# Spin Squeezing of a Cold Atomic Ensemble with the Nuclear Spin of One-Half

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In order to establish an applicable system for advanced quantum information processing based on the interaction between light and atoms, we have demonstrated a quantum nondemolition measurement with a collective spin of cold ytterbium atoms ($^{171}$Yb), and have observed $1.8^{+2.4}_{-1.5}$ dB spin squeezing. Since $^{171}$Yb atoms have only a nuclear spin of one-half in the ground state, the system constitutes the simplest spin ensemble and is thus robust against decoherence. We used very short pulses with a width of 100 ns, and as a result the interaction time became much shorter than the decoherence time, which is important for multistep quantum information processing.

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Controlling the interaction between light and atoms is the primary objective for progress in atomic physics and precise measurements as well as for quantum information processing using atoms. One may think that quantum fluctuations degrade the performance of the measurements. The quantum nondemolition (QND) measurement method has been developed in order to manage the quantum noise, and is also useful for quantum-state preparation devices and producing a quantum entanglement [1] as well as a feasible model for capturing basic features of the quantum measurement process [2,3]. QND measurements of the photon number and the amplitude quadrature of light have been realized [4–7]. QND measurements of the collective spin of an atomic ensemble (spin-QNDM) via the Faraday rotation (FR) interaction with linearly polarized off-resonant light have been proposed [8,9]. Recently, there has been a growing interest in the application of spin-QNDM to studies in condensed matter physics, metrology, particle physics, and quantum information. For example, such measurements can be applied to the detection of strongly correlated states of quantum many-body systems simulated by ultracold atoms in optical lattices [10], and squeezed spin states [11] generated by spin-QNDM can improve the measurement precision of the atomic clock transition [12,13] and of the permanent electric dipole moment [14]. Furthermore, spin-QNDM is also useful for implementing continuous-variable quantum interfaces between light and atoms, such as quantum memory, quantum teleportation, and so on [1,15–18].

Previous experimental approaches to spin-QNDM [1,19] were based on thermal alkali atoms and continuous-wave light or long light pulses (typically 1 ms), which is comparable to the decoherence time of the atomic spin [1,19]. In most cases of the application of this method, the use of cold atoms is preferable. In order to realize a practical quantum interface, it is essential to accomplish the interaction on a time scale much shorter than the decoherence time. In addition, it should be noted that the description of the FR interaction is based on a standard spin model composed of spin-one-half atoms [20,21]. However, the cesium atoms used in the previous experiments have a more complicated multilevel structure, which causes serious difficulties, as pointed out in [20,21]. Therefore, it is valuable to demonstrate spin-QNDM with short pulses and cold spin-one-half atoms.

In this Letter, we report a successful experimental realization of spin-QNDM with laser-cooled ytterbium ($^{171}$Yb) atoms and short light pulses. The important difference between our system and that based on alkali atoms is the fact that $^{171}$Yb atoms have the simplest ground state with a nuclear spin of one-half and have no electron spin. Therefore, the system is robust against stray magnetic fields since the magnetic moment of nuclear spin is a thousandth of that of electron spin. By using short light pulses with a width of 100 ns, more than a hundred time operations are expected to be performed within the coherence time. In order to show the realization of spin-QNDM, we have investigated the correlations between the light pulses which sequentially interact with the atoms.

We note that the spin-QNDM for $^{171}$Yb atoms realized in this work is especially important since cold $^{171}$Yb atoms in an optical lattice are considered to be one of the promising candidates for the future optical standard [22,23]. The performance of the $^{171}$Yb-based optical lattice clock, which is limited by quantum noise is expected to be significantly improved by mapping the ground-state spin squeezing onto the clock transition $^1S_0 \leftrightarrow {}^3P_0$ [13]. In this regard, we have recently learned that spin squeezing for the clock transition between the hyperfine states of alkali atoms has been realized with a somewhat different method [24].

