Order and Creep in Flux Lattices and Charge Density Wave Pinned by Planar Defects

Aleksandra Petković and Thomas Nattermann
Institut für Theoretische Physik, Universität zu Köln, Zülpicher Strasse 77, 50937 Köln, Germany
(Received 15 May 2008; published 31 December 2008)

The influence of randomly distributed point impurities *and* planar defects on the order and transport in type-II superconductors and related systems is considered theoretical. For random planar defects of identical orientation, the flux line lattice exhibits a new glassy phase with diverging shear and tilt modulus, a transverse Meissner effect, large sample to sample fluctuations of the susceptibility, and an exponential decay of translational long range order. The flux creep resistivity for currents J parallel to the defects is $\rho(J) \sim \exp-(J_0/J)^{\mu}$ with $\mu = 3/2$. Strong disorder enforces an array of dislocations to relax shear strain.

DOI: 10.1103/PhysRevLett.101.267005 PACS numbers: 74.25.Qt, 64.70.Rh, 71.55.Jv, 74.62.Dh

Introduction.—Type-II superconductors can be penetrated by an external magnetic field in the form of quantized magnetic flux lines (FLs). Under the influence of a transport current J, FLs will move and hence give rise to dissipation. The resulting linear resistivity is proportional to the magnetic induction **B** [1]. To stabilize superconductivity, it is therefore essential to pin FLs. One source of pinning is point disorder. In high-T_c materials, point disorder is practically always existing because of the nonstoichiometric composition of most materials. Then the system regains superconductivity in the sense that the linear resistivity vanishes [2]. However, thermal fluctuations lead to flux creep resulting in a nonzero *nonlinear* resistivity of the form $\rho(J) \sim e^{-(J_P/J)^{\mu}}$ where $\mu = 1/2$ [3]. $J_P(\gg J)$ is a function of B, temperature T, and the concentration and strength of the pinning centers. This response of the system on an external current is closely related to the order of the FL lattice (FLL) in the presence of point pinning centers, which shows a power law decay of its translational order parameter in the ''Bragg glass'' phase [3–5]. Although the linear conductivity is now zero, there is still a finite resistivity for a finite current. It is therefore indicated to look for a more effective pinning mechanism corresponding to larger values of the creep exponent μ. One option is columnar defects which lead to a ''Bose glass'' phase with stronger pinning properties (see, e.g., [6]).

An even more pronounced effect can be expected from planar defects like twin planes or grain boundaries, which will be considered in the present Letter. Twins are ubiquitous in superconducting yttrium barium copper oxide and La_2CuO_4 where they are needed to accommodate strains arising from tetragonal to rhombic transformations. But other causes are also possible (see Fig. 1). Planar defects occur frequently in families with the same orientation but random distances [7,8] or in orthogonal families of lamella (''colonies'') [9]. The mean distance ℓ_D of the defect planes is of the order of 10 nm [7] to μm [10]. Pinning of individual FLs by planar defects has been investigated in the past both for clean and disordered systems [2,11,12]. Recently, it was shown that depending on the mutual orientation of the FLL and the defects, dilute planar defects are indeed a relevant perturbation even in the presence of point disorder [13], provided they are parallel to the main lattice planes of the FLL. In systems with parallel defect planes, this is the generic situation since the FLL will rotate in such a position to reach maximum overlap with the defects (provided **B** is aligned with the defect planes). It turns out that the Bragg and the Bose glass phase are unstable with respect to the presence of many random planar defects and will be substituted by a new type of a *planar glass phase* (see Fig. 2). In the present Letter, we will discuss the nature of this phase. The latter is characterized by a complex energy landscape with many metastable states and diverging energy barriers leading to a new creep law with $\mu = 3/2$, large sample to sample fluctuations of the magnetic susceptibility, an exponential suppression of translational order in the direction perpendicular to the defects, a resistance against shear deformations, as well as the occurrence of a transverse Meissner effect. If only displacements perpendicular to the defects are considered, as in the main part of this Letter, our results apply also to a wide class of systems which exhibit regular lattices of domain walls like magnetic slabs, charge density waves [14], and incommensurate systems [15].

