

## Self-Induced Transparency Modelocking of Quantum Cascade Lasers

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The possibility of using the self-induced transparency effect to passively modelock lasers has been discussed since the late 1960s, but has never been observed. It is proposed that quantum cascade lasers are the ideal tool to create this modelocking, due to their rapid recovery times and relatively long coherence times and because it is possible to interleave gain and absorbing layers. Conversely, it is possible to use the self-induced transparency effect to create midinfrared pulses that are less than 100 fs in duration in a semiconductor laser.

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With the almost simultaneous discovery of self- or passive modelocking of optical pulses [1,2] and self-induced transparency (SIT) [3,4], it is natural that there was speculation that the observed modelocking was due to SIT [5,6]. However, subsequent work made it clear that SIT could not account for the observed modelocking [7,8]. To date, passively modelocked systems operate in regimes in which the pulse bandwidth is smaller than the gain bandwidth, so that typically coherence times  $T_2$  are short compared to the pulse duration. With the development of the standard theory of passive modelocking [9], work on SIT modelocking almost entirely ceased. An exception is work by Kozlov [10], who pointed out the importance of including an absorbing medium, in which the modelocked pulse is a  $2\pi$  pulse, along with a gain medium, in which the pulse is a  $\pi$  pulse, suppressing the growth of the continuous waves and the Risken-Nummedal-Graham-Haken instability [11]. The absorbing medium acts as a saturable absorber.

In this Letter, we will suggest that quantum cascade lasers (QCLs) are an ideal tool for creating the long-predicted, but never observed, SIT modelocking. Conversely, the use of SIT modelocking makes it possible to obtain modelocked pulses from a midinfrared semiconductor laser that are less than 100 fs in duration.

In order to obtain SIT modelocking, it is necessary to have two highly coherent media with nearly equal resonant frequencies, in one of which the dipole strength is twice the other. It is hard to find naturally occurring media that fulfill these conditions, but the possibility of engineering QCLs makes it possible to circumvent this difficulty by interleaving gain and absorbing layers in the desired ratio, as shown schematically in Fig. 1. As long as there are many gain and absorbing layers within the transverse wavelength of the confined laser mode, the gain and absorbing layers will experience the same field intensity. One must then engineer the absorbing layers so that they have twice the dipole moment of the gain layers. The goal of the design is to inject electrons into the upper resonant state in the gain layers in the usual way, while injecting them into the lower

resonant state in the absorbing layers, where they have a lifetime comparable to the lifetime in the gain layer before tunneling into the next injector. Using now-standard procedures [12], we have designed a structure that satisfies these criteria and operates at  $8\ \mu\text{m}$ . One gain period and one absorbing period are shown in Fig. 2. For the structure shown here, the gain transition is between the levels labeled  $3g$  and  $2g$  and has a dipole moment  $\mu_g/e = 1.55\ \text{nm}$ . The lifetime of level  $3g$  is  $\sim 3\ \text{ps}$ . The absorbing transition is between levels  $3a$  and  $4a$ , and the lifetime of level  $4a$  is  $\sim 0.8\ \text{ps}$ . We have also found a design at  $12\ \mu\text{m}$ , and it appears possible to find designs for a wide range of wavelengths. The details of the designs and the design procedures will be presented elsewhere.

A second key property of QCLs is their narrow linewidths and hence relatively long coherence times  $T_2$ , compared to other semiconductor lasers. Values of  $T_2$  on the order of 100 fs are common, and values on the order of 200 fs are achievable [12]. Another important feature is

