How Precisely Will the Total Cross Section Be Measured at the Large Hadron Collider?

J.-R. Cudell[1] and O. V. Selyugin[2]

[1]*IFPA, AGO Department B5a, Université de Liège, 4000 Liège, Belgium*

[2]*BLTP, Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia*

(Received 27 August 2008; published 22 January 2009)

It is very likely that hadronic scattering will enter a new regime at the CERN Large Hadron Collider, as the black-disk limit is reached. This will lead to a severe change in the t dependence of the real part and of the slope of the elastic scattering amplitude, and in turn, this may bias the measurement of the total cross section. We examine this issue and suggest new strategies to test the reliability of the total cross-section measurements.

DOI: 10.1103/PhysRevLett.102.032003 PACS numbers: 13.85.Lg, 12.40.Nn

Many models predict that soft interactions will enter a new regime at the LHC: given the huge energy, unitarization may play a crucial role as the central part of the protons becomes black. Indeed, all simple-power fits to lower energy data will violate unitarity in some partial-wave amplitudes before the LHC energy. This means that something will happen that will restore it—call it saturation, unitarization, or emergence of cuts—and this will modify the expectations one has from Regge models based on simple poles. While conventional models predict a total cross section from 90 to 125 mb [1], the presence of a hard Pomeron gives around 150 mb [2–4] and U-matrix unitarization can give 230 mb [5]. This clearly shows that the uncertainties due to the underlying models, and especially due to the unitarization scheme, are very large. One may hope to select the true models by a measurement of the total cross section.

The LHC will be well equipped to study in depth the diffractive processes, as it will have a superb rapidity coverage, and two experiments—TOTEM [6] and ATLAS [7]—plan to measure the total cross section. They intend to reach an accuracy on $\sigma_{\rm tot}$ of the order of 1%, which would indeed give a very stringent test of theory. A few assumptions underlie this estimate of the accuracy. First of all, the ratio ρ of the real part to the imaginary part of the elastic scattering amplitude, is assumed to be small and to vary little with t: $\rho(s,t)\approx 0.14$ [1]. Second, the elastic cross section is assumed to fall exponentially with t: $dN/dt \sim \exp(Bt)$.

We want to show here that, if the elastic pp amplitude reaches a new regime at the LHC, it will invalidate the above assumptions, and the measurement of the total cross section will be biased and much more uncertain than foreseen. Indeed, most unitarization schemes lead to novel properties of the elastic amplitude. The problems that we talk about here concern a large class of models, in which the elastic amplitude contains a fast-rising component that needs to be unitarized. We have checked that they are present in models saturating the profile function, in models using analytic unitarization schemes [2] or in the Dubna Dynamical Model [8].

To illustrate our point, we shall consider a simple unitarized two-component model, which includes a soft Pomeron and a hard Pomeron, and which we shall call the eikonalized two-Pomeron model (ETPM). This model is based on a fit to soft data which includes a hard Pomeron component [9] of intercept 1.4 that accounts for the growth of the gluon density at small x in deeply inelastic processes [10]. Although the coefficient of the hard-Pomeron term is small in soft data, it grows like $s^{0.4}$ so that the amplitude will reach the black-disk limit at small impact parameter b before the LHC energy [2,11]. The amplitude must then be unitarized and, in this simple model, we consider a 1-channel eikonal in the impact-parameter representation [2]. The net effect of this unitarization is to make $B(s,t)$ increase with $|t|$ at small $|t|$ for LHC energies, as shown in Fig. 1. We also show in that figure that the t-dependence of $\rho(s,t)$ changes drastically. We can now explore the consequences of these effects on the experimental measurement itself. We shall compare in the following the situation at 2 TeV with that at 14 TeV. We insist that the curves we give for the cross sections and for the ρ parameter are only illustrative of an effect present in many models. The essential ingredients are a sizeable value of ρ, and the strong dependence of ρ and B on t once the black-disk limit is reached.

