Thermal Memory: A Storage of Phononic Information

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Memory is an indispensible element for a computer in addition to logic gates. In this Letter we report a model of thermal memory. We demonstrate via numerical simulation that thermal (phononic) information stored in the memory can be retained for a long time without being lost and more importantly can be read out without being destroyed. The possibility of experimental realization is also discussed.

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Unsatiable demands for thermal management or control in our daily life ranges from thermal insulating to efficient heat dissipation have driven us to better understand heat conduction on a molecular level. Recent years have witnessed a fast development including theoretical proposals in functional thermal devices such as a thermal diode that rectifies heat current [1], thermal transistor that switches and modulates heat current [2], heat pump that carries heat against temperature bias [3], and experimental works such as nanotube phonon waveguide [4], thermal conductance tuning [5] and nanoscale solid state thermal rectifier [6]. More importantly, logic operations with phonons or heat have been demonstrated theoretically [7], which, in principle, has opened the door for a brand new subject—phononics—a science and engineering of processing information with heat [8]. The question arises naturally and promptly: whether a thermal (phononic) memory that can store information is possible?

In this Letter, we would like to give a definite answer by studying the transient process, which in fact exhibits much richer phenomena than asymptotical stationary state does, of a nonequilibrium system. This topic has been, however, rarely studied so far.

Like an electronic memory that records data by maintaining voltage in a capacitance, a thermal memory stores data by keeping temperature somewhere. Therefore, any thermally insulated system might be a candidate for thermal memory since it keeps its temperature (thus data) for a very long time. However, perturbation is unavoidable in such a thermal system, especially when the data are read out, namely, the local temperature is measured. Generally, without external energy source or sink, any thermally insulated system will not be able to recover its original state after the data reading (temperature measuring) process, because there exists energy exchange between the system and the reader (thermometer) during this process. We thus have to turn to a thermal-circuit with power supply, i.e., driven by external heat bath.

In any linear thermal circuit, i.e., all thermal resistances are fixed, independent of temperature and/or temperature drop, the steady state that satisfies Kirchhoff's laws must be unique. This unique state must be stable, namely, under any perturbation the system will eventually return to this state. However in order to work as a memory, a thermal-circuit must have more than one stable steady states, such a bistable thermal-circuit is only possible in the presence of nonlinear thermal devices.

To make such a nonlinear thermal-circuit, we consider a one dimensional *nonlinear* lattice that consists of two Frenkel-Kontorova (FK) segments [9] which describe harmonic chains (mimic a layer of atoms) subject to a spatially periodic potential (mimic the substrate), sandwich a central particle in between. Its configuration is shown in Fig. 1(a) and its Hamiltonian reads

$$H = H_L + H_R + H_{\rm int}, \tag{1}$$

and the Hamiltonian of each segment can be written as $H_W = \sum_{i=1}^{N_W} \frac{\dot{x}_{W,i}^2}{2} + \frac{k_W}{2}(x_{W,i} - x_{W,i-1})^2 - \frac{V_W}{(2\pi)^2}\cos 2\pi x_{W,i}$. W stands for L or R, which denotes left or right segment, respectively. $x_{W,i}$ denotes the displacement from equilibrium position of the ith particle in segment W. For convenience, we have set the mass of all particles to unity. The parameters k and V are the harmonic spring constant and the on-site potential of the FK segments. We couple the last particles of segments L and R to particle O via harmonic springs. Thus $H_{\rm int} = \frac{\dot{x}_O^2}{2} + \sum_W \frac{k_{{\rm int}W}}{2}(x_{W,N_W} - x_O)^2 - \frac{V_O}{(2\pi)^2}\cos 2\pi x_O$. In our numerical simulation, segment L and R each contains 50 particles. Fixed boundary conditions are applied. Other parameters are: $K_L = K_{{\rm int}L} = 0.45$, $K_R = 0.2$, $K_{{\rm int}R} = 0.05$, $V_L = 10$, $V_R = 0$, $V_O = 10$. The deep valley of on-site potential of particle O, V_O, and the weak coupling between segment R and particle O, $K_{{\rm int}R}$, induce a negative differential thermal resistance (NDTR) [2] between O and segment R, which is the essential for thermal memory model. We use a wavy curve to stress this in Fig. 1(a). T_L and T_R, temperatures of the power supplies (heat baths) that contacted to the ends of the two segments, are fixed to 0.04 and 0.3, respectively. All heat baths are simulated by Langevin heat baths [10].