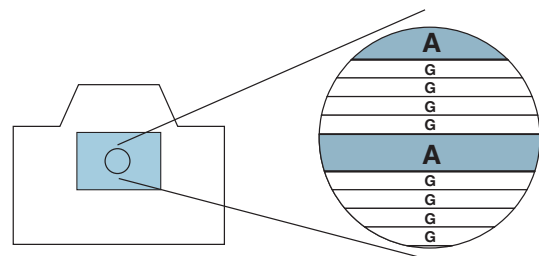


FIG. 1 (color online). Schematic illustration of a QCL structure with gain and absorbing layers. On the left, we show a cutaway view of the QCL structure. The active region is shown as a filled-in rectangle. We are looking in the direction along which light would propagate. Electrodes would be affixed to the top and bottom, so that electrons flow vertically. The heterostructure would also be stacked vertically as shown on the right. We show one absorbing layer for every four gain layers, corresponding schematically to the case in which  $N_g \approx 4N_a$ , and we show absorbing layers that are twice as large as gain layers to indicate schematically that  $\mu_a \approx 2\mu_g$ .

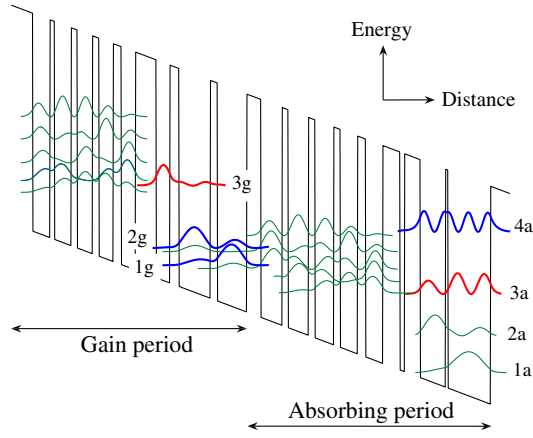


FIG. 2 (color online). Conduction band diagram and moduli-squared wave functions for one gain and one absorbing period of the modelocking QCL structure. The layer sequences are (in Å, from left): **42**, 34, **9**, 33, **12**, 30, **13**, 28, **16**, 28, **41**, 27, **18**, 62, **14**, 58, **28**, 42, **12**, 40, **13**, 37, **13**, 34, **16**, 34, **34**, 9, **31**, 50, **5**, 84. The numbers in bold type correspond to  $\text{In}_{0.52}\text{Al}_{0.48}\text{As}$  barrier layers, and those in roman type correspond to  $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$  well layers.

their rapid gain recovery time  $T_1$ , so that  $T_1 \ll T_{\text{rt}}$ , where  $T_{\text{rt}}$  is the round-trip time. Typical values of  $T_1$  are on the order of 1 ps [12], while values of  $T_{\text{rt}}$  are on the order of 50 ps.

In work to date, Wang *et al.* [13] and Gordon *et al.* [14] observed the Risken-Nummedal-Graham-Haken instability in a QCL with only gain layers. They showed evidence for Rabi oscillations and demonstrated that the two-level Maxwell-Bloch equations, with an additional contribution from a saturable nonlinearity, apply to QCLs in some parameter regimes. Additionally, Wang *et al.* [15] obtained 3 ps pulses in an actively modelocked system, and Choi *et al.* [16] demonstrated SIT-related coherence by injecting 200 fs pulses into a QCL.

Our analysis begins with the Maxwell-Bloch equations in the two-level approximation that we write in the form

$$\frac{n}{c} \frac{\partial E}{\partial t} + \frac{\partial E}{\partial z} = -i \sum_{x=a,g} \frac{kN_x \Gamma_x \mu_x}{2\epsilon_0 n^2} \eta_x - \frac{1}{2} l E, \quad (1a)$$

$$\frac{\partial \eta_{g,a}}{\partial t} = \frac{i\mu_{g,a}}{2\hbar} \Delta_{g,a} E - \frac{\eta_{g,a}}{T_{2g,a}}, \quad (1b)$$

$$\frac{\partial \Delta_{g,a}}{\partial t} = \frac{i\mu_{g,a}}{\hbar} \eta_{g,a} E^* - \frac{i\mu_{g,a}}{\hbar} \eta_{g,a}^* E + \frac{\Delta_{g,a0} - \Delta_{g,a}}{T_{1g,a}}, \quad (1c)$$

