delta\mathcal{L}'}{c}$, where $\Delta\mathcal{L}' = 2[d(\text{Source}, BS2) + d(BS2, M1)]$ (see Fig. 2), the dead time of the detectors must be smaller than $\frac{\Delta\mathcal{L}'}{c}$. For detectors with a dead time of 1 ns, $\Delta\mathcal{L}' > 30$ cm. (IV′) The probability of two two-photon events in $\frac{\Delta\mathcal{L}'}{c}$ must be negligible. This naturally occurs when using standard nonlinear crystals pumped continuously. (V′) To prevent that the local phase setting at one side can affect the outcome at the other side, the local phase settings must randomly switch (ϕ_A between A_0 and A_1, and ϕ_B between B_0 and B_1) with a frequency of the order c/D', where $D' = d(\text{Source}, \phi_A) \gg \Delta\mathcal{L}'$.

There is a trade-off between the phase stabilization of the apparatus (which requires a short interferometer) and the prevention of reciprocal influences between the two local phase settings (which requires a long interferometer). By considering a random phase modulation frequency of 300 kHz, an interferometer about 1 km long would be needed. Current technology allows us to stabilize interferometers of up 4 km long (for instance, one of the interferometers of the LIGO experiment is 4 km long). With these stable interferometers, the experiment would be feasible.

The predictions of quantum mechanics for the setup of Fig. 2 are similar to those in Franson's proposal: Eqs. (1a) and (1b) hold, there is 25% of events in which both photons are detected on the left at times t and $t + \frac{\Delta\mathcal{L}'}{c}$, 25% of events in which both photons are detected on the right, and 50% of coincident events for which (2) holds. The observers must keep the coincident events and reject those giving two detections on detectors of the same side. The main advantages of this setup are: (i) The rejection of events is local and does not require communication between the observers. (ii) The selection and rejection of events is independent of the local phase settings. This is the crucial difference with Franson's experiment and deserves a detailed examination. First consider a selected event: both photons have been detected at time t_D, one in a detector a on the left, and the other in a detector b on the right. t_I is the time a photon takes from ϕ_A (ϕ_B) to a detector a (b). The phase setting of ϕ_A (ϕ_B) at $t_D - t_I$ is in the backward light cone of the photon detected in a (b), but the point is, could a different value of one or both of the phase settings have caused that this selected event would become a rejected event in which both photons are detected on the same side? The answer is no. This would require a mechanism to make one detection to "wait" until the information about the setting in other side comes. However, when this information has finally arrived, the phase settings (both of them) have changed, so this information is useless to base a decision on it.

Now consider a rejected event. For instance, one in which both photons are detected in the detectors a on the left, one at time $t_D = t$, and the other at $t_D = t + \frac{\Delta \mathcal{L}'}{c}$. Then, the phase settings of ϕ_B at times $t_D - t_I$ are out of the backward light cones of the detected photons. The photons cannot have based their decisions on the phase settings of ϕ_B. A different value of ϕ_A cannot have caused that this rejected event would become a selected event. This would require a mechanism to make one detection to wait until the information about the setting arrives to the other side, and when this information has arrived, the phase setting of ϕ_A has changed so this information is useless.

For the proposed setup, there is no physical mechanism preserving locality which can turn a selected (rejected) event into a rejected (selected) event. The selected events are independent of the local phase settings. For the selected events, only the $+1/-1$ decision can depend on the phase settings. This is exactly the assumption under which the Bell-CHSH inequality (3) is valid. Therefore, an experimental violation of (3) using the setup of Fig. 2 and the postselection procedure described before provides a conclusive (assuming perfect detectors) test of local realism using energy-time (or time-bin) entanglement. Indeed, the proposed setup opens up the possibility of using genuine energy-time or time-bin entanglement for many other quantum information experiments.

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