Color Screening by Pions

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(Received 24 September 2008; published 23 January 2009)

Lattice QCD is used to calculate the potential between a static quark and antiquark in the presence of a finite density of π^+'s. Correlation functions of multiple π^+'s are used in conjunction with Wilson-loop correlators to determine the difference between the $Q\bar{Q}$ potential in free space and in the presence of a pion condensate. The modifications to the potential are found to have significant dependence on the $Q\bar{Q}$ separation over the range $r \lesssim 1$ fm explored in this work. Our results are consistent with the pion-condensate behaving as a (nonlinear) chromodielectric.

DOI: 10.1103/PhysRevLett.102.032004 PACS numbers: 12.38.Gc, 25.75.Cj, 25.75.Nq

Introduction.—Flavor dynamics plays a central role in the study of heavy-ion collisions and, ultimately, in unraveling the phase structure of matter at finite temperature and density. The production, evolution, and detection of the heavy flavors, charm, and bottom, in such collisions has long been known to be sensitive to the presence of deconfined phases of quantum chromodynamics (QCD), such as the naive quark-gluon plasma or "the perfect fluid" seen at RHIC [1,2]. Color charges are screened in a deconfined phase, and the production of a given state of quarkonium is expected to be suppressed when the screening length becomes comparable to, or smaller than, its size [3] (for a recent discussion see Ref. [4]). However, quantitative calculations of "J/ψ suppression" and more generally "quarkonium suppression" in heavy-ion collisions are difficult due to the enormous complexity of such collisions.

Understanding the behavior of a static quark, Q, and static antiquark, $\bar{Q}$, pair produced in a heavy-ion collision requires knowledge of its dynamics in both the deconfined and hadronic phases. In the hadronic phase, the potential between a static quark and static antiquark (denoted as the $Q\bar{Q}$ potential) can be screened by the hadrons that participate in the collision. Differences between this "hadronic medium effect" and the effect of a deconfined phase are telltale signatures of a deconfining phase transition in heavy-ion collisions. As a step to improving the description of $Q\bar{Q}$ transport in the hadronic phase we present the results of a Lattice QCD calculation of the modifications of the $Q\bar{Q}$ potential and force resulting from the presence of a condensate of charged pions (either all π^+'s or all π^-'s) in the absence of electromagnetism. The $Q\bar{Q}$ potential is found to be lowered in the condensate, with a nontrivial dependence upon the $Q\bar{Q}$ separation, leading to a reduced $Q\bar{Q}$ force.

It is useful to consider the behavior of the in-medium $Q\bar{Q}$ potential and force in the limiting cases where the $Q\bar{Q}$ separation, R, is much smaller than, or much greater than, the scale of chiral symmetry breaking, Λ_χ, that typifies hadronic interactions. In the $R\Lambda_\chi \ll 1$ limit, it is appropriate to construct an effective field theory (EFT) describing the interactions between the $Q\bar{Q}$ with the gluon fields. The coefficients in this EFT are determined via a multipole expansion of the matrix elements calculated in QCD. The interactions between the $Q\bar{Q}$ state and the hadronic background are thereby factorized into short-distance parts encapsulated in the coefficients of the EFT, and low-energy matrix elements of local operators composed of quark and gluon fields (explicitly independent of R). This method [5] has been used to calculate the binding energy of quarkonium to infinite nuclear matter [6], and quarkonium scattering from nucleons [7]. The interactions of a spatially averaged and orientation-averaged Wilson-loop with quark and gluon fields, at leading order in the strong-coupling and derivative expansion, are described by an effective Lagrange density of the form

$$\mathcal{L} = R^3 \mathcal{S}^\dagger \mathcal{S} G_{\mu\alpha} G^\alpha_\nu [c_1 g^{\mu\nu} + c_2 v^\mu v^\nu], \qquad (1)$$

where $\mathcal{S}$ is the operator that annihilates the $Q\bar{Q}$ pair, $G_{\alpha\beta}$ is the gluon-field strength tensor, v_α is the $Q\bar{Q}$-pair four-velocity. The renormalization-scale dependent coefficients c_i that appear in Eq. (1) are dimensionless and are expected to be of order unity by naive dimensional analysis. The Lagrange density in Eq. (1) makes explicit the separation dependence of the in-medium component of the potential ($\propto R^3$) and force ($\propto R^2$) for $R\Lambda_\chi \ll 1$.

