Extracting Primordial Non-Gaussianity without Cosmic Variance

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Recent work has emphasized the possibility to probe non-Gaussianity of local type by measuring the power spectrum of highly biased tracers of large scale structure on very large scales. This method is limited by the cosmic variance, by the finite number of structures on the largest scales, and by the partial degeneracy with other cosmological parameters that can mimic the same effect. We propose an alternative method based on the fact that on large scales, halos are linearly biased, but not stochastic, tracers of dark matter: by correlating a highly biased tracer of large scale structure against an unbiased tracer, one eliminates the cosmic variance error, which can lead to a significant increase in signal to noise. For an ideal survey out to $z \sim 2$, the error reduction can be as large as a factor of 7, which should guarantee a detection of non-Gaussianity from an all-sky survey of this type.

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One of the major unsolved mysteries in cosmology is the creation of structure in the Universe. There are many competing theories that differ in their predictions, some of which are accessible to cosmological observations today. Inflation, the oldest and most successful of these theories, predicts that the two-point correlation function of initial structures is nearly scale invariant and adiabatic. However, ever since the first detection of cosmic microwave background anisotropies, it has been recognized that cosmic variance, a finite number of structures on the largest scales, provides a fundamental limitation to how well we can distinguish among the competing models using the two-point function correlations [1,2]. This is because the primordial density field is a single realization of a random process, and only its two-point function is specified by the theoretical predictions. For many discriminating tests such as the shape of the power spectrum on large scales, the cosmic variance limit is fundamental.

Our desire to discriminate among the models motivates searching for other probes. One of these is non-Gaussianity, and many of the models predict the non-Gaussianity of local type, $\Phi = \phi + f_{\rm nl}\phi^2$, where Φ is the gravitational potential in the matter era and ϕ is the corresponding primordial Gaussian case. In the simplest models of inflation, primordial non-Gaussianity is predicted to be very small, $|f_{\rm nl}| \ll 1$, but more complex models of inflation, as well as its alternatives, naturally predict large non-Gaussianity, $|f_{\rm nl}| \gg 1$, leading to the non-Gaussianity as the smoking gun for alternatives to the simplest models of inflation.

All of the tests of the non-Gaussianity proposed so far also suffer from the cosmic variance limit. Until recently, the most powerful method to place limits on $f_{\rm nl}$ was based on the bispectrum of the cosmic microwave background (CMB), with the latest WMAP constraint $-9 < f_{\rm nl} < 111$ (95% C.L.) [3] (but see [4] for a claim of a detection at 2.8 sigma level). With a better angular resolution, one can sample more modes, and the limits should be improved by a factor of 4–6 with the higher angular resolution Planck satellite [5]. By measuring high redshift 3-dimensional matter distribution with 21 cm transitions, the limits could be improved further [6], but at the moment, this method remains unproven.

An alternative approach using clustering of biased tracers of structure on very large scales has recently been proposed [7]. It was shown that the non-Gaussianity leads to a very unique scale dependence of the large scale bias, one that increases strongly towards the large scales, and whose amplitude scales with the bias of the tracer relative to the dark matter. One can therefore place the limits on $f_{\rm nl}$ by comparing the scale dependence of the power spectrum of the biased tracer to the one expected in cosmological models under the assumption of a scale independent bias. Simulations suggest this is a reasonable assumption for $k < 0.1\ h/{\rm Mpc}$ [8], which covers the range of interest for non-Gaussianity studies. Subsequent work explored further theoretical issues and prospects for the future [9–11]. A first application of this method, which we will call the power spectrum method, has recently been presented using the large scale clustering of quasar and luminous red galaxies (LRG) galaxy data from the Sloan Digital Sky Survey (SDSS) [10]. The result, $-29 < f_{\rm nl} < 69$ (95% C.L.), is already better than the latest CMB constraints from WMAP, suggesting this is a competitive method compared to the bispectrum from CMB and should be pursued further.

An all-sky sample of highly biased tracers out to $z \sim 2$ with 10^8 galaxies, such as those contemplated for the dark energy mission [12], could reduce the current errors by a factor of 10 [11,13,14]. While this is impressive, it would be great if one could improve them further to reach the realm of a guaranteed detection, which is at the level of

$|f_{\rm nl}| \sim 1$ [15]. There are two main obstacles to this. First, this method suffers significantly from the cosmic variance: this is because the signal is strongest on the largest scales where the variance due to the finite number of realizations within a given volume makes the error on the power spectrum large. Relative error from each mode is of order unity; hence, we can only detect the $f_{\rm nl}$ signal if the relative change in power due to $f_{\rm nl}$ is of order unity. Second limitation pointed out in [10] is that for the current data the effect of $f_{\rm nl}$ is correlated with other cosmological parameters, such as the matter density and the primordial slope of the power spectrum, which also change the shape of the power spectrum on large scales.