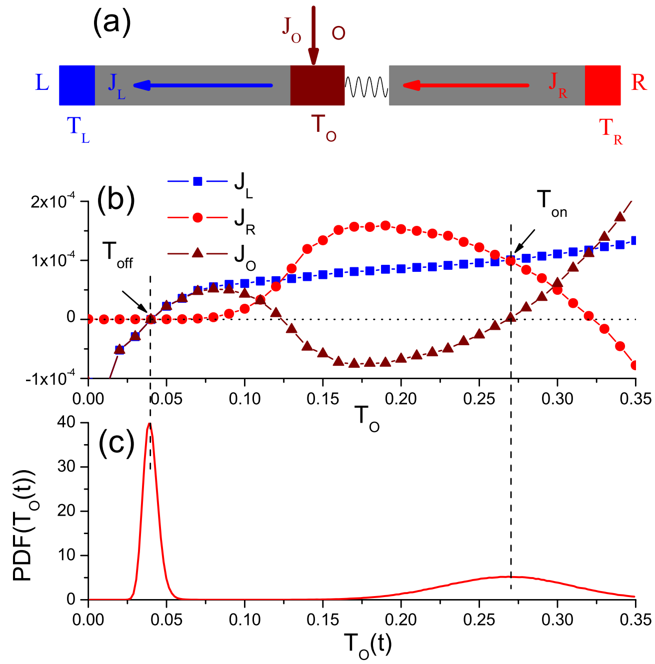

FIG. 1 (color online). (a) Configuration of a thermal memory. Black wavy curve indicates the weak coupling between particle O and segment R, which results in the negative differential thermal resistance (NDTR) here. (b) Heat currents, J_L, J_R and J_O versus temperature T_O. The NDTR between particle O and segment R makes J_R increases in a wide region when T_O is increased, thus makes the curve cross with J_L at more than one values of temperature T_O. (c) PDF of the finite time temperature $T_O(t)$ of the central particle O in the absence of control heat bath. Notice the clear correspondence of the two stable steady states in (b) and the two peaks in (c), which are indicated by the two vertical dot lines. Since $T_O(t)$ can only change continuously, the very low PDF in between clearly points out the low transition probability, thus implies the stability of the two states on and off.

Because of NDTR, with fixed T_L and T_R, and a control heat bath with adjustable temperature T_O is coupled to the particle O, there exist more than one values of T_O with which the heat current from control heat bath to particle O, J_O, is zero [2], as shown in Fig. 1(b). Suppose the temperature of O is initially set to either "T_{on}" or "T_{off}" by the control heat bath. After the control heat bath is removed the state will remain unchanged for a long time, in spite of the thermal fluctuation. As the consequence of the small perturbation which slightly changes the temperature T_O, the changes of heat currents δJ_L and δJ_R always pull T_O back; thus, these two states are "stable." It is easy to check the other steady state in between is, however, unstable.

In the equilibrium state of a Hamiltonian system with ergodicity, temperature can be defined as twice of the average kinetic energy per degree of freedom. In order to study transient process, the "finite time temperature" is naturally defined as, for a 1D system, twice of the average kinetic energy in a time window δt: i.e., $T(t) \equiv \frac{1}{\delta t} \times \int_{t-(\delta t/2)}^{t+(\delta t/2)} v^2(t')dt'$. Hereafter, any temperature which is a function of time t means the finite time temperature. Because of the thermal fluctuation, even in a steady state, finite time temperature has different value at different time window. The probability density function (PDF) of the finite time temperature is thus defined accordingly. If the time window δt is chosen to be too long, the PDF approaches a δ function located at the "infinite time" temperature T, while if δt is too short, big thermal fluctuation dominates; thus, the distribution is more or less a Maxwell energy distribution. In the intermediate cases PDF presents much more information than the value of temperature T itself. In this Letter, δt is fixed to 10^4 dimensionless time units. Since the frequency of the system we studied is about 0.2 [2], this period of time covers about 2000 oscillations, thus is a microscopical long time.