In order to describe the FR interaction, let us define the normalized collective spin operator of the atoms $\vec{\tilde{J}} = (\tilde{J}_x, \tilde{J}_y, \tilde{J}_z) = (1/\sqrt{N_A/2}) \sum_{i=1}^{N_A} \vec{j}_i$, where $\vec{j}_i$ is a spin operator of a single atom and $N_A$ is the number of the atoms. The normalized Stokes operator of a pulsed light $\vec{\tilde{S}} = (\tilde{S}_x, \tilde{S}_y, \tilde{S}_z)$ is defined by $\tilde{S}_x = (\sqrt{2N_L})^{-1}\int_0^t (a_+^\dagger a_- + a_-^\dagger a_+)dT$, $\tilde{S}_y = (i\sqrt{2N_L})^{-1}\int_0^t (a_+^\dagger a_- - a_-^\dagger a_+)dT$, $\tilde{S}_z = (\sqrt{2N_L})^{-1}\int_0^t (a_+^\dagger a_+ - a_-^\dagger a_-)dT$, where $N_L$ is the mean photon number of the pulse, $t$ is the pulse duration, and $a_\pm$ is the annihilation operator of $\sigma_\pm$ circular polarization mode, respectively [25]. In our experiment, we consider the situation that the initial states of the light and atoms are coherent states and polarized in the $x$-direction, namely, $\tilde{J}_x \simeq \sqrt{J} \equiv \sqrt{N_A/2}$ and $\tilde{S}_x \simeq \sqrt{S} \equiv \sqrt{N_L/2}$ hold. Then, the angular momentum commutation relation for each of $\vec{\tilde{J}}$ and $\vec{\tilde{S}}$ implies $[\tilde{J}_y, \tilde{J}_z] = i$ and $[\tilde{S}_y, \tilde{S}_z] = i$, and they satisfy the uncertainty relation, $\langle\Delta\tilde{J}_y^2\rangle\langle\Delta\tilde{J}_z^2\rangle \geq 1/4$ and $\langle\Delta\tilde{S}_y^2\rangle\langle\Delta\tilde{S}_z^2\rangle \geq 1/4$. The variances of coherent states are $\langle\Delta\tilde{S}_y^2\rangle = \langle\Delta\tilde{S}_z^2\rangle = 1/2$ and $\langle\Delta\tilde{J}_y^2\rangle = \langle\Delta\tilde{J}_z^2\rangle = 1/2$. We say the state is spin squeezed in the $z$ direction if $\langle\Delta\tilde{J}_z^2\rangle < 1/2$ is satisfied [11].

The Hamiltonian of the FR interaction is given by $H_{\text{int}} = \alpha\sqrt{SJ}\tilde{S}_z\tilde{J}_z$, where $\alpha$ is a real constant and $z$ means the propagation direction of the light [8]. This interaction causes the time evolution of $\vec{\tilde{J}}$ and $\vec{\tilde{S}}$, so that, $\tilde{S}_y \to \tilde{S}_y + \kappa\tilde{J}_z$, $\tilde{S}_z \to \tilde{S}_z$, $\tilde{J}_y \to \tilde{J}_y + \kappa\tilde{S}_z$, $\tilde{J}_z \to \tilde{J}_z$, where the interaction strength is given by $\kappa \equiv \alpha t\sqrt{JS}$. Note that this interaction satisfies a back-action evading condition $[H_{\text{int}}, \tilde{J}_z] = 0$, and makes a quantum correlation between $\tilde{J}_z$ and $\tilde{S}_y$. Thereby, the measurement of $\tilde{S}_y$ will essentially project the spin state into an eigenstate of $\tilde{J}_z$ and the variance of $\tilde{J}_z$ will be squeezed. The measurement of $\tilde{S}_y$ is said to be the QND measurement of $\tilde{J}_z$, and induces spin squeezing in the $z$ direction [8].