Model.—We consider an Abrikosov FLL in the presence of randomly distributed point impurities and random defect

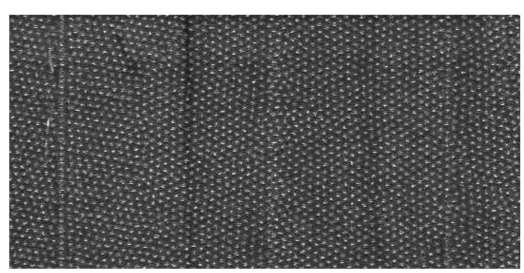

FIG. 1. Planar crystalographic defects in bismuth strontium calcium copper oxide (bright vertical lines).[31].

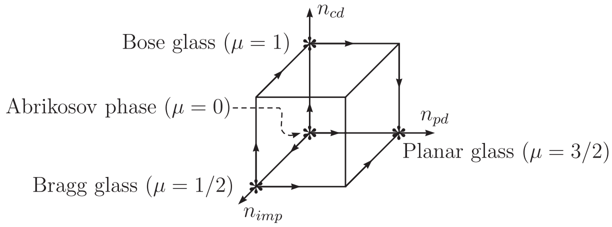

FIG. 2. Disordered vortex lattices resulting from impurities, columnar and planar defects of concentration $n_{\rm imp}$, n_{cd} and n_{pd}, respectively. In the presence of planar defects, the planar glass phase is ultimately stable. μ denotes the creep exponent.

planes, aligned with the magnetic field. Since in both types of imperfections superconductivity is suppressed, they will attract FLs. Then the Hamiltonian reads [16]

$$\mathcal{H} = \int d^3 r \frac{1}{2} \Big\{ \sum_{\alpha\beta\gamma\delta} c_{\alpha\beta\gamma\delta}(\partial_\alpha u_\beta)(\partial_\gamma u_\delta) + \sum_\alpha c_{44}^{(\alpha)} (\partial_z u_\alpha)^2 + 2[V_P(\mathbf{r}) + V_D(\mathbf{r})]\rho(\mathbf{u}, \mathbf{r}) \Big\} \quad (1)$$

where α, β, γ, δ run over x, y. $\mathbf{u}(\mathbf{r}) = (u_x, u_y)$ denotes the FL displacement. Only components of the elastic constants, $c_{\alpha\beta\gamma\delta}$, with pairwise equal indices are nonzero [17]. $\rho(\mathbf{u}, \mathbf{r}) = \rho_0\{-\nabla_\perp \mathbf{u} + \sum_{\mathbf{G}} e^{i\mathbf{G}(\mathbf{r}_\perp - \mathbf{u})}\} \equiv \rho_s + \rho_p$ is the FL density with $\rho_0 = B_0/\phi_0$; ϕ_0 is the flux quantum. $\mathbf{G}$ is a reciprocal lattice vector of the FLL and $\mathbf{r}_\perp = (x, y)$. $V_P(\mathbf{r})$ denotes the pinning potential resulting from randomly distributed point impurities. We will first consider the (realistic) case that all defect planes have the same orientation but random distances [7]. Then the FLL will orient itself such that its main lattice planes will be parallel to the planar defects to allow for their maximal overlap [13]. The defect pinning potentials then have the form $V_D(\mathbf{r}) = -v_D \sum_{\rm d} \delta(x - x_{\rm d})$ [2] where we assumed that the defect planes are parallel to the yz plane. The δ functions are considered to have a finite width of the order of the superconductor coherence length, ξ_c. A rough estimate for the defect strength is given by $v_D \approx H_c^2 \xi_c^3$; H_c is the thermodynamic critical field. The statistical properties of the pinning energies are then encoded in their pair correlation functions, $R_P(\mathbf{u})$ and $R_D(u_x)$, for point disorder and planar defects, respectively. Since the FLL density includes a slowly varying and a periodic part, ρ_s and ρ_p, respectively, we decompose the pinning energy density accordingly. From the periodic part we get $R_D(u_x) = (v_D\rho_0)^2/l_D \sum_{n\neq 0} e^{{\rm i}n2\pi u_x/\ell}$; n is an integer [18]. $\ell \ll \ell_D$, where ℓ and ℓ_D are the mean spacing between the FLs and the defect planes, respectively. The contributions from ρ_s do not contribute to the glassy properties of the system since they can be eliminated by a simple transformation [19].