In this Letter, we propose a method to probe the non-Gaussianity from the large scale structure that circumvents the cosmic variance limit and also eliminates the problem of its degeneracy with other cosmological parameters. Instead of measuring the power spectrum of a tracer and comparing it to predictions based on cosmological models, we propose to compare directly the density field of two tracers with different bias. The relative bias of the two tracers is scale independent in the absence of non-Gaussianity, but picks up a scale dependence in its presence. The main advantage is that tracers are generally biased, but not stochastic, on very large scales. This means that the precision with which the relative bias between the two tracers can be determined is only limited by the noise, which is given by the Poisson sampling of the field, and not by the cosmic variance. Moreover, because we are directly comparing two density fields, any scale dependence of relative bias can only be caused by the non-Gaussianity and not by the other cosmological parameters, which cannot affect the amplitude ratio even if they can affect the power spectrum. Hence, there is no degeneracy between the $f_{\rm nl}$ and the other cosmological parameters with this method. In the following, we expand on these ideas.

Let us assume we measure two tracers of matter density field, δ_1 and δ_2. They are both biased tracers of the underlying matter density field δ, $\delta_i = b_i\delta$, where b_1, b_2 is the large scale bias of tracers 1 and 2, respectively. We can introduce the relative bias $\alpha = b_1/b_2$, and the corresponding covariance matrix elements in the Fourier domain are $C_{22} = \langle\delta_2^2\rangle = (P_2 + \bar{n}_2^{-1})/V$, $C_{12} = \langle\delta_1\delta_2\rangle = r\alpha P_2/V$, and $C_{11} = \langle\delta_1^2\rangle = (\alpha^2 P_2 + \bar{n}_1^{-1})/V$, where r is the cross-correlation coefficient between the two fields, P_2 is the power spectrum of second tracer, $\bar{n}_1$ and $\bar{n}_2$ are the densities of the two tracers, respectively, and V is the volume of the survey.

We want to compare the errors on non-Gaussianity parameter $f_{\rm nl}$ as extracted from the power spectrum analysis to the one from the analysis of the relative amplitude of the two tracers. To answer this, we need to compute the errors of the corresponding parameters, P_2 and α, and their dependence on $f_{\rm nl}$. We will first compute the errors for these two parameters and then combine with their dependence on $f_{\rm nl}$ to derive the final error predictions on $f_{\rm nl}$. Fisher matrix plays a key role in describing the ability of a survey to constrain parameters such as $f_{\rm nl}$. Its inverse gives the expected covariance matrix of parameters one wishes to estimate. It is defined as $F_{\lambda\lambda'} = \frac{1}{2}\,{\rm Tr}[C_{,\lambda}C^{-1}C_{,\lambda'}C^{-1}]$, where C is the covariance matrix of the data defined above and λ is the set of parameters one is estimating.

Let us begin with the estimated error of the power spectrum P_2 for a single mode. Applying equation above, we find in the limit of zero noise, $X_i \ll 1$, where $X_i = (\bar{n}_i P_2)^{-1}$, that cosmic variance dominates the error and the irreducible error for one mode is $F^{-1}_{P_2P_2} = \sigma^2_{P_2} = 2P_2^2$; i.e., the relative error is of order of unity.

We can also apply the above expressions to compute the diagonal component of the Fisher matrix for the relative amplitude α,

$$F_{\alpha\alpha} = \frac{\alpha^2 X_2(1+2X_2) + r^2 X_1(1+X_2) + \alpha^2(1-r^2)(2-r^2+3X_2)}{[\alpha^2(1-r^2) + \alpha^2 X_2 + X_1 + X_1X_2]^2}, \tag{1}$$

which in the limit $X_1 \ll 1$, $X_2 \ll 1$, $1 - r^2 \ll 1$ becomes $F^{-1}_{\alpha\alpha} = \sigma^2_\alpha = \alpha^2 X_2 + X_1 + \alpha^2(1 - r^2)$. Thus, the error on α from a single mode can be much less than unity if there is little stochasticity ($r \sim 1$) and the field is oversampled, X_1, $X_2 \ll 1$. Therefore, there is no fundamental cosmic variance limit here, and if α depends on $f_{\rm nl}$, then there is no cosmic variance limit on the latter either.