Suppose that two successive pulses interact with the atoms. Then, the Stokes operator of the first pulse ($\tilde{S}_{1,y}$, $\tilde{S}_{1,z}$) and that of the second pulse ($\tilde{S}_{2,y}$, $\tilde{S}_{2,z}$) are transformed as $\tilde{S}_{1,y}^{(f)} = \tilde{S}_{1,y}^{(i)} + \kappa\tilde{J}_{1,z}$, $\tilde{S}_{1,z}^{(f)} = \tilde{S}_{1,z}^{(i)}$, $\tilde{S}_{2,y}^{(f)} = \tilde{S}_{2,y}^{(i)} + \kappa\tilde{J}_{2,z}$, $\tilde{S}_{2,z}^{(f)} = \tilde{S}_{2,z}^{(i)}$, where $\tilde{J}_{1,z}$ and $\tilde{J}_{2,z}$ are the collective spin operators for the first pulse and second pulse, respectively. If both of the initial states are coherent states and the QND condition $\tilde{J}_{1,z} = \tilde{J}_{2,z}$ is satisfied, we have

$$\sigma_{1(2)} \equiv \langle\Delta(\tilde{S}_{1(2),y}^{(f)})^2\rangle = (1 + \kappa^2)/2, \tag{1}$$

$$\sigma_z \equiv \langle\Delta(\tilde{S}_{1,z}^{(f)})^2\rangle = \langle\Delta(\tilde{S}_{2,z}^{(f)})^2\rangle = 1/2. \tag{2}$$

This implies that the FR interaction increases the individual variances $\sigma_{1,2}$ by the same factor of $\kappa^2/2$ and it does not change the variances in the $z$ direction, $\sigma_z$. In addition, we have the following relations about the positive correlation $\sigma_+$ and the negative correlation $\sigma_-$ as

$$\sigma_\pm \equiv \langle\Delta(\tilde{S}_{1,y}^{(f)} \pm \tilde{S}_{2,y}^{(f)})^2\rangle/2 = (1 + \kappa^2 \pm \kappa^2)/2. \tag{3}$$

As one can see, $\sigma_+$ increases by a factor of $\kappa^2$, while $\sigma_-$ does not change. In contrast with the QND measurement in optics [3,7], it is difficult to directly measure the spin states, and hence, we experimentally investigate these relations to confirm the achievement of spin-QNDM. Specifically, we show the behavior of the variances and correlations for various incident photon number of the pulses $N_L$. In order to further check the validity of our results, we investigate the behavior when the QND condition $\tilde{J}_{1,z} = \tilde{J}_{2,z}$ does not hold, by performing the same measurement with the atomic spins reinitialized during the interval of the two interactions. In this case, we do not expect any correlation between the first and second probe polarization measurements and $\sigma_\pm$ are expected to approach $\sigma_{1,2}$. We also estimate the measurement-conditioned variance and the degree of spin squeezing obtained by the experiments.

The experimental setup is shown in Fig. 1(a) and is basically the same as our previous experiment [26]. By several improvements, the FR angle of $\phi = \alpha tJ/2 \simeq 143$ mrad was achieved with the relatively small fluctuation $\sigma(\phi) \simeq 10$ mrad, where $\sigma(X)$ means the standard variance of $X$. For the atomic number, we typically have $J \simeq 3.4 \times 10^5$ and $\sigma(J) \simeq 2.4 \times 10^4$ [27]. The probe system was the same as our another experiment [28]. The probe beam was focused in the atomic region and the frequency of the light was locked to the center of the two hyperfine-splitted optical lines of the $^1S_0 \leftrightarrow {}^1P_1$ transitions of $^{171}$Yb. $\kappa$ is calculated as

$$\kappa = \frac{\Gamma\sigma_0\sqrt{SJ}}{3\pi w_0^2}\left(\frac{\delta - \delta_0}{(\delta - \delta_0)^2 + (\Gamma/2)^2} - \frac{\delta}{\delta^2 + (\Gamma/2)^2}\right), \tag{4}$$

