

## Parametric Normal-Mode Splitting in Cavity Optomechanics

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Recent experimental progress in cavity optomechanics has allowed cooling of mesoscopic mechanical oscillators via dynamic backaction provided by the parametric coupling to either an optical or an electrical resonator. Here we analyze the occurrence of normal-mode splitting in backaction cooling at high input power. We find that a hybridization of the oscillator's motion with the fluctuations of the driving field occurs and leads to a splitting of the mechanical and optical fluctuation spectra. Moreover, we find that cooling experiences a classical limitation through the cavity lifetime.

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**Introduction.**—Recently, cavity optomechanical systems that parametrically couple a driven high-frequency mode to a high- $Q$ , low-frequency mechanical mode have been subject to increasing investigation [1]. They have been implemented in multiple ways. Optomechanical systems have been demonstrated or proposed that couple the mechanical motion to an optical field directly via radiation pressure buildup in a cavity [2–6] or indirectly via quantum dots [7] or ions [8]. On the other hand, in the electromechanical domain, this has been realized or proposed using devices such as (superconducting) single-electron transistors [9,10],  $LC$  circuits [11], a sapphire parametric transducer [12], Cooper pair boxes [13,14], or a stripline microwave resonator [15]. Importantly, the parametric coupling can not only be used for highly sensitive readout of mechanical motion [2] but also by virtue of *dynamical backaction* be used to cool the mechanical oscillator. Indeed, recent progress has enabled the observation of radiation pressure dynamical backaction cooling [4–6], as predicted decades ago [2,16]. Enabled by this work, one emerging goal in this context is ground state cooling, which may open up the possibility of studying nonclassical states of motion or entanglement in mechanical objects [8,17,18]. For both electro- and optomechanical systems, it has been shown that ground state cooling is possible only in the resolved sideband regime (RSB) where the mechanical resonance frequency exceeds the bandwidth of the driving resonator [19,20]. This result is analogous to the laser cooling of ions in the “strong binding” regime [21]. RSB cooling has recently been demonstrated [22,23].

Here we show that the cooling of mechanical oscillators in the RSB regime at high driving power can entail the appearance of normal-mode splitting (NMS). NMS—the coupling of two degenerate modes with energy exchange taking place on a time scale faster than the decoherence of each mode—is a phenomenon ubiquitous in both quantum and classical physics. A prominent realization occurs when atoms are coupled to a cavity field, which leads to the splitting of the cavity transmission into a doublet [24]. In

addition to atom-photon interactions, NMS also arises in exciton-photon and phonon-photon interactions [25]. NMS has also been observed with “artificial atoms” in circuit QED [26] and single quantum dot cavity QED [27] settings. In these examples, the NMS corresponds to a splitting in the energy spectrum of the coupled two-mode system which may be accessed via linear response. In contrast, the optomechanical NMS studied here involves driving two parametrically coupled nondegenerate modes out of equilibrium. Hence, as will be discussed further below, only in a “shifted” [19] rotating-frame representation does the Hamiltonian become analogous to the one characterizing the aforementioned examples. Concomitantly, the splitting, rather than appearing directly in the cavity transmission, manifests itself in the *fluctuation* spectra. This scenario is reminiscent of the single trapped ion realization of the Jaynes-Cummings model [28] with the role of the pseudospin now played by the optical (or electrical) mode. Since this type of normal-mode splitting occurs during RSB cooling, we analyze how the onset of NMS affects and limits cooling in the RSB regime.

**Theoretical model.**—We start from the rotating-frame Hamiltonian  $H' = -\hbar\Delta' a_p^\dagger a_p + \hbar\Omega_m a_m^\dagger a_m + \hbar\eta\Omega_m a_p^\dagger a_p \times (a_m + a_m^\dagger) + \hbar(s_+ a_p + s_+^* a_p^\dagger)$ , which provides a unified treatment of both a coherently driven optical and an electrical resonator (frequency  $\omega_p$ ) coupled to a mechanical