To connect these expressions to the precision with which one can determine $f_{\rm nl}$, we need to look at the dependence of P_2 and α on $f_{\rm nl}$. For a given tracer with the large scale bias b, we have [7,9–11] $P(k) = [b + \Delta b(k) f_{\rm nl}]^2 P_{\rm dm}(k)$, where $P_{\rm dm}(k)$ is the dark matter power spectrum and $\Delta b(k) = 2(b-p)\delta_c\frac{\phi}{\delta} = \frac{3(b-p)\delta_c\Omega_m H_0^2}{c^2k^2T(k)D(z)}$, where $\delta_c = 1.686$ is the spherical collapse linear overdensity, ϕ is the primordial potential in matter domination, δ is the matter overdensity, H_0 is the Hubble parameter, c is the speed of light, $T(k)$ is the transfer function, $D(z)$ is the linear growth rate normalized to $(1+z)^{-1}$ for $z \gg 1$ and we use $p = 1$ assuming mass selected halos, but can be as large as $p = 1.6$ for recent merger selected halos [10].

The Fisher matrix for $f_{\rm nl}$ is $F_{f_{\rm nl}f_{\rm nl}} = \sum_{\lambda,\lambda'} F_{\lambda\lambda'}\frac{\partial\lambda}{\partial f_{\rm nl}}\frac{\partial\lambda'}{\partial f_{\rm nl}}$. Let us continue to treat the two ways of estimating $f_{\rm nl}$ separately. From the power spectrum estimator for a single mode, we find $F_{f_{\rm nl}f_{\rm nl}}(P_2) = F_{P_2P_2}[2\Delta b(k)/bP_2]^2$, which in the sampling variance limit gives $\sigma_{f_{\rm nl}}(P_2) = \frac{b}{2^{1/2}\Delta b(k)}$ in the limit of the non-Gaussian correction being small. We have assumed that the two samples have been combined optimally into a single sample with an overall bias b and non-

Gaussianity dependence Δb, the details of which will depend on their number density and bias. For example, the simplest case is if $b_2 = 1$, in which case we can assume this tracer contains no information on $f_{\rm nl}$ and the overall sample is just the biased sample with $b = b_1$ and $\Delta b = \Delta b_1$. The overall error is obtained by integrating over all the modes, $\sigma_{f_{\rm nl}}^{-2}(P_2) = \frac{V}{\pi^2}\int_{k_{\rm min}}^{k_{\rm max}}[\Delta b(k)/b]^2 k^2 dk$, where we assumed the sampling variance limit ignoring the Poisson term. Here, $k_{\rm min} \sim 2\pi/V^{1/3}$ is the smallest wave vector accessible in such a survey, and $k_{\rm max}$ is the largest wave vector for which we can assume validity of this expression. Its value is often not very important since most of the sensitivity comes from the large scales, i.e., small wave vectors. Note that we have ignored any degeneracies between $f_{\rm nl}$ and other cosmological parameters in this expression, so it is bound to be overly optimistic, although for anything more quantitative, one would need to do the actual analysis with a given survey geometry, and choosing the cosmological parameters, one is varying in the analysis.

In the oversampling, low stochasticity limit the Fisher matrix for $f_{\rm nl}$ from the relative amplitude of two tracers for a single mode is given by

$$\sigma_{f_{\rm nl}}(\alpha) = (X_2 + X_1\alpha^{-2} + 1 - r^2)^{1/2}\left(\frac{\Delta b_1}{b_1} - \frac{\Delta b_2}{b_2}\right)^{-1}. \tag{2}$$

Integrating over all the modes gives $\sigma_{f_{\rm nl}}^{-2}(\alpha) = \frac{V}{2\pi^2} \times \int_{k_{\rm min}}^{k_{\rm max}} F_{\alpha\alpha}\alpha^2(\frac{\Delta b_1(k)}{b_1} - \frac{\Delta b_2(k)}{b_2})^2 k^2 dk$. Here, we have assumed that $\alpha = b_1/b_2$ is known perfectly from the smaller scales where $f_{\rm nl}$ effects are negligible but linear bias still applies so that information on α from large scales is used for the estimation of $f_{\rm nl}$ rather than the relative bias itself. A more detailed analysis in [16] provides further support of results presented here.