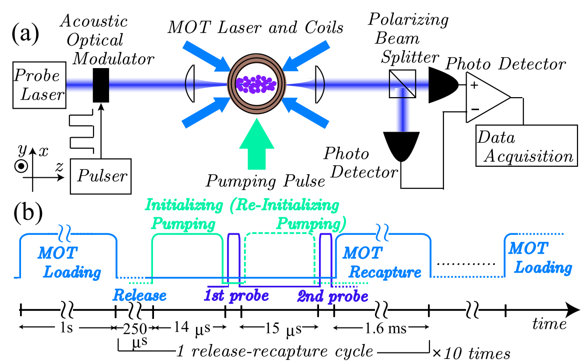


FIG. 1 (color online). (a) Experimental setup. Two successive linearly-polarized 100 ns probe pulses pass through the polarized $^{171}$Yb atoms released from MOT and the polarizations of the pulses are measured. (b) Experimental time sequence. At first, typically $10^6$ atoms are loaded in MOT and are released in the next period, during which the trapping system is switched off. Second, the atoms are polarized by the circularly-polarized resonant pumping pulse. Then, the two probe pulses pass through the atoms and go into the polarization detector. The atoms can be reinitialized when necessary, by applying the optical pumping pulse between the two pulses, represented by the dashed line.

where $\Gamma = 2\pi \times 29$ MHz is the natural full line width, $\sigma_0 = 7.6 \times 10^{-14}$ m$^2$ is the photon-absorption cross section of $^{171}$Yb atom, $w_0 = 58$ $\mu$m is the beam waist, $\delta = 2\pi \times 160$ MHz is the detuning from the $^1S_0 \leftrightarrow {}^1P_1$ ($F' = 3/2$) states, and $\delta_0 = 2\pi \times 320$ MHz is the frequency difference between the $F' = 1/2$ and $F' = 3/2$ states in the $^1P_1$ state [25,26]. In our experimental conditions, the maximum value of $\kappa$ is 0.62. At this value, $N_L$ is $3.2 \times 10^6$ and the loss parameter $\epsilon \equiv rt/2$ is $9.3 \times 10^{-2}$, where $r$ is the absorption rate [29].

The time sequence is shown in Fig. 1(b). At first, typically $10^6$ atoms are loaded in a magneto-optical trap (MOT) in 1 s and are released in the next 250 $\mu$s, during which the MOT light, the MOT magnetic field, and the Zeeman Slower light are switched off. Secondly, the atoms are polarized by the circularly-polarized resonant pumping pulse whose width is 14 $\mu$s. Then, two linearly-polarized probe pulses pass through the atoms and go into the polarization detector. The pulses have the same width of 100 ns and the interval between them is 15 $\mu$s. The atoms can be reinitialized when necessary, by applying the optical pumping pulse during this 15 $\mu$s period. This process is represented by the dashed line in Fig. 1(b). The detection of the second pulse completes the single measurement process of our experiment. Then, the atoms are recaptured by MOT and reused for the next cycle. For the single loading of atoms, we repeated 10 release-and-recapture cycles as shown in Fig. 1(b). In this manner, we measured about 2600 pairs of the Stokes operators of the pulses for each arrangement of the experimental parameters [30].

Figure 2(a) and 2(b) show the joint distribution of the measured $\tilde{S}_{1,y}^{(f)}$ and $\tilde{S}_{2,y}^{(f)}$ for the cases with (a) no atoms and (b) $3.4 \times 10^5$ atoms at $\kappa = 0.62$ in the QND condition, respectively. We observe the increase of $\sigma_{1,2}$ in (b) from the ones in (a) and the positive correlation in (b). Figure 2(c) shows the measured joint distribution of $\tilde{S}_{1,z}^{(f)}$ and $\tilde{S}_{2,z}^{(f)}$ at $\kappa = 0.62$. As a check of the validity of our result, the measurement of $\tilde{S}_{1,z}^{(f)}$ and $\tilde{S}_{2,z}^{(f)}$ was performed by inserting a $\lambda/4$ plate before the polarizing beam splitter. We can see that $\tilde{S}_{1,z}^{(f)}$ and $\tilde{S}_{2,z}^{(f)}$ do not show any specific correlation and the distribution is essentially the same as that of Fig. 2(a), as expected from Eq. (2).