where the subscripts  $g$  and  $a$  in Eqs. (1b) and (1c) represent gain and absorption, respectively. The independent variables  $z$  and  $t$  are length along the light-propagation axis of the QCL and time. The dependent variables  $E$ ,  $\eta_{g,a}$ , and  $\Delta_{g,a}$  refer to the envelopes of the electric field, gain polarization, and gain inversion. The parameters  $\Delta_{g0} \approx 1.0$  and  $\Delta_{a0} \approx -1.0$  refer to the equilibrium inversion away from

the modelocked pulse. The parameters  $n$  and  $c$  denote the index of refraction and the speed of light. The parameters  $N_{g,a} \Gamma_{g,a}$  denote the effective electron density multiplied by the overlap factor. The parameters  $k$ ,  $l$ ,  $\epsilon_0$ , and  $\hbar$  denote the wave number in the active region, the linear loss including the mirror loss, the vacuum dielectric permittivity, and Planck's constant. The notation closely follows that of Wang *et al.* [13], with the differences that we have an absorbing as well as a gain medium, and we are considering unidirectional propagation, as is appropriate for a modelocked pulse [17]. As a consequence, spatial hole burning will not be present. Equation (1) does not include the effects of dispersion [18] and saturable nonlinearity [13,14]. In Refs. [13,14], it was not found necessary to include the dispersion, and Gordon *et al.* [14] attributed the saturable nonlinearity to effects that depend sensitively on the device geometry and can only be accurately modeled for specific devices. In practice, there are more than two levels involved in the transitions in both the gain and absorbing media. However, as long as all transition times satisfy  $T_2 \ll T_1 \ll T_{\text{rt}}$ , as is typically the case, we do not expect the results to be strongly affected.

In order to achieve modelocking, the gain of the continuous waves must be below threshold. In the case of a fully inverted medium, so that  $\Delta_g = \Delta_{g0} = 1$  and  $\Delta_a = \Delta_{a0} = -1$ , the steady-state condition may be obtained by substitution into Eq. (1) as  $g - a - l = 0$ , where  $g, a = kN_{g,a} \Gamma_{g,a} \mu_{g,a}^2 T_{2g,a} / 2\epsilon_0 n^2 \hbar$ . Physically, the parameter  $g$  corresponds to the gain per unit length, and the parameter  $a$  corresponds to the absorption per unit length in the absorbing layers of the QCL. The condition for the linear gain to remain below threshold is  $g - a - l < 0$ .

Assuming that  $T_{1g}$  and  $T_{1a}$  are large enough so that they may be set equal to  $\infty$  in Eq. (1), and focusing on the special case in which  $\mu_a = 2\mu_g$ , Eq. (1) has an exact analytical solution that we may write

$$\begin{aligned} E &= \frac{\hbar}{\mu_g \tau} \mathbf{S}, & \eta_g &= \frac{iB_g}{2} \mathbf{S}, \\ \Delta_g &= B_g \left( \frac{\tau}{T_{2g}} - \mathbf{T} \right), & \eta_a &= \frac{iB_a}{2} \left( \mathbf{S} \mathbf{T} - \frac{\tau}{3T_{2a}} \mathbf{S} \right), \\ \Delta_a &= -\frac{B_a}{2} \left( 1 + \frac{\tau^2}{3T_{2a}^2} \right) + B_a \left( \mathbf{S}^2 + \frac{2\tau}{3T_{2a}} \mathbf{T} \right), \end{aligned} \quad (2)$$

where  $\mathbf{S} \equiv \text{sech} x$  and  $\mathbf{T} \equiv \tanh x$ , with  $x = (t/\tau) - (z/v\tau)$ ,  $B_g = (1 + \tau/T_{2g})^{-1}$ , and  $B_a = 2(1 + \tau/T_{2a})^{-1} \times (1 + \tau/3T_{2a})^{-1}$ . This solution incorporates the initial conditions  $\Delta_g \rightarrow 1$  and  $\Delta_a \rightarrow -1$  as  $t \rightarrow -\infty$ , corresponding physically to complete recovery of the gain and absorption media after one round trip of the modelocked pulse through the laser. The parameters  $v$  and  $\tau$  that correspond to the pulse velocity and duration may be determined from the equations  $(1/v) = (n/c) + a\tau(\tau/2T_{2a}) \times (1 + \tau/T_{2a})^{-1} (1 + \tau/3T_{2a})^{-1}$ , and