Fitting procedure with luminosity-dependent method.—The number of elastic events is related to the total hadronic cross section through the following formula:

$$\frac{dN}{dt} = \mathcal{L}\left[\frac{4\pi\alpha^2}{|t|^2}G^4(t) - \frac{2\alpha[\rho(s,t)+\phi_{\rm CN}(s,t)]\sigma_{\rm tot}G^2(t)e^{-[B(s,t)|t|i/2]}}{|t|} + \frac{\sigma_{\rm tot}^2[1+\rho(s,t)^2]e^{-B(s,t)|t|}}{16\pi}\right] \quad (1)$$

where $\mathcal{L}$ is the luminosity, the first term is the Coulomb term—α is the electromagnetic coupling constant and $G(t)$ the electromagnetic form factor given by $G^2(t) = (4m_p^2 - \mu t)(4m_p^2 - t)/[\Lambda^2(\Lambda - t)^2]$ with m_p the proton mass,

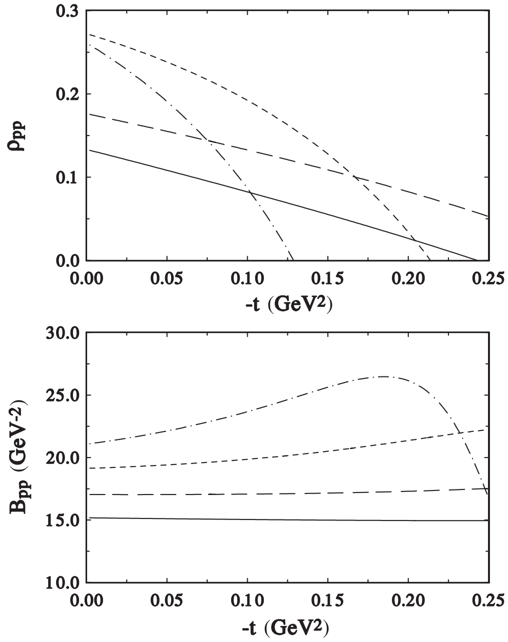

FIG. 1. Results of the ETPM model: $\rho(s,t)$ (upper panel) and $B(s,t)$ (lower panel) at 100 GeV (plain curve), 500 GeV (long dashes), 5 TeV (short dashes) and 14 TeV (dash-dotted curve).

$\Lambda = 0.71\ \mathrm{GeV}^2$ and $\mu = 2.79$—the second term is the interference term between the Coulomb amplitude and the hadronic amplitude—$\phi_{\mathrm{CN}}(s,t)$ is the phase of the Coulomb-Nucleon Interference (CNI) term [12]—from which one can extract ρ, and the third term is the purely hadronic contribution.

We can use this formula to generate simulated data for two energies $\sqrt{s} = 2$ TeV and $\sqrt{s} = 14$ TeV, using $B(s,t)$ and $\rho(s,t)$ calculated in the ETPM. We assume that 90 points will be measured in a t interval identical to that of the UA4/2 experiment $-0.1\ \mathrm{GeV}^2 \leq t \leq -0.0006\ \mathrm{GeV}^2$ [13]. We then randomize the theoretical ("true") curve assuming Gaussian errors similar to those of UA4/2. The resulting simulated data are shown in Fig. 2 for $\sqrt{s} =$ 14 TeV, and correspond to the parameters given in Table I. One can then fit these simulated data according to Eq. (1) but assuming constant B and ρ. One has 2 extra parameters besides ρ and B: $\mathcal{L}$, the luminosity, and the total cross section σ_{tot}, which is what one aims to measure.