At extremely large distances, the ground state of the system is a heavy meson-antimeson pair in the pion condensate interacting through the exchange of hadrons described by an effective field theory analogous to that used to describe the interactions of nucleons. In QCD, the force between these heavy mesons is Yukawa-like with a mass scale set by the pion mass.

At intermediate distances and in vacuum, the ground state of the system has the $Q\bar{Q}$ potential increasing with separation, and the gluonic field configuration between the $Q\bar{Q}$ is tending toward a flux tube with constant force between the $Q\bar{Q}$ pair (for recent lattice calculations see Ref. [8]). It is not obvious how the presence of a pion condensate will modify the interactions between the $Q\bar{Q}$ pair. If instead, the system under consideration was a collection of neutral hadrons in the presence of an electric field, the energy shift of the system would depend upon the electric polarizabilities of the hadron and the strength and volume intersected by the electric field. An overall reduction in the $Q\bar{Q}$ force would result, encapsulated in the dielectric function of the medium.

The lattice QCD calculation.—The ground-state energy of a system composed of a $Q\bar{Q}$ pair separated by a distance R and n-π^+'s can be extracted from the correlation functions

$$\begin{aligned} C_n(t_\pi, t) &= \langle 0| \Big[\sum_{\mathbf{x}} \chi_{\pi^+}(\mathbf{x}, t) \chi^\dagger_{\pi^+}(0, t_\pi) \Big]^n |0\rangle, \\ C_W(R, t_w, t) &= \langle 0| \sum_{\mathbf{y}, |\mathbf{r}|=R} \mathcal{W}(\mathbf{y}+\mathbf{r}, t; \mathbf{y}, t_w) |0\rangle, \\ C_{n,W}(R, t_\pi, t_w, t) &= \langle 0| \Big[\sum_{\mathbf{x}} \chi_{\pi^+}(\mathbf{x}, t) \chi^\dagger_{\pi^+}(0, t_\pi) \Big]^n \\ &\quad \times \sum_{\mathbf{y}, |\mathbf{r}|=R} \mathcal{W}(\mathbf{y}+\mathbf{r}, t; \mathbf{y}, t_w) |0\rangle, \end{aligned} \tag{2}$$

where $\chi_{\pi^+}(x) = u^a(x)\gamma_5 \bar{d}_a(x)$ is a pseudoscalar interpolating operator for the π^+ (a is a color index), and $\mathcal{W}(\mathbf{y}, t_0; \mathbf{y}+\mathbf{r}, t)$ is the Wilson-loop operator formed from products of gauge links joining the vertices at ($\mathbf{y}$, t_0), ($\mathbf{y}+\mathbf{r}$, t_0), ($\mathbf{y}+\mathbf{r}$, t), and ($\mathbf{y}$, t). At large times, the static $Q\bar{Q}$ potential in vacuum is extracted using

$$\sum_{t_w} C_W(R, t_w, t_w + t) \xrightarrow{t\to\infty} A e^{-V(R)t}. \tag{3}$$

It is useful to define the ratio of the three correlation functions in Eq. (2),

$$G_{n,W}(R, t_\pi, t_w, t) = \frac{C_{n,W}(R, t_\pi, t_w, t)}{C_n(t_\pi, t) C_W(R, t_w, t)}, \tag{4}$$

from which the in-medium modification to the potential can be extracted, for $t_w \gg t_\pi$ (the pion system takes a longer time to reach its ground state than the Wilson loop and so its source is earlier in time),

$$\left\langle \log\left(\frac{(G_{n,W}(R, t_\pi, t_w, t_w + t)}{G_{n,W}(R, t_\pi, t_w, t_w + t + 1)} \right) \right\rangle_{t_w} \xrightarrow{t\to\infty} \delta V(R, n), \tag{5}$$

where the $\langle \ldots \rangle_{t_w}$ denotes an average over a number of initial time slices for the Wilson loop, $t_w^{\min}$ to $t_w^{\max}$. It is possible that the uncertainties in our analysis can be slightly decreased by accounting for correlations among different t_w.