$$g \frac{\tau/T_{2g}}{1 + \tau/T_{2g}} - a \frac{\tau^2/3T_{2a}^2}{(1 + \tau/T_{2a})(1 + \tau/3T_{2a})} - l = 0. \quad (3)$$

We now consider in more detail the special case  $T_{2g} = T_{2a} \equiv T_2$ . Writing  $\bar{g} = g/l$ ,  $\bar{a} = a/l$ , and  $\bar{\tau} = \tau/T_2$ , we find that the condition to avoid the growth of continuous waves becomes  $\bar{g} < \bar{a} + 1$  and the equation for the pulse duration becomes

$$\frac{3}{\bar{\tau}} = \frac{3\bar{g} - 4}{2} + \left[ \left( \frac{3\bar{g} - 4}{2} \right)^2 + 3(\bar{g} - \bar{a} - 1) \right]^{1/2}. \quad (4)$$

A solution is only possible when  $\bar{g} > (4\bar{a}/3)^{1/2} + 2/3$ . We have found by solving Eq. (1) computationally that when an initial pulse is injected into a medium that violates this condition, it damps away. The condition  $\bar{g} < \bar{a} + 1$  and Eq. (4) define a parameter regime in which stable operation is possible, shown in Fig. 3. In this figure, we also show contours of constant values of  $\bar{\tau}$ . Stable operation is only possible when  $\bar{a} > (1/3) + (1/\bar{\tau})$ , implying that  $\bar{g} > (4/3) + (1/\bar{\tau})$ . Moreover, to obtain pulse durations that are on the order of  $T_2$ , we should have values of  $\bar{a} \geq 2$ , corresponding to  $\bar{g} \geq 2.5$ . We note from the definitions of  $\bar{g}$  and  $\bar{a}$  that they will both be proportional to the current. Hence, a larger gain and absorption require a larger current. At the same time, we note that  $\bar{g}$  and  $\bar{a}$  are proportional to  $T_2$ . Hence, it is possible to reduce the required current by increasing  $T_2$ . Another point to note is that the pulse durations can be made arbitrarily small, but at the cost of increasing  $\bar{g}$  and hence the required current.

In order for the solution reported in Eq. (2) to be of any practical interest, it must be robust when  $\mu_a$  differs from  $2\mu_g$ , when  $T_{1g}$  and  $T_{1a}$  are on the order of a picosecond or less, when the inversion of the gain and absorbing media is

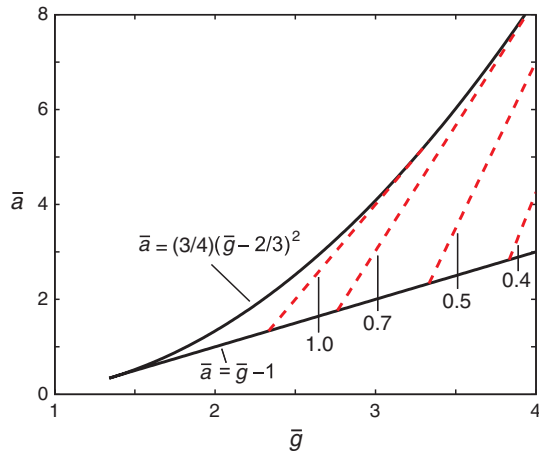


FIG. 3 (color online). Stability limits of the normalized absorption  $\bar{a}$  vs the normalized gain  $\bar{g}$ . Below the lower limit, multipulsing occurs due to continuous wave growth. Above the upper limit, the gain cannot compensate for the loss, and any initial pulse will damp. Dashed lines show contours of constant normalized pulse duration  $\bar{\tau}$ . The values of  $\bar{\tau}$  are marked.