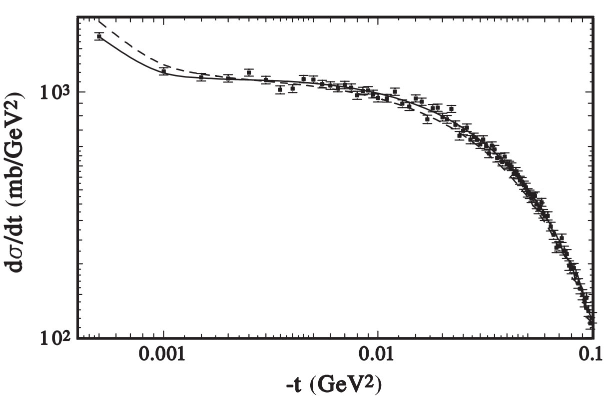

FIG. 2. The simulated data at $\sqrt{s} = 14$ TeV, the theoretical curve from which the data are generated (plain line) and the fit to them for ρ fixed at 0.1 (dashed line).

TABLE I. Input parameters for the simulated data at $\sqrt{s} =$ 2 TeV and $\sqrt{s} = 14$ TeV obtained in EPTM model, and the results of fits to these data with a simple exponential form of the scattering amplitude.

$\sqrt{s}$	$\mathcal{L}$ (fb^{-1})	σ_{tot} (mb)	$\rho(s,0)$	B(s,0) (GeV^{-2})
		input		
2 TeV	1	82.7	0.23	18.7
14 TeV	1	152.5	0.24	21.4
		output for $\mathcal{L}$ and ρ fixed		
2 TeV	1	$83.61 \pm .44$	0.15	23.6 ± 0.2
2 TeV	0.95	$85.80 \pm .45$	0.15	23.6 ± 0.2
		output for all parameters free		
2 TeV	0.93 ± 0.07	85.2 ± 3.0	0.15 ± 0.04	18.10 ± 0.25
14 TeV	1.15 ± 0.05	142.3 ± 2.8	0.29 ± 0.06	23.6 ± 0.2

The result of the fitting procedure at 2 TeV is shown in Fig. 3, where the correlation between the value of ρ and σ_{tot} is shown. If $\mathcal{L}$ is fixed at 1 fb^{-1} (i.e., if we know the luminosity), the difference between the central values of the fitted σ_{tot} is small ≈ 0.3 mb; the errors from the fitting procedure are 1 mb, and the obtained value of σ_{tot} differs

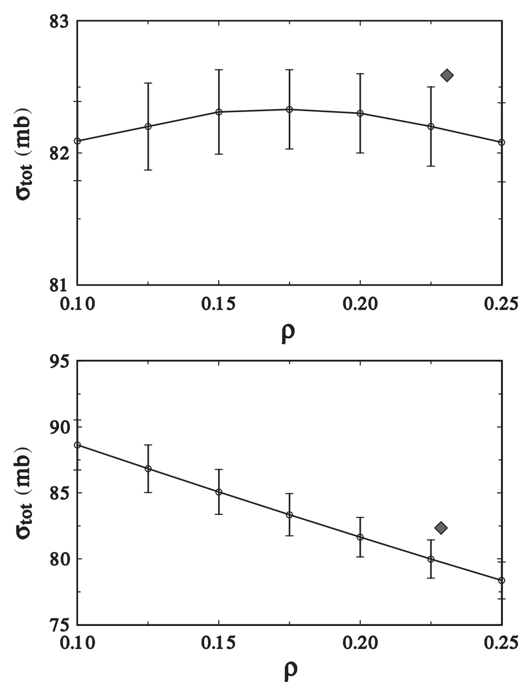

FIG. 3. The size of σ_{tot} obtained by fitting the simulated data assuming a fixed value of ρ at $\sqrt{s} = 2$ TeV with $\mathcal{L}$ fixed at the input value (upper panel) and with free $\mathcal{L}$ (lower panel); the diamond gives the true value of σ_{tot} and $\rho(s,0)$.