We have computed the correlators in Eq. (2) in mixed-action lattice QCD, using domain-wall valence quark propagators from a Gaussian smeared source on rooted-staggered MILC gauge configurations (see Refs. [9,10] for details). Here we focus on calculations on an ensemble of 1001 lattices with a pion mass of $m_\pi \sim 320$ MeV, and a lattice spacing of $b = 0.087(1)$ fm with dimension $28^3 \times 96$ giving a spatial dimension of $\sim$2.5 fm. Propagators were calculated with both periodic and antiperiodic boundary conditions in the time direction and combined to effectively double the length of the time direction leading to long plateaus in effective energy plots of the mesonic correlators. The light quark propagators were computed after the gauge field had undergone a single level of hypercubic (HYP) smearing [11], while Wilson loops were calculated with $N_{\rm HYP} = 0$, 1, 2, and 4 levels of HYP smearing of the gauge field. Further, the spatial links of the Wilson loops were APE smeared [12,13] in the transverse-spatial directions to optimize the signals for $C_W(R, t_w, t)$. The increasing levels of HYP smearing result in improved signal-to-noise ratios and enable the potential, and in-medium modifications to the potential, to be determined over a range of $Q\bar{Q}$ separations. The correlation functions $C_n(t_\pi, t)$ have been previously calculated on these lattices, enabling a study of the properties of the pion and kaon condensates, and the three-meson interactions [14–16].

Correlated fits are performed to the effective energies (or energy differences), e.g., Eq. (5), for each $Q\bar{Q}$ separation and number of pions in the volume. Separately, jackknife and bootstrap procedures were used to generate the covariance matrix over the given fitting interval of time slices, t in Eq. (5), and correlated χ^2 minimization was performed to extract the energy and its associated statistical uncertainty. A systematic uncertainty is determined by a comparison of the various fit procedures and various fitting ranges (including the choice of $t_w^{\min/\max}$; typically $t_w^{\min} \sim t_\pi + 20$ and $t_w^{\max} - t_w^{\min} \sim 40$).

The $Q\bar{Q}$ potential in vacuum is determined from C_W, and the $Q\bar{Q}$ force is determined by correlated finite differences of the potential. Our results for these two quantities are shown in Fig. 1. By comparing the force in vacuum calculated with different levels of HYP smearing, we ascertain the separations at which the force (and potential) cease to be significantly contaminated by the smearing procedure. We conclude from Fig. 1 that the potential and force calculated at separations $R > bN_{\rm HYP}$ are close to the result of an unsmeared calculation. In our analysis, we assume this also holds in the presence of the pion medium and only present measurements that satisfy this criterion. In every case where multiple different smearings satisfy the criterion, there is complete agreement between them. Three representative effective energy plots associated with the in-medium contributions to the potential are shown in Fig. 2. Typically, signals become noisier

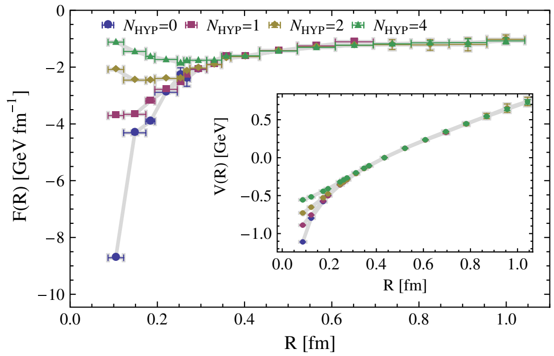

FIG. 1 (color online). The $Q\bar{Q}$ force in vacuum determined with different levels of HYP smearing. Inset is the $Q\bar{Q}$ potential with these same smearings, each normalized to $V(R_0) = 0$ at $R_0 = 5b$. Horizontal uncertainties in $F(R)$ arise from its construction by finite differences.