Since our main concern is the defect planes, it seems to be justified to start with a simplified model in which only the displacements $u_x \equiv u$ of the FLs *perpendicular* to the defect planes are considered. Then only the elastic terms with the coefficients $c_{xxxx} \equiv c_{11}$, $c_{yxyx} \equiv c_{66}$, and $c_{44}^{(x)} \equiv c_{44}$ remain in the Hamiltonian. From a technical point of view, it is convenient to consider a generalization of our model in d dimensions by replacing x by a $(d-2)$-dimensional vector $\mathbf{x}$.

Weak disorder.—In the absence of defect planes, point impurities are relevant in less than 4 dimensions. The FLL exhibits a phase with quasilong range order: the Bragg glass [3–5], which exhibits a power law decay of $S_{\mathbf{G}}(\mathbf{r}) = \overline{\langle e^{i\mathbf{G}[\mathbf{u}(\mathbf{r})-\mathbf{u}(\mathbf{0})]}\rangle} \sim |\mathbf{r}|^{-(4-d)}$. The Fourier transform of $S_{\mathbf{G}}(\mathbf{r})$ is the structure factor which has Bragg peaks.

It was recently shown in [13] that dilute planar defects can be a relevant perturbation also in the presence of point disorder. Indeed, distorting the initially ordered FLL in volume $L^{d-2}L_yL_z$, the energy gain is of the order $-[R_D''''(0)L^{d-2}]^{1/2}L_zL_y$, whereas, the elastic energy loss is $c_{11}L_zL_yL^{d-4}$ since distortions are aligned parallel to the defects. For $L \gg L_D \sim [c_{11}^2/R_D''''(0)]^{1/(6-d)}$ the pinning energy gain wins and the FLL starts to disorder in the directions perpendicular to the defects. The critical dimension below, which weak planar defects, are relevant is $d = 6$.

For a more detailed study, we now use a functional renormalization group approach in $d = 6 - \epsilon$ dimensions. We follow closely a related approach for columnar disorder [20,21], but keep the unrescaled quantities which correspond to the effective parameters measured on scale L. To lowest order the flow equations for $\epsilon \ll 1$ read

$$d\ln c_{ii}/d\ln L = 2R_D''''(0)L^\epsilon/(4\pi c_{11})^2, \qquad i = 4, 6,$$
$$dR_D(u)/d\ln L = R_D''(u)L^\epsilon[R_D''(u) - 2R_D''(0)]/(4\pi c_{11})^2. \quad (2)$$

Thermal fluctuations and point disorder are irrelevant for $\epsilon < 4$ and $\epsilon < 2$, respectively. There is no renormalization of c_{11} because of a statistical tilt symmetry [22]. For $L \to L_D$, many metastable states appear and $R_D''(0)$ develops a slope discontinuity at the origin which results in diverging elastic constants, c_{44} and c_{66}. The renormalization can however be continued to $L \gg L_D$ if one imposes a small but *finite tilt* of the FLL such that $R''''(0)$ has to be replaced by $R''''(0^+)$ in Eq. (2). In this case, c_{44} and c_{66} remain finite but new terms of the form $\int_0^{2\pi} d\phi |\Sigma_y \cos\phi(\partial_y u) + \Sigma_z \sin\phi(\partial_z u)|/4\ell$ are generated in the energy density which dominate the energy for small u. The fixed point function $R_D^{*''}(u, L)L^\epsilon = (2\pi c_{11})^2\epsilon[\frac{\ell^2}{36} - \frac{1}{3}(u - \frac{\ell}{2})^2]$ for $0 \leq u < \ell$ is periodic in u with period ℓ. The newly generated terms renormalize according to

$$c_{66}^{-1/2} d\Sigma_y/d\ln L \approx c_{44}^{-1/2} d\Sigma_z/d\ln L \approx \epsilon\sqrt{c_{11}}\ell^2/12L. \quad (3)$$