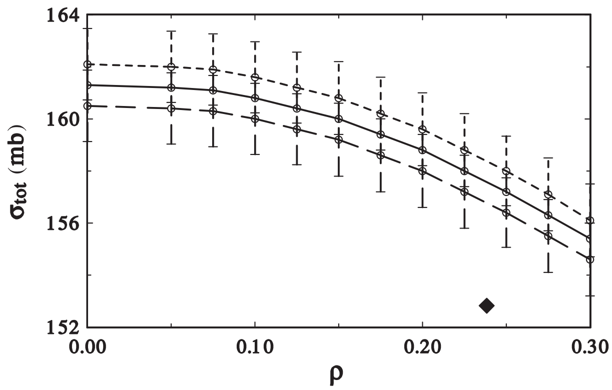

FIG. 4. σ_{tot} at $\sqrt{s} = 14$ TeV with fixed normalization ($n = 1$) (central curve) and $n = 1 \pm 0.1$ (exterior curves). The diamond indicates the input σ_{tot} and $\rho(s, 0)$ from which the data were simulated.

from the input by 0.5 mb. A very different picture appears if we fit the luminosity. We obtain then an error $\Delta\sigma_{tot} = 1.9$ mb and the correlation between σ_{tot} and the assumed value of ρ is very high, as seen in Fig. 3 and Table I. The result of a joint fit to $\mathcal{L}$, B, σ_{tot} and ρ is also shown in Table I. All parameters are within 1σ, except $\rho(0)$ which is 2σ from the input.

The situation at $\sqrt{s} = 14$ TeV is much worse: one sees from Fig. 4 that the result of the correlation between ρ and σ_{tot} increases drastically: even if one knows the luminosity, the dependence of σ_{tot} on ρ is very strong. The difference between the true σ_{tot} and the fitted value reaches 2.5 mb, as shown in Table I. All the parameters are now several standard deviations from their true value. Note also that, because the CNI term is negative, a decrease in ρ from its true value 0.24 to 0.1 leads to an increase in the value of $d\sigma/dt$, as seen in Fig. 2.

Fitting procedure with the luminosity-independent method.—Another way to extract the total cross section (for example, see [14]) is the luminosity-independent method, which gives

$$\sigma_{tot} = \frac{16\pi}{1 + \rho^2} \frac{(dN_{el}/dt)|_{t=o}}{N_{el} + N_{inel}}. \quad (2)$$

To obtain the information needed for this method, one must measure the elastic rate at values of $|t|$ large enough to neglect the Coulomb amplitude [15]. Hence, one hopes to obtain a more accurate result with less information.

This method relies on the hope that $N_{el} + N_{inel}$ can be measured accurately. However, there are three problems. The first one concerns the Coulomb and CNI regions: one needs to cut them off, but $N_{el} + N_{inel}$ is the total number of hadronic events, so one must compensate somehow for that cut. The second problem comes from the fact that part of the inelastic events (such as N^* production) will escape the detector. The third problem, which we shall address here, concerns the extrapolation of dN/dt from a minimum $|t|$ far away from the CNI region to $t = 0$.

To simulate this analysis, we take $N_{el} + N_{in} = n\sigma_{tot}$, and let n go from 0.9 to 1.1. We adopt the cuts planned for the TOTEM experiment at the LHC, $-t$ in $[0.03, 0.1]$ GeV^2, so we are left with in 51 simulated data points.

For the analysis at 2 TeV, if we take $n = 1$, the dependence of σ_{tot} over ρ will be comparable to that shown in Fig. 2, but the errors on σ_{tot} will increase. For example, if we fix $\rho = 0.15$, then $\sigma_{tot} = 83.61 \pm 0.44$ mb. σ_{tot} changes by 1.9 mb when ρ goes from 0.05 to 0.25. If n can go as low as 0.95, then the measurement of σ_{tot} increases, e.g., $\sigma_{tot} = 85.8$ mb when $\rho = 0.15$.