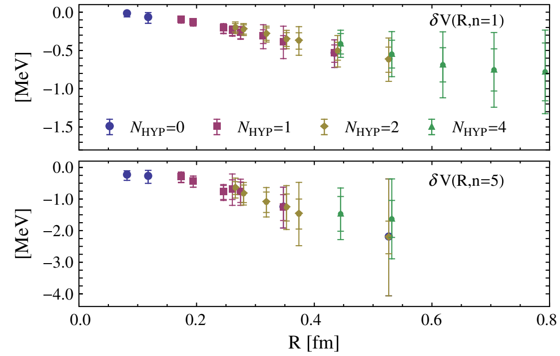

FIG. 3 (color online). The in-medium contributions to the $Q\bar{Q}$ potential at a pion densities of ρ_0 and $5\rho_0$. The inner uncertainty associated with each point is statistical, while the outer is the statistical and systematic uncertainties combined in quadrature. Different HYP smearings are offset for clarity.

as either R or n increases, restricting the present analysis to $R \lesssim 1$ fm.

The in-medium contributions to the $Q\bar{Q}$ potential, $\delta V(R, n)$, resulting from both a single pion in the lattice volume [a pion number density of $\rho_0 \sim 1/(2.5\text{ fm})^3 = 0.064\text{ fm}^{-3}$], and from five pions in the lattice volume ($\rho = 5\rho_0 = 0.32\text{ fm}^{-3}$), are shown in Fig. 3. For $\rho \lesssim 7\rho_0$ they are found to be linear in the density within the uncertainties of the calculations, as can be seen for a representative R in Fig. 4. We note that in-medium contributions to the potential as small as $\delta V(b, 1) \sim 100$ keV have been determined. For the system containing a single pion in the lattice volume, the energy shift can be directly related to the scattering phase shift using Lüscher's method [17]. Therefore, the $\delta V(R, 1)$ shown in Fig. 3 can be used to determine the scattering length associated with a $Q\bar{Q}$ pair of fixed separation and a pion.

The effects of the medium on the radial $Q\bar{Q}$ force, $\delta F(R, n)$, are determined from the effective energy-differences derived from finite differences of $\delta V(R, n)$ with respect to R. The modifications to the force at densities ρ_0 and $5\rho_0$ are shown in Fig. 5. The $Q\bar{Q}$ force is seen to be reduced by an approximately R-independent amount over the separations and pion densities we have been able to explore.

The in-medium potential and force have also been calculated on MILC lattices of the same spatial volume as the configurations used here, at four different quark masses, but at a coarser lattice spacing ($b \sim 0.125$ fm). We find the results at the corresponding pion mass are consistent, suggesting that lattice discretization errors are not large. However, the uncertainties in the medium modifications to the $Q\bar{Q}$ potential and force are somewhat larger than those on the current ensemble. A mild variation with the pion mass was observed, however, more precise calculations are required in order to quantify this dependence.

Discussion.—In-medium effects play an important role in the diagnostics used to explore new phases of matter in heavy-ion collisions, and more generally emerge as useful

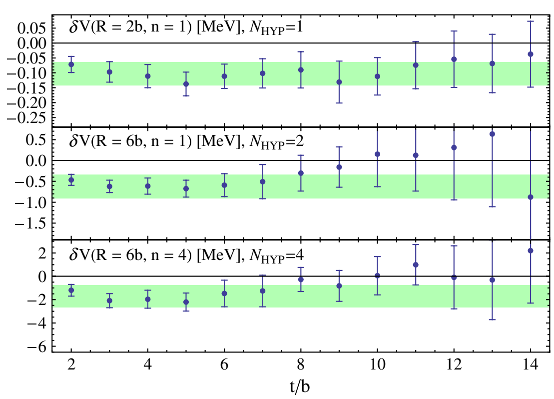

FIG. 2 (color online). Representative effective energy plots for the in-medium contributions to the potential. The horizontal band corresponds to the combined statistical and systematic uncertainty of the respective fit.

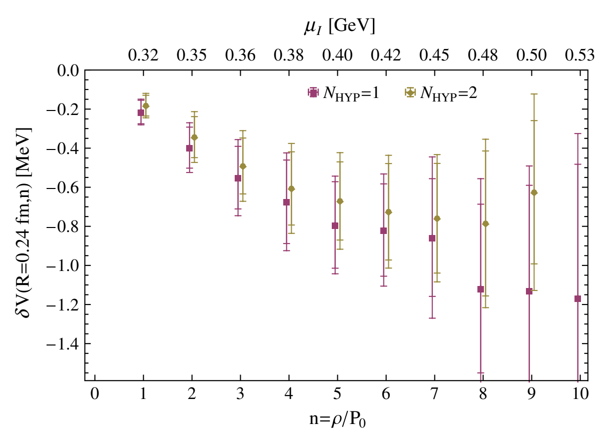

FIG. 4 (color online). The in-medium contribution to the $Q\bar{Q}$ potential at $R \sim 0.24$ fm as a function of the number of pions. The uncertainties are as described in Fig. 3. The isospin chemical potential of the system is shown on the upper axis [16].