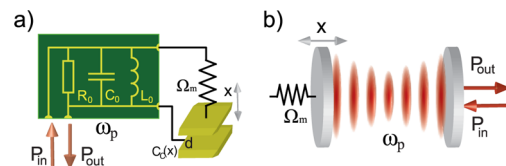


FIG. 1 (color online). (a) Electromechanical realization of parametric coupling of a mechanical oscillator to an  $LC$  circuit, where the coupling is determined by  $\frac{d\omega_p}{dx}|_{x=0} = \frac{\omega_p C_c}{2dC_{\text{tot}}}|_{x=0}$  ( $C_{\text{tot}}$  is the total capacitance). (b) Optomechanical realization of parametric coupling of a mechanical oscillator to a Fabry-Perot optical mode with  $\frac{d\omega_p}{dx}|_{x=0} = -\frac{\omega_p}{L}|_{x=0}$  ( $L$  is the cavity length).

oscillator (frequency  $\Omega_m \ll \omega_p$ ) via the dimensionless parameter  $\eta = (x_0/\Omega_m) \frac{d\omega_p}{dx}|_{x=0}$ . Here  $x_0 = \sqrt{\hbar/2m_{\text{eff}}\Omega_m}$  is the zero point motion of the mechanical mode,  $m_{\text{eff}}$  its effective mass,  $\Delta'$  the detuning of the drive from  $\omega_p$ , and  $a_m$  ( $a_p$ ) the annihilation operator for the mechanical (optical or electrical) mode. The dependence of the resonant frequency  $\omega_p$  on the mechanical oscillator's deflection  $x$  determines the strength of the coupling via  $\frac{d\omega_p}{dx}|_{x=0}$  [cf. Fig. 1]. The driving rate is given by  $|s_+| = \sqrt{P/\hbar\omega_p\tau_{\text{ex}}}$ , where  $P$  denotes the launched input power and  $\tau_{\text{ex}}^{-1}$  is the external coupling rate.

We derive the Heisenberg equations of motion for the canonical variables and introduce noise operators  $\xi_m(t)$  and  $\xi_p(t)$  weighted with the rates  $\Gamma_m$  and  $\kappa$  that characterize, respectively, the dissipation of the mechanical and optical (or electrical) degree of freedom. Subsequently, we shift the canonical variables to their steady-state values (i.e.,  $a_p \rightarrow \alpha + a_p$  and  $a_m \rightarrow \beta + a_m$ ) and linearize to obtain the following Heisenberg-Langevin equations [18,20,29]:

$$\begin{aligned} \dot{a}_p &= \left(i\Delta - \frac{\kappa}{2}\right)a_p - i\frac{g_m}{2}(a_m + a_m^\dagger) + \sqrt{\kappa}\xi_p(t), \\ \dot{a}_m &= \left(-i\Omega_m - \frac{\Gamma_m}{2}\right)a_m - i\frac{g_m}{2}(a_p + a_p^\dagger) + \sqrt{\Gamma_m}\xi_m(t). \end{aligned} \quad (1)$$

Here  $\Delta$  is the detuning with respect to the renormalized resonance, and  $\Delta < 0$  leads to cooling [19]. The optomechanical coupling rate is given by  $g_m = 2\alpha\eta\Omega_m$ , which is positive by an appropriate choice for the phase of  $s_+$ , and  $|\alpha|^2$  gives the mean resonator occupation number. In the case of the mechanical degree of freedom, the rotating wave approximation in the coupling to its environment implied by Eqs. (1) is warranted only for high  $Q$  values (and small  $g_m/\Omega_m$ ) [18]—conditions that are satisfied in the parameter regime of interest for ground state cooling. The latter also requires  $\Gamma_m \ll \kappa$ , which we will assume throughout our treatment. Equations (1) and their Hermitian conjugates constitute a system of four first-order coupled operator equations, for which the Routh-Hurwitz criterion implies that the system is stable only for  $g_m < \sqrt{(\Delta^2 + \kappa^2/4)\Omega_m/|\Delta|} \approx \Omega_m$  (if  $\Omega_m \gg \kappa$  and  $|\Delta| \approx \Omega_m$ ).