At $\sqrt{s} = 14$ TeV, the changes are more pronounced, as seen in Fig. 4. Cutting off the CNI region removes the possibility to measure ρ. As the normalization is inversely proportional to $1 + \rho^2$, it is rather obvious that if one allows ρ to range from 0.05 [1] to 0.3, one will get a 10% change in σ_{tot}, which will only be added to the uncertainty coming from the estimate of $N_{el} + N_{inel}$. So it seems to us that it is illusory to hope for an accuracy on σ_{tot} better than 10% from this method.

The large discrepancy between the measurements of σ_{tot} by CDF [14] and E710/E811 [16] probably has its origin in the uncertainty on $N_{el} + N_{in}$ and on the dependence of ρ and B on t, and not in some experimental mistake. Hence, this large difference reflects the real error one is to expect from this method.

Conclusion.—Our analysis shows that both methods introduce large correlations between ρ and σ_{tot}. As it is very likely that at the LHC unitarization will play an important role, one should not assume that $B(s, t)$ and $\rho(s, t)$ are constant with t, and their exact behavior with t is model-dependent. Hence, the inescapable conclusion is that a 1% measurement of σ_{tot} will be possible only if one measures $B(s, t)$ and $\rho(s, t)$ as well.

$|t|$ should go from very small values, as close as possible to zero, to about 0.1 GeV^2, with sufficiently small bins. And it will be important to allow all parameters to vary. Indeed, the measurement of ρ performed by UA4/2 (0.135 ± 0.015) [13] seemed to contradict that of UA4 (0.24 ± 0.02) [17] only because σ_{tot} was fixed. Allowing σ_{tot} to be fitted to the data leads to an agreement between the two measurements [18].

We also believe that the luminosity-dependent method is preferable, as it uses more information. The measurement of ρ performed at the Tevatron [19] used the luminosity-independent method with very large bins in t, and the interval considered was $0.00095 \leq |t| \leq 0.1431$ GeV^2. On the lower side, one reached very small t, so that the behavior of the amplitude cannot be taken as a single exponential because of the CNI effect. On the upper side, the intervals in t were too big to measure the specific properties of the CNI region. Hence, the ρ parameter extracted is very uncertain, and it could be that it varied appreciably with t.

As the standard fitting procedure can give misleading results, we remind the reader that additional methods have been proposed to check the validity of the assumptions entering the fits. First, it is possible to extract the value of $\rho(s, t)$ at small but nonzero t [20,21], using the fact that the Coulomb amplitude $F_C(t)$ has an opposite sign to that of the real part of the pp amplitude $F_n(t)$, so that there is a value $t = t_C$ for which $F_C(t_C) = -\Re e F_n(t_C)$, so that the differential cross section $d\sigma/dt$ has a local minimum. The position of this minimum depends strongly on the form assumed for $\rho(s, t)$ and extracting its value would show whether ρ varies quickly with t or not. Using this method, it was found in [20,21] that already at $\sqrt{s} = 52.8$ GeV, ρ was not constant with t.

It is also possible, at small $|t|$, to determine the elastic hadronic cross section via an iterative method, which takes advantage of the expression of the total elastic cross section in the CNI region [22].

Finally, one can adapt a method that was first designed to study eventual oscillations in $d\sigma/dt$ [23]. The idea is to compare two statistically independent samples built by binning the whole t range in small intervals, and by keeping, e.g., one interval out of two. The deviations of the experimental values from theoretical expectations, weighted by the experimental error, are then summed for each sample k,

$$\Delta R^k(t) = \sum_{|t_i|<|t|} [(d\sigma^k/dt_i)^{\rm exp} - (d\sigma/dt_i)^{\rm th}]/\delta_i^{\rm exp}, \quad (3)$$

where $\delta_i^{\rm exp}$ is the experimental error. If the theoretical curve does not precisely describe the experimental data (for example, if the physical hadron amplitude does not have an exactly exponential behavior with momentum transfer), the sum $\Delta R^k(t)$ will differ from zero, going beyond the size of the statistical error.

Using these methods will help to test the assumptions entering the future experimental analyses of TOTEM and ATLAS and may lead to a much more reliable measurement of ρ and $\sigma_{\rm tot}$.