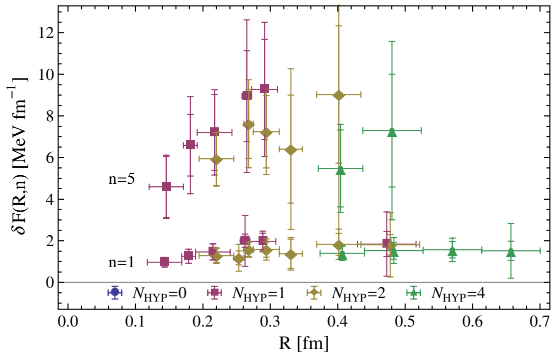

FIG. 5 (color online). The in-medium contribution to the radial $Q\bar{Q}$ force, $\delta F(R, n)$, at a pion number density of ρ_0 and $5\rho_0$. The uncertainties are as described in the caption of Fig. 3.

quantities in the context of mean-field constructions in many-body systems. We have performed the first QCD calculation (without electromagnetism) of modifications to the $Q\bar{Q}$ potential and force in a hadronic medium by calculating the energy of a Wilson loop in the presence of a condensate of charged pions. The attractive $Q\bar{Q}$ interaction is found to be reduced by the hadronic medium over the range of separations we were able to explore, $R \lesssim 1$ fm. This is a first step toward a more systematic exploration of in-medium effects with lattice QCD, with the ultimate goal of looking for in-medium modifications of hadronic observables in backgrounds of baryons. Such calculations will require significantly more computational resources than are currently available, and precise calculations of multibaryon systems are a prerequisite.

At ρ_0, our calculations are analogous to those of $J/\psi - \pi$ scattering lengths (with the J/ψ replaced by a Wilson loop) which have been performed previously in QCD [18] and quenched QCD [19]. At higher densities, the calculations involve multipion backgrounds and are the first of their kind. A nonzero three-pion interaction is required to describe the volume dependence of the energy levels of $n > 2$ π^+'s in these volumes, see Refs. [14–16]. However, within the uncertainties of our calculations, the multipion–$Q\bar{Q}$-pair interactions are found to be consistent with zero over the range of $Q\bar{Q}$ separations we have explored as the medium modifications depend linearly on n. It is important to refine this work by performing higher-statistics calculations in order to determine the multipion interactions with the $Q\bar{Q}$ pair. In addition, with a corresponding calculation of the pionic matrix elements of the gluonic operators in Eq. (1), the coefficients, c_i, in Eq. (1) could be determined.

From our analysis we observe that the medium modification of the force is independent of the $Q\bar{Q}$ separation in the region where the force in vacuum is becoming constant. This is consistent with the pion-condensate behaving as a dielectric in the volume of the color flux tube between the $Q\bar{Q}$ pair. It implies that the pion and, collectively, the condensate has a chromosusceptibility, which is expected to be highly nonlinear in the gluon-field strength. At small separations, the modification to the force appears consistent with the behavior expected from Eq. (1), but calculations at smaller lattice spacing are required to confirm this.

We thank our NPLQCD collaborators for their contributions to this work, Barak Bringoltz, David Kaplan, Jerry Miller, and Steve Sharpe for useful conversations, and R. Edwards and B. Joo for help with the QDP++/Chroma programming environment [20]. We are indebted to the MILC for use of their configurations. This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG03-97ER4014. The computations for this work were performed at Jefferson Lab, Fermilab, Centro Nacional de Supercomputación, the University of Washington (the Athena cluster), NERSC (Office of Science of the U.S. Department of Energy, No. DE-AC02-05CH11231), and the NSF through TeraGrid resources provided by the NCSA.

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