Since "on" and "off" are two stable states, the finite time temperature of the particle O in the absence of control heat bath is expected to stay around T_{on} and T_{off} with much higher probability than elsewhere. In Fig. 1(c) the locations of the two peaks clearly confirm this expectation. By adjusting parameters of the system, the values of T_{on} and T_{off} and the peaks shift simultaneously but always coincide with each other.

We have so far confirmed the stability of the two states on and off of the model. In the following, we shall demonstrate a complete, four-stage, writing-reading process of the thermal memory, as is shown in Fig. 2(a). During the whole process the left and right ends of the memory are always connected to two power supply heat baths with temperatures $T_L = 0.04$ and $T_R = 0.3$, respectively. Stage 1: initializing the memory. Each particle is initially set a velocity chosen from a Gaussian distribution corresponding to a temperature in (T_L, T_R). After some time the system reaches an asymptotic state; thus, $T_O(t)$ saturates, see Fig. 2(b). Stage 2: writing the datum into the memory. In our simulation, the "writer" is simulated by a FK lattice with $N = 10$ particles and identical parameters with segment L. It is connected via a linear spring with $k = 1.0$, at its one end, to the particle O of the memory. The temperature of the writer is initially set to T_{off} and the other end is connected to a heat bath with temperature T_{off} also. In a short time the writer cools down $T_O(t)$ to T_{off}; namely, the datum off has been written into the memory. Stage 3: maintaining the datum in the memory. In this stage, the writer is removed from the memory. And we can see from Fig. 2(b) that $T_O(t)$ remains almost unchanged during this stage, which means that the datum stored in the memory can last for a long time. Stage 4: reading out the datum from memory. A "reader"(thermometer) is used to read the datum out from the memory. Here the reader is simulated by the same FK lattice as the writer. It is connected to the particle O of the memory. The temperature of the reader is initially set to a middle temperature between T_{on} and T_{off} (in this Letter it is 0.11). Different from the writer, the reader is not connected to any heat bath. This

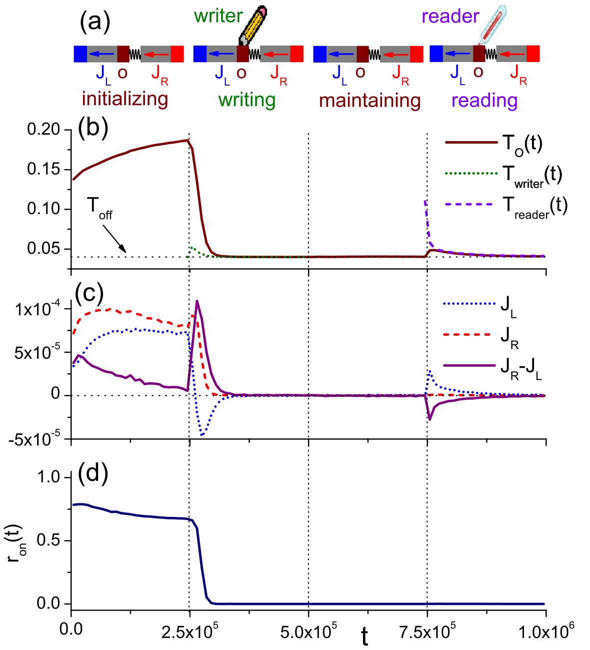

FIG. 2 (color online). A writing-reading process for an off state. Vertical dot lines separate different stages to guide eyes. (a) Illustration of the four stages. (b) Temperatures of the central particle $T_O(t)$, writer $T_{\mathrm{writer}}(t)$ and reader $T_{\mathrm{reader}}(t)$. (c) Heat currents through the two segments, J_L and J_R. It can be seen that at the beginning of the reading stage, the net current $J_R - J_L$ changes to negative, namely, energy is absorbed from O; thus, $T_O(t)$ is cooled down back to the low temperature T_{off}. (d) After the writing stage $r_{\mathrm{on}}(t)$ keeps very close to zero, namely, nearly all samples stay at the off state, even under the not-very-small perturbation from the hot reader.