incomplete, and when an initial pulse that is long compared to its final, stable duration is injected into the medium. We have carried out an extensive set of simulations of Eq. (1) to investigate these issues. The details will be reported elsewhere, but we will summarize the results here. Taking as our base parameter set,  $T_{1g} = T_{1a} \equiv T_1 = 10T_2$ , corresponding, for example, to a  $T_1$  of 1 ps and a  $T_2$  of 100 fs, which are physically reasonable values [12], we find that when  $\mu_a/\mu_g = 2$ , the upper stability boundary in Fig. 3 is slightly lowered, so that when  $\bar{g} = 4.0$ , we obtain  $\bar{a} = 7.3$  for stable operation, rather than 8.3, as predicted by Eq. (4). The lower stability limit is unchanged. When we further reduce  $T_1/T_2$  so that it equals 5, we find that when  $\bar{g} = 4.0$ , the upper stability boundary has moved to  $\bar{a} = 6.65$ . At a ratio of  $T_1/T_2 = 10$ , we find that an initial sech pulse can propagate stably with  $\mu_a/\mu_g > 1.4$  when  $\bar{g} = 4$  and  $\mu_a/\mu_g > 1.8$  when  $\bar{g} = 3$ , with  $\bar{a} = 3$  in both cases. There is no evident upper limit of  $\mu_a/\mu_g$  for stable operation, though pulses increasingly distort as  $\mu_a/\mu_g$  increases above 2. We have investigated what happens if we reduce  $\Delta_{g0}$  to 0.8 and increase  $\Delta_{a0}$  to  $-0.8$ . Both the lower and upper stability lines in Fig. 3 are lowered somewhat, but a large stability region still exists. Finally, we have investigated the result of changing the initial pulse energy and duration. As long as the initial pulse is shorter than  $T_1$  and is sufficiently energetic, it will converge to the stable pulse shape and duration predicted in Eqs. (2) and (4). For example, when we launch an initial sech pulse,  $E_0 = \text{sech}(t/\tau_i)$ , with  $\tau_i = 5T_2 = 0.5T_1$ , and with  $\bar{g} = 3.5$ ,  $\bar{a} = 3.0$ , and  $\mu_a/\mu_g = 2$ , we find that the initial value of  $\Theta = (\mu_g/\hbar) \int_{-\infty}^{\infty} E dt$  must be above  $0.84\pi$ . There is no clear upper limit in this case. Input pulses split when  $\Theta > 3\pi$ , but only one pulse is stable and all other pulses decay up to at least  $\Theta = 10\pi$ . We have also investigated the effects of level detuning, which requires a modification of Eq. (1). The detuning must be small compared to  $T_2^{-1}$ .

In order to use SIT modelocking, one must have  $T_1 \ll T_{\pi}$ . Since the modelocked pulse is a  $\pi$  pulse in the gain medium, the gain medium is uninverted after the pulse passes through. The gain medium must reinvert before the pulse passes through again. There are short regions near the mirrors where the pulse passes through the same region twice in rapid succession; however, since  $T_1 \sim 1$  ps and  $T_{\pi} \sim 50$  ps, it is reasonable to assume that this effect can be neglected. Because the laser must operate in this regime, we must make the absorption  $a$  sufficiently large so that the laser cannot self-start; if the laser can self-start, then there is nothing to prevent multipulsing. To start the modelocking, an external source is therefore needed. We suggest two possibilities. First, it may be possible to combine SIT modelocking with active modelocking [14]. If the time duration in which the gain is large enough for continuous wave growth to occur is kept short compared to  $T_1$ , then it should be possible to seed the SIT modelocking

process. Second, it may be possible to use injection locking [15]. This second approach requires a source of pulses that are short compared to  $T_1$  with a repetition rate that can be tuned to a multiple of  $T_{rt}$ , but this multiple can be very large in principle. We are currently investigating the feasibility and quantitative requirements for both approaches.