We have performed this measurement for a various interaction strength $\kappa$ by changing the incident photon number $N_L$ and observed the behavior of the variances and correlations. Figure 3(a) shows the individual variances $\sigma_{1,2}$ with and without atoms. At a large $\kappa$, it is apparent that $\sigma_{1,2}$ with atoms became larger than $\sigma_{1,2}$ without atoms. Figure 3(b) shows the values of $\sigma_\pm$ with atoms. As expected from Eq. (1) and (3), we successfully observed larger value of $\sigma_+$ than $\sigma_{1,2}$ with atoms in Fig. 3(a), while $\sigma_-$ remained the value of $1/2$ corresponding to the light-shot noise. Figure 3(c) shows the case with reinitialized atoms, where the above correlation observed in Fig. 3(b) almost disappeared and $\sigma_\pm$ approached the value of $\sigma_{1,2}$ in Fig. 3(a), as expected. The deviations from the theoretical curve might be caused by imperfect polarization of the reinitializing pulse. Here, the solid curve, dash-dotted

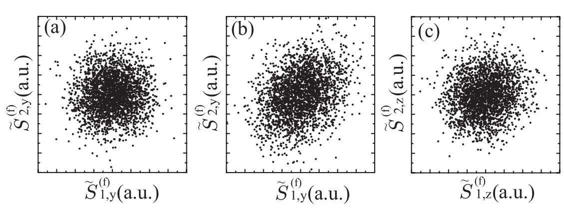


FIG. 2. Joint distribution of the measured polarization $\tilde{S}_{1,y}^{(f)}$ and $\tilde{S}_{2,y}^{(f)}$. (a) With no atoms. The variances were limited by the light shot noise and the distribution was isotropic. (b) With $3.4 \times 10^5$ atoms in the probe region. The correlation was observed. (c) $\tilde{S}_{1,z}$ and $\tilde{S}_{2,z}$ with $3.4 \times 10^5$ atoms. The distribution is essentially the same as that of Fig. 2(a), as expected from Eq. (2).

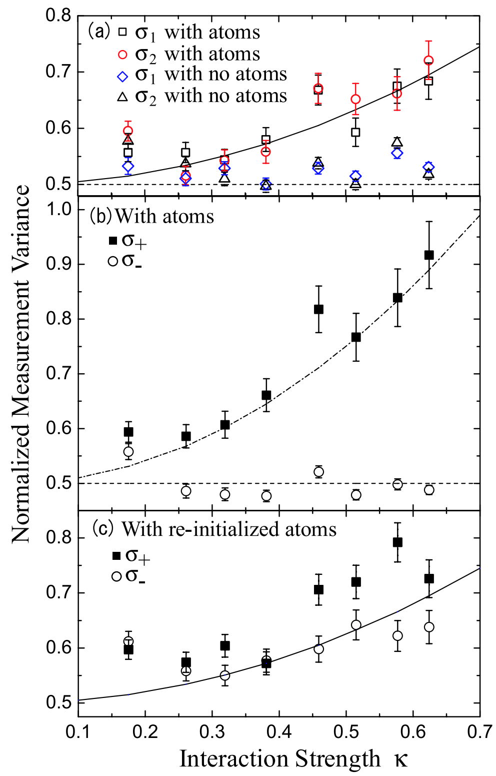


FIG. 3 (color online). Measured variance as a function of the interaction strength $\kappa$. The solid curve, dash-dotted curve, and dashed curve are given by $(1 + \kappa^2)/2$, $(1 + 2\kappa^2)/2$, and $1/2$, respectively. (a) Individual variances $\sigma_{1,2}$ with and without atoms. At a large $\kappa$, it is apparent that $\sigma_{1,2}$ with atoms became larger than $\sigma_{1,2}$ without atoms. (b) $\sigma_\pm$ with atoms. As expected from Eq. (1) and (3), we observed larger value of $\sigma_+$ than $\sigma_{1,2}$ with atoms in Fig. 3(a), while $\sigma_-$ remains the value of $1/2$ corresponding to the light shot noise. (c)$\sigma_\pm$ with reinitialized atoms. The significant difference between $\sigma_\pm$ observed in (b) almost disappeared.