Here we follow a semiclassical theory by considering *noncommuting* noise operators for the input field, i.e.,  $\langle \xi_p(t) \rangle = 0$ ,  $\langle \xi_p^\dagger(t')\xi_p(t) \rangle = n_p\delta(t' - t)$ , and  $\langle \xi_p(t')\xi_p^\dagger(t) \rangle = (n_p + 1)\delta(t' - t)$ , and a *classical* thermal noise input for the mechanical oscillator, i.e.,  $\langle \xi_m(t) \rangle = 0$  and  $\langle \xi_m^\dagger(t')\xi_m(t) \rangle = \langle \xi_m(t')\xi_m^\dagger(t) \rangle = n_m\delta(t' - t)$ , in Eqs. (1). The quantities  $n_m$  and  $n_p$  are the equilibrium occupation numbers for the mechanical and optical (or electrical) oscillators, respectively. We transform to the quadratures (i.e.,  $x/x_0 = a_m + a_m^\dagger$ ) and solve the Langevin equations in Fourier space [20]. Thus we recover a steady-state displacement spectrum [29] given (for  $n_p = 0$ ) by  $S_x(\omega) = \frac{x_0^2}{2\pi}\Omega_m^2|\chi(\omega)|^2[\Gamma_m n_m - \frac{\Delta^2 + \omega^2 + \kappa^2/4}{2\Delta\Omega_m}\Gamma_s(\omega)]$ , with

$$\begin{aligned} \chi^{-1}(\omega) &= \Omega_m^2 + 2\Omega_m\Omega_s(\omega) - \omega^2 - i\omega[\Gamma_m + \Gamma_s(\omega)], \\ \Omega_s(\omega) &= \frac{g_m^2}{4} \left[ \frac{\omega + \Delta}{(\omega + \Delta)^2 + \kappa^2/4} - \frac{\omega - \Delta}{(\omega - \Delta)^2 + \kappa^2/4} \right], \\ \Gamma_s(\omega) &= \frac{g_m^2}{4\omega} \left[ \frac{\Omega_m\kappa}{(\omega + \Delta)^2 + \kappa^2/4} - \frac{\Omega_m\kappa}{(\omega - \Delta)^2 + \kappa^2/4} \right]. \end{aligned} \quad (2)$$

This spectrum is characterized by a mechanical susceptibility  $\chi(\omega)$  that is driven by thermal noise ( $\propto n_m$ ) and by the quantum fluctuations of the radiation pressure (quantum backaction). In linear cooling theory, the susceptibility is approximated by evaluating the terms  $\Gamma_s(\omega)$  and  $\Omega_s(\omega)$  at the (bare) mechanical frequency [4,6,30]. Then  $\Gamma_s(\Omega_m)$  coincides with the cooling rate and is linear in the input power ( $g_m^2 \propto P$ ).

*Parametric NMS.*—The above approximation is adequate only for weak driving such that  $g_m \ll \kappa$  [19,20]. To obtain an understanding of the mechanical susceptibility beyond this linear regime, we return to the linearized Heisenberg-Langevin equations (1) and calculate the corresponding eigenfrequencies that determine the dynamics of the system. Though there exists an analytical solution, it is rather opaque and does not provide physical insight, so we will use instead an approximation scheme appropriate for the parameter regime relevant for the observation of NMS and to attain ground state cooling. Along these lines, we focus in the following on: (i) the RSB regime ( $\kappa \leq \Omega_m/2$ ) necessary for ground state cooling [19,20,22], (ii) optomechanical coupling  $g_m \leq \Omega_m/2$ , and (iii)  $\delta^2 \ll \Omega_m^2$  ( $\delta \equiv -\Delta - \Omega_m$ , the frequency detuning from the lower sideband). In the shifted representation corresponding to Eqs. (1) [19], the relevant part of the parametric interaction in Hamiltonian  $H'$  is described by an effective dipolelike interaction term, i.e.,  $\hbar\eta\Omega_m a_p^\dagger a_p (a_m + a_m^\dagger) \rightarrow \frac{\hbar g_m}{2}(a_p + a_p^\dagger)(a_m + a_m^\dagger)$  after neglecting the nonlinear term. This interaction term is analogous to the Jaynes-Cummings setting (with  $a_p \rightarrow \sigma^-$ ) and naturally leads to resonance splitting when the modes have matching frequencies. The off-resonant counterrotating terms (CRT)  $\propto a_p^\dagger a_m^\dagger$ ,  $a_p a_m$  induce a small frequency shift analogous to the Bloch-Siegert shift in atomic physics [31]. These CRT terms, which are responsible for the mixing between the creation and annihilation operators in the Heisenberg-Langevin equations (1), can be treated in perturbation theory within the parameter range defined by (i)–(iii). The first nonvanishing order in this perturbative expansion is quadratic in the CRT and yields a correction to the decoupled eigenvalues  $\omega_\pm \approx \omega_\pm^{(0)} + \omega_\pm^{(2)}$  [note that we take  $\Gamma_m = 0$  in  $\omega_\pm^{(2)}$ ]:

$$\begin{aligned} \omega_\pm^{(0)} &= \Omega_m + \frac{\delta}{2} - i\frac{\kappa + \Gamma_m}{4} \\ &\pm \frac{1}{2}\sqrt{g_m^2 - (\kappa/2 - \Gamma_m/2 + i\delta)^2}, \end{aligned} \quad (3)$$

$$\omega_\pm^{(2)} \approx -\frac{g_m^2/4}{2\Omega_m + \delta \pm \sqrt{g_m^2 - (\kappa/2 + i\delta)^2}}. \quad (4)$$

Naturally, there is another pair of eigenfrequencies given by  $-\omega_{\pm}^*$ . In Fig. 2, the real and imaginary parts of the eigenvalues are plotted. The inset shows the frequency shift ( $\omega_{\pm}^{(2)}$ ) due to the CRT. If we choose the value  $\delta = 0$  (i.e.,  $\Delta = -\Omega_m$ ) relevant for  $\kappa \ll \Omega_m$  (see below) and neglect  $\Gamma_m$ , the square root term of  $\omega_{\pm}^{(0)}$  leads to two regimes. While for  $g_m < \kappa/2$  the term is fully imaginary and modifies the decay rate of the modes, for  $g_m > \kappa/2$  it becomes real instead and the real parts of the eigenfrequencies exhibit the splitting that signals NMS (Fig. 3). The latter is associated to a mixing between the mechanical mode and the *fluctuation* around the steady state of the resonator field. Classically, this fluctuation can be understood as a beat of the pump photons with the photons scattered on resonance which leads to oscillations with frequency  $|\Delta|$  in the intensity time-averaged over  $2\pi/\omega_p$ . For  $\kappa^2/4 \ll g_m^2$ , the splitting ( $\approx g_m$ ) is proportional to the square root of the mean cavity photon number ( $\alpha^2$ ). This is analogous to NMS in atomic physics where the splitting of the cavity resonance is proportional to the square root of the number of atoms coupled to the cavity mode [24]. When detecting the phase fluctuations in the transmitted light with a homodyne detection scheme, the signal at  $\Omega_m$  splits [cf. Fig. 3(b)], but the (suppressed) scattered light at the carrier frequency exhibits no splitting. It is important to note that the splitting in the displacement spectrum is not observed unless  $g_m > \kappa/\sqrt{2}$  due to the finite width of the peaks. Because of the requirements on the cavity bandwidth and the detuning, the parameter regime in which NMS may appear implies cooling. In turn, for a positive detuning (which entails amplification) the observation of NMS is prevented by the onset of the parametric instability [3]. Therefore, a discussion of NMS

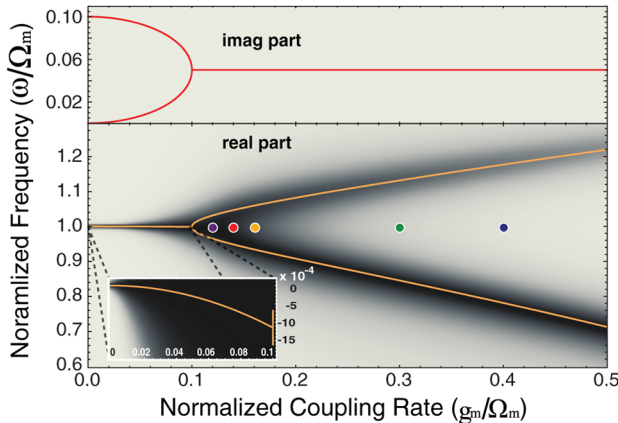


FIG. 2 (color online). Real and imaginary parts of the eigenvalues [cf. Eqs. (3) and (4)] of the linearized cooling problem corresponding, respectively, to the eigenfrequency and mode damping for  $\Delta = -\Omega_m$  and  $\kappa/\Omega_m = 0.2$ . The inset magnifies the resonance shift before the mode splitting. The real part is underlaid with the normalized classical displacement spectrum (contribution  $\propto n_m$ ) [cf. Eq. (2)]; thereof sample curves are highlighted in Fig. 3 for the rates marked by the dots.

cannot be decoupled from an analysis of the associated cooling. We also show below that the CRT in the interaction leads to the quantum limit of backaction cooling [19,20].