$\Sigma_{z(y)}$ has the meaning of a interface tension of a domain wall parallel to the $\mathbf{x}$ and y (z) axes. Σ_z can be measured by changing the external magnetic field by $H_x\hat{\mathbf{x}}$ which changes the Hamiltonian by $-(B_0/4\pi)\int d^3 r H_x \partial_z u$. To tilt the flux lines with respect to the z axis, H_x has to overcome the interface energy $\sim\Sigma_z$ which results in a threshold field $H_{x,c} = 8\pi\Sigma_z\ell/(\phi_0\sqrt{3})$ below which FLs remain locked parallel to the planes. This is the *transverse Meissner effect*: a weak transverse magnetic field H_x is screened from the sample. In this case c_{44} is infinite. Only for $H_x > H_{x,c}$, the average tilt of the FLs becomes nonzero and c_{44} stays finite. Moreover, there is a *resistance against shear* of the FLL: the shear deformation $\partial_y u_x$ is nonzero (and c_{66} finite) only if the shear stress σ_{xy} is larger then a critical value Σ_y/ℓ, otherwise c_{66} is infinite. The divergence of c_{66} is a new property which does not exist in Bose glass.

An infinitesimal change $\delta H_z\hat{\mathbf{z}}$ in the longitudinal field allows to measure the longitudinal susceptibility $\chi = B_0\partial\langle\partial_x u\rangle/\partial H_z$. The disorder averaged susceptibility $\bar{\chi} = B_0^2/4\pi c_{11}$ is independent of the disorder as a result of the statistical tilt symmetry. The glassy properties of the systems can most easily be seen by the sample to sample fluctuations of the magnetic susceptibility, $\overline{\chi^2} - \bar{\chi}^2$. Perturbation theory gives $(\overline{\chi^2} - \bar{\chi}^2)/\bar{\chi}^2 = R_D''''(0)L^\epsilon/(5c_{11}^2) \sim (L/L_D)^\epsilon$, i.e., the sample to sample fluctuations of the susceptibility grow with the scale $L \lesssim L_D$, $d < 6$ which is a signature of a glassy phase [19].

The structural correlations in this phase are obtained in the standard way from $R_D''(u, L)$ [3] which gives $S_{\mathbf{G}}(\mathbf{x}, y, z) \sim |\mathbf{x}|^{-(6-d)}$. In $d \leq 4$ dimensions also the part of the pinning potential related to ρ_s becomes relevant which gives the dominating contribution to the FL displacements. Both a Flory argument [3] and more detailed calculations for a related one-dimensional problem [23–25] give in $d = 3$ dimensions, $S_{\mathbf{G}}(x, y, z) \sim e^{-|x|/L_D}$. In the related study [24], Villain and Fernandez found from a nonperturbative renormalization-group method that for $d \leq 4$ the disorder renormalizes to strong coupling. We will show below that this case gives qualitatively the same results.

To get more information about a real three-dimensional system, we consider next the stability of this glassy phase with respect to point disorder by using an Imry-Ma argument [26]. The energy gain from the point disorder in a region $L^{d-2}L_yL_z$ is of the order $-(\langle R_P(u)\rangle L_yL_zL^{d-2})^{1/2}$ [27] which has to be compared with the elastic energy loss $L^{d-2}(c_{11}\ell^2L_yL_z/L^2 + \Sigma_zL_y + \Sigma_yL_z) \sim L^d$. If one ignores the fluctuations of u and replaces $\langle R_P(u)\rangle$ by a constant, one finds that point disorder is irrelevant above $d = 2$ dimensions. This critical dimension is further decreased to zero if the fluctuations of u are taken into account by using $R_P(u) \sim L^{(d-6)/2}$. A similar argument shows the irrelevance of columnar disorder. This argument applies for $L \gg L_D$ where $\Sigma_{y/z}$ has developed.