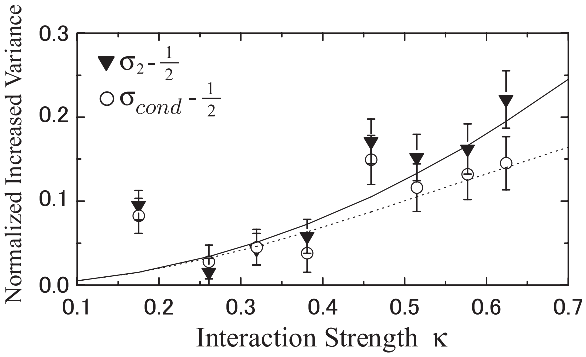


FIG. 4. Conditioned variance $\sigma_{\rm cond} - 1/2$ and $\sigma_2 - 1/2$ as a function of a various interaction strength $\kappa$. Solid curve shows the theoretical value of the total variance $\kappa^2/2$ and dotted curve shows the one of the conditioned variance, which is given as $\kappa^2/\{2(1+\kappa^2)\}$. At a large $\kappa$, it is apparent that $\sigma_{\rm cond}$ became smaller than $\sigma_2$.

curve, and dashed curve are given by $(1+\kappa^2)/2$, $(1+2\kappa^2)/2$, and $1/2$, respectively [see Eq. (1) and (3)]. As one can see, all of the measured variances are almost consistent with the theory.

Finally, we show that the $z$ component of the spin $J_z$ is conditionally squeezed. In order to observe this spin squeezing, one may count the data of $\tilde{S}^{(f)}_{2,y}$ only when $\tilde{S}^{(f)}_{1,y}$ takes a specific value. Accordingly, we divided the data around the center into 21 bins of $\tilde{S}^{(f)}_{1,y}$ and took the average of the variance of $\tilde{S}^{(f)}_{2,y}$ of each bin. In Fig. 4, we showed the conditioned variance $\sigma_{\rm cond} - 1/2$ and $\sigma_2 - 1/2$ for various interaction strength $\kappa$. Note that the error bars are calculated from the statistical error of the variance measurement and the atomic number fluctuation. At a large $\kappa$, it is apparent that $\sigma_{\rm cond}$ became smaller than $\sigma_2$. Here, the solid curve shows the theoretically expected dependence of the total variance $\kappa^2/2$, and the dotted curve shows the theoretically expected one of the conditioned variance $\kappa^2/\{2(1+\kappa^2)\}$ [8]. As one can see, the experimental results have the values near to the above theoretical estimation and spin squeezing was achieved with the degree of $1.8^{+2.4}_{-1.5}$ dB when $\kappa = 0.62$. This level of spin squeezing with $\kappa = 0.62$ is large enough to ensure a successful quantum domain operation of quantum memory protocol [18] with a wide range inverse width parameter $0.03 \lesssim \lambda \lesssim 37$. In the case of Yb-based optical lattice clock [23] the accuracy of the clock could be improved by spin squeezing, if any.

In conclusion, we reported spin-QNDM with cold $^{171}$Yb. From the quantitative and qualitative analysis, we concluded that we have achieved spin-QNDM and $1.8^{+2.4}_{-1.5}$ dB spin squeezing. This demonstration with a nuclear spin one-half and short pulses of a 100 ns width could be a milestone toward a future realization of multistep quantum information processing [18], and a dramatic improvement of the Yb-based optical lattice clock [23].

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