It is useful to compare the ratio of expected errors for a single mode in the sampling variance limit assuming $r = 1$. We find $\frac{\sigma_{f_{\rm nl}}^2(\alpha)}{\sigma_{f_{\rm nl}}^2(P_2)} = 2(X_2 + X_1/\alpha^2)[\frac{\Delta b/b}{\Delta b_1/b_1 - \Delta b_2/b_2}]^2$. If we assume again, we have two tracers, one unbiased, $b_2 = 1$ and one biased with $b_1 = b$ this simplifies to

$$\frac{\sigma_{f_{\rm nl}}(\alpha)}{\sigma_{f_{\rm nl}}(P_2)} = \sqrt{2(X_2 + X_1/\alpha^2)}. \tag{3}$$

Let us apply the above derived relations to some examples of interest. SDSS-III plans to do a spectroscopic survey of about 1.5×10^6 luminous red galaxies (LRGs) out to $z \sim 0.7$ over a quarter of the sky. The expected number density is about 3×10^{-4} $(h/\mathrm{Mpc})^3$ and average bias about $b = 1.7$ [17], so $P_1(k = 0.01\ h/\mathrm{Mpc}) = 0.8 \times 10^5$ $(\mathrm{Mpc}/h)^3$, and from Eq. (3), we may expect about a factor of 3 error improvement if the unbiased tracers are available and their shot noise can be neglected. This latter assumption is by no means easy to achieve, since lower bias galaxies live in lower mass halos, are typically fainter, and SDSS-III will not target them spectroscopically. One way to pursue is with photometric samples, which is discussed further below. Another is to split the LRG sample into two luminosity bins, where we may expect for the brighter bin $\bar{n}_1 = 10^{-4}$ $(h/\mathrm{Mpc})^3$ and $b \sim 2$, so $P_1(k = 0.01\ h/\mathrm{Mpc}) = 1.2 \times 10^5$ $(\mathrm{Mpc}/h)^3$ at $z \sim 0.5$, while for the fainter bin, we may expect $n_2 = 2 \times 10^{-4}$ $(h/\mathrm{Mpc})^3$ and $b = 1.4$, in which case $d\alpha/df_{\rm nl}$ is reduced by more than a factor of 2 and the noise is increased since $X_1 = X_2 = 0.1$. In this case, this method gives a comparable signal to noise to the power spectrum method. This may still be advantageous since there is no degeneracy with other cosmological parameters and one can extend the analysis to small scales.

Looking further into the future, there are a number of planned spectroscopic surveys that will measure a high number of redshifts to higher redshifts [18,19]. In this case, one may expect larger improvements, specially if all of the most biased halos can be identified. For example, at $z \sim 1.8$, we may have $\bar{n}_1 \sim 3 \times 10^{-3}$ $(h/\mathrm{Mpc})^3$ halos with average bias $b = 2$ and $P_1(k = 0.01\ h/\mathrm{Mpc}) \sim 3 \times 10^4$ $(\mathrm{Mpc}/h)^3$, and most of these should be detectable with the above mentioned surveys. Hence in this case, the improvement can be a factor of 7 over the power spectrum method. Note that with the power spectrum method, such an improvement would require a 50-fold increase in the volume of the survey. This is a particularly exciting prospect since the predicted 95% confidence level interval for a full sky survey to $z \sim 2$ is of order $\Delta f_{\rm nl} \sim 5$–10 [11,13,14] and with the additional factor of 7 reduction of error, we may be able to achieve $\Delta f_{\rm nl} \sim 1$ (95% C.L. interval), at which point we enter into regime of a guaranteed detection [20].

The benefits may be even greater for photometric surveys, where no spectroscopy is needed and hence can be done with a significantly smaller investment. On very large scales, the advantages of a spectroscopic survey become less important if the photometric redshift error is well below the largest scale of the survey. Indeed, currently the strongest constraints come from the power spectrum method using the LRG photometric sample and the quasar photometric sample, both from the SDSS [10]. As mentioned above, the main limitation of photometric samples is that the projection along the line of sight causes the non-Gaussianity effect to extend over a wide range of multipole moments, making the effect difficult to distinguish from the other cosmological parameters using the power spectrum method [10]. This problem is eliminated using the method proposed here. It is also possible to combine spectroscopic and photometric samples. For example, one can use a high bias spectroscopic sample, such as the above mentioned LRG spectroscopic survey from SDSS-III, and apply the radial weights to reproduce the redshift distribution of a photometric sample. Finally, we should also note that the ultimate unbiased tracer insensitive to $f_{\rm nl}$ to correlate against can be the dark matter itself, as measured from weak lensing, although in this case the radial window is rather broad [21].

These predictions are based on the assumption that there is no stochasticity in the tracers, i.e., we assumed $r = 1$ in the analysis. Observational constraints suggest $r > 0.95$ on scales above 10 Mpc [22]. As shown in Eq. (3), stochasticity becomes an issue when $1 - r^2$ exceeds $(\bar{n}P)^{-1}$, so this may be a limitation for $\bar{n}P > 10$. More detailed studies using very large volume simulations with biased tracers will be needed to see what the ultimate limit is.