*Effect of NMS on backaction cooling.*—We now use the approximate eigenfrequencies to perform contour integration on the normal ordered mechanical spectrum in order to obtain the final occupancy of the mechanical oscillator  $n_f = \langle a_m^\dagger(\tau)a_m(0) \rangle|_{\tau=0}$ . In this treatment we take both the thermal and the vacuum noise of the driving resonator into account. A finite value for  $n_p$  may be relevant for electromechanical systems [11,15]. Within our approximation scheme we can introduce a formal parameter that tags the CRT terms and expand  $n_f$  in its powers. To zeroth order the poles are determined by the approximate eigenfrequencies  $\omega_{\pm}^{(0)}$ ,  $-\omega_{\pm}^{(0)*}$  given in Eqs. (3), and it is straightforward to evaluate  $n_f^{(0)}$  (including  $\Gamma_m$ ). To second order we use instead the poles  $\omega_{\pm}^{(0)} + \omega_{\pm}^{(2)}$  and  $-\omega_{\pm}^{(0)*} - \omega_{\pm}^{(2)*}$ . Subsequently,  $n_f^{(2)}$  is expanded in the *small* parameters  $g_m/\Omega_m$ ,  $\kappa/\Omega_m$ , and  $|\delta|/\Omega_m$  up to second order with  $\Gamma_m \rightarrow 0$ . Both  $n_f^{(0)}$  and  $n_f^{(2)}$  do not contain terms linear in  $\delta$ , allowing one to directly minimize the result with respect to  $\delta$  by setting  $\delta \rightarrow 0$ . This yields

$$\begin{aligned} n_f^{(0)} &= n_m \frac{\Gamma_m}{\kappa} \frac{g_m^2 + \kappa^2}{g_m^2 + \Gamma_m \kappa} + \frac{g_m^2}{g_m^2 + \Gamma_m \kappa} n_p, \\ n_f^{(2)} &= n_m \frac{\Gamma_m}{\kappa} \frac{g_m^2}{4\Omega_m^2} + \left(n_p + \frac{1}{2}\right) \frac{\kappa^2 + 2g_m^2}{8\Omega_m^2}. \end{aligned} \quad (5)$$

The final occupancy  $n_f = n_f^{(0)} + n_f^{(2)}$  consists of three contributions. One is proportional to the occupancy of the thermal bath  $n_m$  and displays linear cooling for  $\Gamma_m \ll g_m \ll \kappa$ , i.e.,  $n_f \approx \frac{\Gamma_m}{g_m/\kappa} n_m$ . When  $g_m$  approaches  $\kappa$ , deviations from the linear cooling regime become apparent. Indeed, the final occupancy is *always limited* by  $n_f \geq n_m \frac{\Gamma_m}{\kappa}$ , which implies that the largest temperature reduction

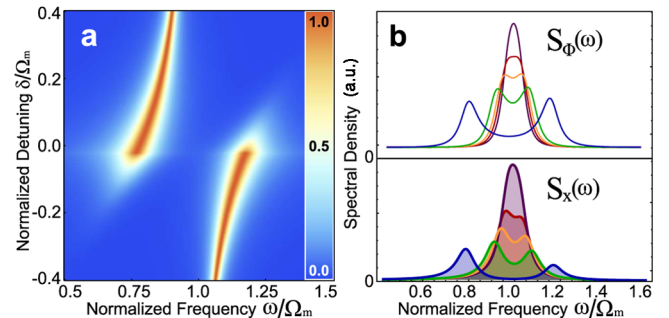


FIG. 3 (color online). (a) Normalized logarithm of the classical displacement spectrum (contribution  $\propto n_m$ ) [cf. Eq. (2)] as a function of the normalized detuning for  $g_m/\Omega_m = 0.4$  and  $\kappa/\Omega_m = 0.2$ . (b) Typical displacement spectra for coupling rates that are represented by the dots in Fig. 2. The solid curves above correspond to the phase spectral density  $S_\Phi(\omega)$  measured in homodyne detection.