In conclusion, we have shown that by combining absorbing with gain layers in a QCL, one can create nearly ideal conditions to observe the long-predicted, but never observed, phenomenon of SIT modelocking. Conversely, this approach opens a path to obtaining pulses less than 100 fs long from a midinfrared semiconductor laser. We have presented an analytical, modelocked solution to the Maxwell-Bloch equations, we have determined the conditions under which this solution is stable, and we have calculated the stable pulse durations. We have summarized computational results that demonstrate that this solution is robust when the system parameters are changed. We have shown that this laser cannot self-start and reach a stable operating regime, and we have suggested two possible methods for seeding the modelocking.

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- [1] M. H. Crowell, IEEE J. Quantum Electron. **1**, 12 (1965).  
 [2] T. Uchida and A. Ueki, IEEE J. Quantum Electron. **3**, 17 (1967).

- [3] S. L. McCall and E. L. Hahn, Phys. Rev. Lett. **18**, 908 (1967).  
 [4] S. L. McCall and E. L. Hahn, Phys. Rev. **183**, 457 (1969).  
 [5] A. G. Fox and P. W. Smith, Phys. Rev. Lett. **18**, 826 (1967).  
 [6] P. W. Smith, IEEE J. Quantum Electron. **3**, 627 (1967).  
 [7] A. Frova, M. A. Duguay, C. G. B. Garrett, and S. L. McCall, J. Appl. Phys. **40**, 3969 (1969).  
 [8] V. V. Kozlov, JETP Lett. **69**, 906 (1999) [Pis'ma Zh. Eksp. Teor. Fiz. **69**, 856 (1999)].  
 [9] H. A. Haus, J. Appl. Phys. **46**, 3049 (1975).  
 [10] V. V. Kozlov, Phys. Rev. A **56**, 1607 (1997).  
 [11] H. Risken and K. Nummedal, J. Appl. Phys. **39**, 4662 (1968); R. Graham and H. Haken, Z. Phys. **213**, 420 (1968).  
 [12] C. Sirtori and R. Teissier, in *Quantum Cascade Lasers: Overview of Basic Principles of Operation and State of the Art*, edited by R. Paiella, Intersubband Transitions in Quantum Structures (McGraw-Hill, New York, 2006), p. 1.  
 [13] C. Y. Wang, L. Diehl, A. Gordon, C. Jirauschek, F. X. Kärtner, A. Belyanin, D. Bour, S. Corzine, G. Höfler, M. Troccoli, J. Faist, and F. Capasso, Phys. Rev. A **75**, 031802(R) (2007).  
 [14] A. Gordon, C. Y. Wang, L. Diehl, F. X. Kärtner, A. Belyanin, D. Bour, S. Corzine, G. Höfler, H. C. Liu, H. Schneider, T. Maier, M. Troccoli, J. Faist, and F. Capasso, Phys. Rev. A **77**, 053804 (2008).  
 [15] C. Y. Wang, L. Kuznetsova, L. Diehl, F. Kärtner, M. A. Belkin, H. Schneider, H. C. Liu, and F. Capasso, in Proceedings of the Conference on Lasers and Electro-Optics, San Jose, 2008.  
 [16] H. Choi, V.-M. Gkortsas, F. X. Kärtner, L. Diehl, C. Y. Wang, F. Capasso, D. Bour, S. Corzine, J. Zhu, G. Höfler, and T. B. Norris, in Proceedings of the Conference on Lasers and Electro-Optics, San Jose, 2008.  
 [17] H. A. Haus, IEEE J. Sel. Top. Quantum Electron. **6**, 1173 (2000).  
 [18] H. Choi, L. Diehl, F. Capasso, D. Bour, S. Corzine, J. Zhu, G. Höfler, and T. B. Norris, Opt. Express **15**, 15 898 (2007).