Flux creep.—Next we consider the flux creep under the influence of a transport current parallel to the defect planes which creates a driving force density, $\mathbf{f} = \mathbf{J} \wedge \mathbf{B}/c$, perpendicular to them. $\mathbf{J}$ is the current density. The motion of the FL bundles under the influence of $\mathbf{f}$ occurs then by nucleation of critical droplets in which FLs are moved by a distance ℓ. This droplet is a saddle point, as usual in nucleation phenomena. In the presence of planar defects, the energy of the nucleus has the from

$$E_{\text{nucl}} \approx L^{d-2}L_yL_z\left(\frac{c_{11}\ell^2}{L^2} + \frac{\Sigma_z}{L_z} + \frac{\Sigma_y}{L_y} - f\ell\right). \quad (4)$$

Here we have taken into account that the elastic energy and the energy from the disorder scale in the same way. The saddle point $L_y/\Sigma_y = L_z/\Sigma_x \sim L^2/c_{11} \sim f^{-1}$ gives for the nonlinear resistivity in $d = 3$ ($J \ll J_D$)

$$\rho(J) \sim e^{-(J_D/J)^{3/2}}, \qquad J_D = \mathcal{C}\frac{(\Sigma_y\Sigma_z)^{2/3}(c_{11}/\ell)^{1/3}c}{BT^{2/3}}. \quad (5)$$

Thus the nonlinear resistivity is reduced considerably with respect to the case of point impurities. A similar consideration for the Bose glass gives $\mu = 1$ which is, as far as we are aware, also a new result [2].

To summarize the results obtained so far, we remark that the new phase described here is characterized by (i) diverging elastic constants c_{44} and c_{66} but a finite compressibility c_{11}, (ii) a transverse Meissner effect as well as a resistance against shear deformation, (iii) large sample to sample fluctuations of the susceptibility, (iv) an exponential decay of the structural correlations (in $d = 3$), and (v) a creep exponent $\mu = 3/2$. Since the totality of these properties is different from the Bragg glass or the Bose glass, we will call this new phase a *planar glass*. This phase is also different from that found for equally spaced defects which is incompressible [28].

Strong disorder.—If the disorder is strong, i.e., if $L_D \lesssim \ell_D$, (we ignore for the moment the point disorder) each defect will be completely overlapped by a FLL plane to gain its full energy. Integrating out the displacement field between two adjacent defect planes we get in $d = 3$:

$$\frac{\mathcal{H}}{L_zL_y} = \sum_{i=1}^{N}\left\{\frac{c_{11}}{2}\frac{(u_{i+1} - u_i)^2}{\Delta x_{i+1}} - \rho_0 v_D \sum_n e^{iG_D n(x_i - u_i)}\right\},$$

where $G_D = 2\pi/\ell$, $\Delta x_{i+1} = x_{i+1} - x_i$, and the sum over i is over the defect planes. For $v_D \to \infty$, we have $x_i - u_i = \ell n_i$ with n_i integer to minimizes the pinning potential. Minimizing, subsequently, the elastic energy allows the exact determination of the ground state [25]: $u_i^0 = x_i - \ell\sum_{j=1}^{i}[\Delta x_j/\ell]_G$, where $[x]_G$ denotes the closest integer to x. For $\ell_D \gg \ell$, $S_{\mathbf{G}}(\mathbf{r})$ is again decaying exponentially in the x direction on scale ℓ_D. Considering flux creep due to a driving force, $\mathbf{f}$, perpendicular to the defect planes in $d = 3$, we obtain then the same form of the nonlinear resistivity [Eq. (5)] as in the case of weak disorder. This formula applies for small currents where droplets cover many planar defects. Thus both weak and strong disorder give the same results for the correlations and the flux creep.