is bound by the cavity decay rate  $\kappa$  [32]. This is equivalent to the condition  $Q_m > n_m \frac{\Omega_m}{\kappa}$  for ground state cooling. It is noted that operation in the deeply RSB regime is advantageous to avoid photon-induced heating [22], entailing that the condition on the mechanical  $Q$  is therefore more stringent. A second contribution is proportional to the finite occupancy of the driving circuit ( $n_p$ ) and corresponds to heating from thermal noise in its input. It implies that it is impossible to cool below the equilibrium occupation of the resonator. If we assume that the mechanical and electromagnetic baths are at the same temperature  $T_m$ , it entails  $n_f \geq n_m \frac{\Omega_m}{\omega_p}$ . Last, there is a term in  $n_f^{(2)}$  that is temperature-independent and corresponds to heating from quantum backaction noise. This term determines the quantum limit to the final occupancy and agrees with Refs. [19,20]. Interestingly, in the present analysis the quantum limit arises from the CRT. We note that the trade-off between the quantum limit and the cavity bandwidth limitation leads to an optimal value for  $\kappa$ . Consistent results are obtained with a covariance matrix approach [33].

Finally, we consider appreciable cooling [ $n_f \ll n_m$  so that we can take  $\Gamma_m \rightarrow 0$  in the denominator of Eqs. (5)] and optimize  $n_f^{(0)} + n_f^{(2)}$  with respect to  $g_m$ , which yields

$$n_{\text{opt}} \approx n_m \frac{\Gamma_m}{\kappa} + n_p + \frac{\kappa^2}{16\Omega_m^2} + \sqrt{\frac{n_m \Gamma_m \kappa (n_p + \frac{1}{2})}{\Omega_m^2}} \quad (6)$$

for  $g_{\text{opt}} = \sqrt{4n_m \Gamma_m \kappa \Omega_m^2 / [n_p + (1/2) + n_m \Gamma_m / \kappa]}$ . In the ground state cooling regime, the first three terms of Eq. (6) always give the correct order of magnitude. Thus, a comparison of  $g_{\text{opt}}$  with the condition  $g_m > \kappa/2$  implies that optimal ground state cooling leads to NMS only when the thermal noise [first term in Eq. (6)] is at least comparable to the quantum backaction noise [third term in Eq. (6)]. This is likely to be the case in current endeavors to reach the ground state.

*Experimental realization.*—To demonstrate that the observation of parametric NMS is within experimental reach, we discuss the parameters from Ref. [22]:  $\Omega_m/2\pi = 73.5$  MHz,  $\kappa/2\pi = 3.2$  MHz, and  $\Gamma_m/2\pi = 1.3$  kHz. The cooling rate  $\Gamma_c/2\pi = 1.56$  MHz was extracted from the displacement spectrum's FWHM. A comparison with Eqs. (2) then yields a coupling rate  $g_m/2\pi \approx 2.0$  MHz. Therefore, the observation of parametric NMS is within experimental reach. In the electromechanical domain, using a superconducting coplanar waveguide resonator, Ref. [23] reports coupling rates of  $g_m/2\pi = 6$  kHz for a cavity with a decay rate  $\kappa/2\pi = 230$  kHz.

In summary, we have analyzed a novel instance of NMS that occurs in cavity optomechanics due to the coupling between the fluctuations of the cavity field and the mechanical oscillator mode. Furthermore, we have elucidated

its implications for ground state cooling, namely, the limitation through the cavity bandwidth.

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- [32] Note that  $n_f^{(0)}$  follows from the classical rate equations for two resonant oscillators (frequency  $\Omega_m$ ) connected, respectively, to two reservoirs at temperatures  $T_m$  and  $T_{\text{eff}} = T_m \Omega_m / \omega_p$  via rates  $\Gamma_m$  and  $\kappa$  ( $\Gamma_m \ll \kappa$ ) and coupled via heat diffusion with a rate  $g_m^2 / \kappa$ . In this picture, the deviation from linear cooling corresponds to heat diffusion from the cavity to the mechanical oscillator.
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