stage is the most important and most interesting one. We see that at the beginning the particle O is temporarily warmed up because the reader is hotter. However, since the off state is stable, $T_O(t)$ and also the temperature of the reader are cooled down to T_{off} shortly, implying that the datum has been successfully read out. To gain a clearer picture, we show the heat currents through the two segments J_L and J_R in the Fig. 2(c). At the beginning of the reading stage, since the particle O is warmed up by the reader, as a response J_L increases a lot whereas J_R increases only very little; thus, the net current $J_R - J_L$ changes from zero to negative; i.e., the power supply (left end heat bath) absorbs heat from the particle O. This is the reason that T_O can be cooled down and recovered to T_{off} automatically.

The results shown in Fig. 2 are obtained from an ensemble average over 20 000 independent realizations. For a model of thermal memory, the ratio of samples that keep the correct data is an important feature. To describe this quantitatively, we calculate the ratio of on state: $r_{\mathrm{on}}(t) \equiv N_{\mathrm{on}}(t)/N$, while $N_{\mathrm{on}}(t)$ is the total number of samples stay at on state at time t and N is the total number of samples.

We define, at time t, a sample is at on state if the finite time temperature of its particle O is greater than a critical value $T_c = 0.11$. we show $r_{\mathrm{on}}(t)$ during the whole process in Fig. 2(d). It is clearly seen that after the writing stage, $r_{\mathrm{on}}(t)$ always keeps at a very low level, even under the not-very-small perturbation from the hot reader. The discrepancy between $r_{\mathrm{on}}(t)$ and 0 is in the order of 10^{-4}, which means two or three "errors" among the 20 000 samples, is even indistinguishable in the figure by eye, namely, the memories successfully maintain the data off that are initially set by the writer, the data are hardly destroyed even after being read!

It has been pointed out that, to be a memory, the system must have more than one stable steady states around which the above writing-reading process can be completed; thus, in the following we demonstrate the writing-reading process around the other stable steady state, the on state in Fig. 3. The difference from Fig. 2 is that in the second stage the writer is set to temperature T_{on} and connected to a heat bath with temperature T_{on}. We see in this stage $T_O(t)$ changes to T_{on} shortly. In the consequent maintaining stage $T_O(t)$ keeps unchanged at T_{on}. In the reading stage: the reader is initially set to the same temperature as before. We see at the beginning because this time the reader is colder, the particle O is temporarily cooled down; as a response,