Displacement parallel to the defects, dislocations.— Next we include displacements, u_y, parallel to the defects. In the case of strong disorder, each defect is occupied by a single FL layer and hence $u_x(x_i, y, z; n_i) = x_i - \ell n_i$, $\forall$ y, z, to maximize the pinning energy gain. Even without point disorder, we obtain then a nonzero displacement u_y. This can be seen most easily in the isotropic case where $\sigma \partial_x u_x = -\partial_y u_y$; here, $\sigma = (c_{11} - c_{66})/(c_{11} + c_{66})$ is the Poisson number, $0 < \sigma < 1$ [29]. The strain $\partial_x u_x$ in the segment between the defects at x_{i+1} and x_i is $\partial_x u_x \approx 1 - \ell \Delta n_{i+1}/\Delta x_{i+1}$ where $\Delta n_{i+1} = (n_{i+1} - n_i)$. The difference of the strain $\partial_y u_y$ in neighboring segments is then $\Delta \partial_y u_y \approx \sigma \ell [\Delta n_{i+1}/\Delta x_{i+1} - \Delta n_i/\Delta x_i]$ which is of the order $\pm \sigma \ell/\ell_D$. On the scale L_y, this implies $\Delta u_y \sim \pm \sigma \ell L_y/\ell_D$. To avoid a diverging shear energy, one has to allow for dislocations with Burgers vector *parallel* to the y direction sitting at the defects. Their distance in the y direction is of the order ℓ_D/σ. Comparing the energy of an edge dislocation piercing the crystal to the energy gain from the disorder, we find that dislocations will be present if $\sigma c_{66} \ell^3 \xi_c \ll \ell_D v_D$. In general, the network of additional FLL sheets spanned by the dislocations will be complicated. The resulting state is ordered in the sense that Σ_y, Σ_z are nonzero, and hence the transverse Meissner effect is still present.

Adding weak point impurities will further randomly shift the positions of the dislocation leading most likely to decay of translational correlations in the yz plane. Since the Burgers vector of the dislocations is always parallel to the defects, creep in the x direction is not facilitated. Σ_y and Σ_z are still both nonzero and hence we recover the creep law, Eq. (5). To describe creep parallel to the defects, one has to take into account the interaction between the dislocation, a situation not considered so far [30]. We leave this case for further studies. In the case of weak pinning, qualitatively the same behavior can be expected on scales $L_x \gg L_D$, in particular, if the flow is again to the strong coupling fixed point. If the samples exhibit orthogonal families of (nonintersecting) defects, long range order in the xy plane is destroyed even without point disorder on scales larger than L_D. The creep is now limited by the slowest mechanism and hence Eq. (5) is likely to be valid for all current directions in the xy plane.

The authors acknowledge support from the SFB 608 and helpful comments from F. de la Cruz, J. Kierfeld, V.M. Vinokur, M. Zaiser, and E. Zeldov.