O. V. S. gratefully acknowledges financial support from FRNS and would like to thank the University of Liège where part of this work was done.

[1] J. R. Cudell *et al.* (COMPETE Collaboration), Phys. Rev. Lett. **89**, 201801 (2002).

[2] J. R. Cudell and O. V. Selyugin, Phys. Lett. B **662**, 417 (2008).

[3] O. V. Selyugin and J. R. Cudell, *Proceedings of the 12th International Conference on Elastic and Diffractive Scattering: Forwards Physics and QCD 2007*, edited by J. Bartels, K. Borras, M. Diehl, and H. Jung (Deutsches Elektronen-Synchroton, Hamburg, 1997), pp. 279–284.

[4] P. V. Landshoff, 11th International Conference on Elastic and Diffractive Scattering: Towards High Energy Frontiers: The 20th Anniversary of the Blois Workshops, Chateau de Blois, (The Gió'i Publishers, Hanoi, Vietnam, 2005).

[5] S. M. Troshin and N. E. Tyurin, Int. J. Mod. Phys. A **22**, 4437 (2007).

[6] V. Berardi *et al.* (TOTEM Collaboration), Reports No. CERN-LHCC-2004-002 and CERN-LHCC-2004-020.

[7] C. Sbarra (ATLAS Collaboration), in *Proceedings of the 12th International Conference on Elastic and Diffractive Scattering: Forwards Physics and QCD 2007*, edited by J. Bartels, K. Borras, M. Diehl, and H. Jung (Deutsches Elektronen-Synchroton, Hamburg, 1997), pp. 215–220.

[8] S. V. Goloskokov, S. P. Kuleshov, and O. V. Selyugin, Z. Phys. C **50**, 455 (1991).

[9] J. R. Cudell, E. Martynov, O. V. Selyugin, and A. Lengyel, Phys. Lett. B **587**, 78 (2004).

[10] A. Donnachie and P. V. Landshoff, Phys. Lett. B **550**, 160 (2002).

[11] J. R. Cudell and O. V. Selyugin, Czech. J. Phys. **54**, A441 (2004).

[12] O. V. Selyugin, Mod. Phys. Lett. A **11**, 2317 (1996).

[13] C. Augier *et al.* (UA4/2 Collaboration), Phys. Lett. B **316**, 448 (1993).

[14] F. Abe *et al.* (CDF Collaboration), Phys. Rev. D **50**, 5550 (1994).

[15] Otherwise, one must subtract the electromagnetic and CNI terms, but this means that one should know the luminosity beforehand.

[16] N. A. Amos *et al.* (E710 Collaboration), Phys. Rev. Lett. **63**, 2784 (1989); C. Avila *et al.* (E811 Collaboration), Phys. Lett. B **445**, 419 (1999).

[17] M. Bozzo *et al.* (UA4 Collaboration), Phys. Lett. B **147**, 392 (1984).

[18] O. V. Selyugin, Phys. Lett. B **333**, 245 (1994).

[19] N. Amos *et al.* (E710 Collaboration), Phys. Rev. Lett. **68**, 2433 (1992).

[20] P. Gauron, B. Nicolescu, and O. V. Selyugin, Phys. Lett. B **629**, 83 (2005).

[21] O. V. Selyugin, in *Proceedings of the VIIth Blois Workshop on Elastic and Diffractive Scattering*, edited by P. Chiappetta, M. Haguenauer, and L. Tran Thanh Van (Editions Frontières, Paris, 1995), p. 87.

[22] O. V. Selyugin, Yad. Fiz. **55**, 841 (1992) [Sov. J. Nucl. Phys. **55**, 466 (1992)].

[23] O. V. Selyugin, Ukr. J. Phys. **41**, 296 (1996); P. Gauron, B. Nicolescu, and O. V. Selyugin, Phys. Lett. B **390**, 405 (1997).