It is also possible to look for the signature using the antibiased tracers, such as the low mass halos. For these, the bias is asymptotically approaching $b \sim 0.7$ [8,23], and for this population, the effect of $f_{\rm nl}$ has the opposite sign from the biased population with an $f_{\rm nl}$ dependence that is about a factor of 3-4 smaller than for $b = 2$ population assuming $p = 1$. While such an antibiased tracer has about a factor of 10 smaller amount of power, this can be offset by the higher number density, which can be orders of magnitude larger. More generally, recent studies suggest that bias can depend on quantities other than the halo mass [23], and similarly one can also expect secondary parameters that may enhance or suppress the sensitivity to $f_{\rm nl}$. Indeed, an extended Press-Schechter analysis suggests one such second parameter that suppresses the sensitivity to $f_{\rm nl}$ of highly biased tracers is the recent merger activity [10]. The challenge for the future is to find two tracers, one that exhibits a significant dependence on $f_{\rm nl}$ and one that does not, both of which come with a sufficiently high number density and can be observationally identified within existing and future surveys. Given the potentially huge payoff of finding such tracers for the questions of interest to the fundamental theories of the universe, this is a challenge that is worth exploring further both with observations as well as with numerical simulations and analytic methods.

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[1] R. Scaramella and N. Vittorio, Mon. Not. R. Astron. Soc. **263**, L17 (1993).
[2] U. Seljak and E. Bertschinger, Astrophys. J. Lett. **417**, L9 (1993).
[3] E. Komatsu, J. Dunkley, M. R. Nolta, C. L. Bennett, B. Gold, G. Hinshaw, N. Jarosik, D. Larson, M. Limon, and L. Page *et al.*, arXiv:0803.0547.
[4] A. P. S. Yadav and B. D. Wandelt, Phys. Rev. Lett. **100**, 181301 (2008).
[5] A. Cooray, D. Sarkar, and P. Serra, Phys. Rev. D **77**, 123006 (2008).
[6] A. Cooray, Phys. Rev. Lett. **97**, 261301 (2006).
[7] N. Dalal, O. Doré, D. Huterer, and A. Shirokov, Phys. Rev. D **77**, 123514 (2008).
[8] U. Seljak and M. S. Warren, Mon. Not. R. Astron. Soc. **355**, 129 (2004).
[9] S. Matarrese and L. Verde, Astrophys. J. Lett. **677**, L77 (2008).
[10] A. Slosar, C. Hirata, U. Seljak, S. Ho, and N. Padmanabhan, J. Cosmol. Astropart. Phys. 08 (2008) 031.
[11] N. Afshordi and A. J. Tolley, Phys. Rev. D **78**, 123507 (2008).
[12] A. Albrecht, G. Bernstein, R. Cahn, W. L. Freedman, J. Hewitt, W. Hu, J. Huth, M. Kamionkowski, E. W. Kolb, and L. Knox *et al.*, arXiv:astro-ph/0609591.
[13] P. McDonald, arXiv:0806.1061 [Phys. Rev. D (to be published)].
[14] C. Carbone, L. Verde, and S. Matarrese, Astrophys. J. **684**, L1 (2008).
[15] N. Bartolo, E. Komatsu, S. Matarrese, and A. Riotto, Phys. Rep. **402**, 103 (2004).
[16] P. McDonald and U. Seljak, arXiv:0810.0323.
[17] N. Padmanabhan, D. J. Schlegel, U. Seljak, A. Makarov, N. A. Bahcall, M. R. Blanton, J. Brinkmann, D. J. Eisenstein, D. P. Finkbeiner, and J. E. Gunn *et al.*, Mon. Not. R. Astron. Soc. **378**, 852 (2007).
[18] A. Cimatti, M. Robberto, C. Baugh, S. V. W. Beckwith, R. Content, E. Daddi, G. De Lucia, B. Garilli, L. Guzzo, and G. Kauffmann *et al.*, arXiv:0804.4433 [Experimental Astronomy (to be published)].
[19] J. B. Peterson, K. Bandura, and U. L. Pen, arXiv:astro-ph/0606104.
[20] N. Bartolo, S. Matarrese, and A. Riotto, J. Cosmol. Astropart. Phys. 01 (2004) 003.
[21] U.-L. Pen, Mon. Not. R. Astron. Soc. **350**, 1445 (2004).
[22] M. E. C. Swanson, M. Tegmark, M. Blanton, and I. Zehavi, Mon. Not. R. Astron. Soc. **385**, 1635 (2008).
[23] L. Gao, V. Springel, and S. D. M. White, Mon. Not. R. Astron. Soc. **363**, L66 (2005).