[1] J. Bardeen and M.J. Stephen, Phys. Rev. **140**, A1197 (1965).
[2] G. Blatter, M.V. Feigel'man, V.B. Geshkenbein, A.I. Larkin, and V.M. Vinokur, Rev. Mod. Phys. **66**, 1125 (1994).
[3] T. Nattermann, Phys. Rev. Lett. **64**, 2454 (1990); T. Emig, S. Bogner, and T. Nattermann, Phys. Rev. Lett. **83**, 400 (1999); T. Nattermann and S. Scheidl, Adv. Phys. **49**, 607 (2000).
[4] T. Giamarchi and P. Le Doussal, Phys. Rev. Lett. **72**, 1530 (1994).
[5] T. Klein *et al.*, Nature (London) **413**, 404 (2001); N.D. Daniilidis *et al.*, Phys. Rev. Lett. **99**, 147007 (2007).
[6] D.R. Nelson and V.M. Vinokur, Phys. Rev. Lett. **68**, 2398 (1992); Phys. Rev. B **48**, 13 060 (1993).
[7] G.W. Crabtree *et al.*, Physica (Amsterdam) **185C–189C**, 282 (1991); W.K. Kwok *et al.*, Phys. Rev. Lett. **69**, 3370 (1992).
[8] Y. Fasano, J. Herbsommer, and F. de la Cruz, Phys. Status Solidi B **215**, 563 (1999); Y. Fasano,, M. Menghini, F. de la Cruz, and G. Nieva, Phys. Rev. B **62**, 15 183 (2000).
[9] T. Roy and T.E. Mitchell, Philos. Mag. A **63**, 225 (1991); S. Sanfilippo *et al.*, Europhys. Lett. **39**, 657 (1997).
[10] G.J. Dolan *et al.*, Phys. Rev. Lett. **62**, 827 (1989).
[11] L. Balents and M. Kardar, Europhys. Lett. **23**, 503 (1993); Phys. Rev. B **49**, 13 030 (1994).
[12] M.C. Marchetti and V.M. Vinokur, Phys. Rev. B **51**, 16 276 (1995).
[13] T. Emig and T. Nattermann, Phys. Rev. Lett. **97**, 177002 (2006).
[14] G. Grüner, Rev. Mod. Phys. **60**, 1129 (1988).
[15] A.D. Bruce and R.A. Cowley, J. Phys. C **11**, 3609 (1978); P. Bak, Rep. Prog. Phys. **45**, 587 (1982).
[16] V.G. Kogan and L.J. Campbell, Phys. Rev. Lett. **62**, 1552 (1989).
[17] The dispersion of the tilt and the compression modulus on small length scales [2] is negligible for the present problem since we are interested in the asymptotic properties (large scales, small currents).
[18] The sum is cut at $n \sim \ell/\xi_c$, $\ell = [\sqrt{3}/(2\rho_0)]^{1/2}$.
[19] T. Hwa and D.S. Fisher, Phys. Rev. Lett. **72**, 2466 (1994). Although we were not able to prove it, $\bar{\chi}^2/\bar{\chi}^2 - 1$ will most likely approach a universal value for $L \gg L_D$.
[20] L. Balents, Europhys. Lett. **24**, 489 (1993).
[21] After having done our calculations, we became aware of the following work: A.A. Fedorenko, Phys. Rev. B **77**, 094203 (2008).
[22] U. Schulz *et al.*, J. Stat. Phys. **51**, 1 (1988).
[23] M.V. Feigel'man, Sov. Phys. JETP **52**, 555 (1980).
[24] J. Villain and J.F. Fernandez, Z. Phys. B **54**, 139 (1984). Note that there is a shift of dimension $d \rightarrow d + 2$ from their model to our model.
[25] A. Glatz and T. Nattermann, Phys. Rev. B **69**, 115118 (2004).
[26] J. Imry and S.K. Ma, Phys. Rev. Lett. **35**, 1399 (1975).
[27] $R_P(\mathbf{u}) = n_{\text{imp}}(v_P\rho_0)^2 \sum_{\mathbf{G}\neq\mathbf{0}} e^{i\mathbf{G}\mathbf{u}}$, v_p is the strength of an impurity and $|\mathbf{G}| \leq 1/\xi_c$.
[28] L. Balents and D.R. Nelson, Phys. Rev. Lett. **73**, 2618 (1994).
[29] L.D. Landau and E.M. Lifshitz, *Elasticity Theory* (Butterworth-Heinemann, Oxford, 2004).
[30] J. Kierfeld, H. Nordborg, and V.M. Vinokur, Phys. Rev. Lett. **85**, 4948 (2000); P. Moretti, M.C. Miguel, M. Zaiser, and S. Zapperi, Phys. Rev. B **69**, 214103 (2004).
[31] M. Menghini, Y. Fasano, F. de la Cruz , and E. Zeldov (unpublished).