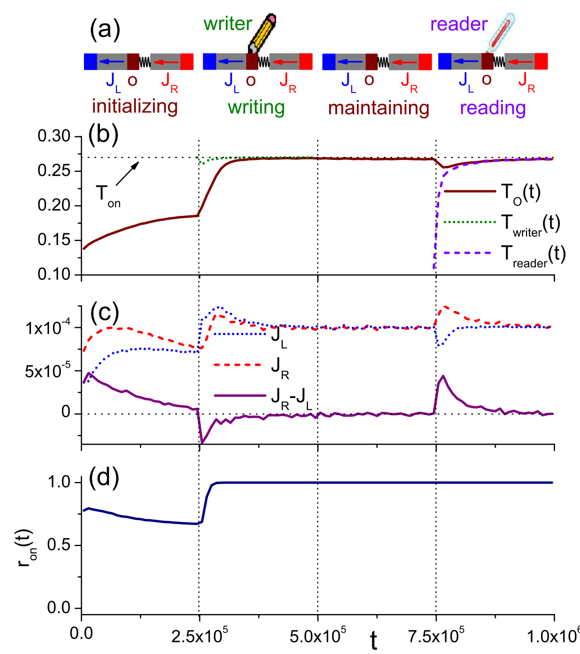

FIG. 3 (color online). A writing-reading process for an on state. (a) Illustration of the four stages. (b) Temperatures of the central particle $T_O(t)$, writer $T_{\mathrm{writer}}(t)$ and reader $T_{\mathrm{reader}}(t)$. (c) Heat currents through the two segments, J_L and J_R. At the beginning of the reading stage, the net current $J_R - J_L$ is positive, namely, provides energy to O thus warms it up back to the high level T_{on}. (d) After the writing stage $r_{\mathrm{on}}(t)$ remains very close to 1, namely, nearly all samples stay at the on state, even under the not-very-small perturbation from the cold reader.

J_L decreases while J_R increases; thus, the net current $J_R - J_L$ becomes positive, which warms up the particle O (also the reader) and recovers its temperature to $T_{\rm on}$. The datum on is again read out successfully. As for $r_{\rm on}(t)$; see Fig. 3(d). This time $r_{\rm on}(t)$ keeps at a very high level. The discrepancy between $r_{\rm on}(t)$ and 1 is again at the order of 10^{-4}; namely, the datum on can be successfully maintained in and read out from memory.

Finally, we would like to discuss the lifetime of data in the thermal memory. In fact an ideal thermal memory is never possible because sooner or later thermal fluctuation will eliminate any record of the history. This is, however, not a serious problem from application point of view since we only need to maintain the data until we refresh or read them. The widely used electronic dynamic random access memory (DRAM) works exactly in this way. In a DRAM, because of the charge leakage of capacitor, the data will eventually fade away unless the capacitor is refreshed periodically ($\sim$100/s). Similarly, we find a few errors among the many samples that we studied at the end of the writing-reading process. Suppose the data fading process is a Poisson process then the average lifetime of the data is roughly 5×10^9 dimensionless time units which contains about 10^9 periods of oscillation. This corresponds to 100 μs if the memory is made of carbon nanostructures such as nanotube [11], in which the key of the model, NDTR has been found numerically [12]. This lifetime is quite short compared with an electronic DRAM. However, this value can be easily and greatly enlarged by parallel combining more identical memories together. The dynamics of finite time temperature of particle O, $T_O(t)$ around either of the two stable steady states can be roughly described by an autoregressive diffusion process, say, the simplest one: $\dot{X} = -\lambda X + \sigma\xi(t)$. Where X corresponds to $T_O(t)$ relative to $T_{\rm on}$ or $T_{\rm off}$. $\lambda > 0$ represents the effect of the response net heat current that pulls X back to zero and $\sigma\xi(t)$ is a Gaussian white noise with zero mean and fixed variance σ^2 which describes fluctuation of the heat current. The mean first passage time (MFPT) through a certain boundary $X = \Delta X$ (which denotes $T_c - T_{\rm off}$ or $T_{\rm on} - T_c$) conditional upon $X(0) = 0$ represents the average lifetime of the data in the memory. In the limiting case that $\frac{\sqrt{\lambda}\Delta X}{\sigma} \gg 1$, MFPT $\propto \frac{1}{\lambda}{\rm erfi}(\frac{\sqrt{\lambda}\Delta X}{\sigma})$, where ${\rm erfi}(x) = \int_0^x e^{x'^2} dx'$ [13]. MFPT diverges rapidly as σ^2 decreases. Parallel combining n identical memories together decreases σ^2 by n times while keeps λ unchanged, thus enlarges the average lifetime of data greatly (since the data fading rate is inversely proportional to the average lifetime, it decreases greatly). Such a fast divergence tendency of MFPT has also been widely found in a class of diffusion processes with steady state distribution [14]; thus, the above conclusion is not limited to the specified process.

In summary, we have demonstrated the feasibility of thermal memory from a bistable thermal circuit. The information stored in such a thermal memory can last very long time and, more importantly it is self-recoverable under the not-very-small perturbation from the thermometer when the data are read. With the rapid developing nanotechnology [4–6], the thermal memory should be realized in nanoscale systems experimentally in a foreseeable future.

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