Constraining Lorentz Violation with Cosmology

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(Received 14 August 2008; published 23 December 2008)

The Einstein-aether theory provides a simple, dynamical mechanism for breaking Lorentz invariance. It does so within a generally covariant context and may emerge from quantum effects in more fundamental theories. The theory leads to a preferred frame and can have distinct experimental signatures. In this Letter, we perform a comprehensive study of the cosmological effects of the Einstein-aether theory and use observational data to constrain it. Allied to previously determined consistency and experimental constraints, we find that an Einstein-aether universe can fit experimental data over a wide range of its parameter space, but requires a specific rescaling of the other cosmological densities.

DOI: 10.1103/PhysRevLett.101.261102 PACS numbers: 98.80.Jk, 11.30.Cp, 95.30.Sf

The spacetime symmetry of local Lorentz invariance is a cornerstone of modern physics [1], but is not inviolate. Violations can occur in quantum gravity theories, with the symmetry emergent and approximate at macroscopic levels [2]. In the particle physics sector the symmetry has been experimentally verified to extremely high precision [3]. On the large scales characteristic of the gravitational sector, however, constraints are much less certain. In this letter we explore the extent to which precision cosmology can constrain a Lorentz-violating theory.

The theoretical workhorse for studying violation of Lorentz symmetry in gravitation is the Einstein-aether theory [4], a simple, elegant proposal for dynamically violating Lorentz invariance within the framework of a diffeomorphism-invariant theory. It is a refinement of the gravitationally coupled vector field theories first proposed by Will and Nordvedt in 1972 [5] and has been explored in exquisite detail by Jacobson, Mattingly, Foster, and collaborators [6–8]. A Lorentz-violating vector field, henceforth called the *aether*, will affect cosmology: it can lead to a renormalization of the Newton constant [9], leave an imprint on perturbations in the early Universe [10,11], modify the propagation of cosmic microwave background (CMB) photons [12], and in more elaborate actions it may even affect the growth rate of stucture [13,14]. Calculations of CMB power spectra have been reported for some aether parameter combinations by [15]. Here we consider general combinations and compare the results to experimental data.

The action for the Einstein-aether is

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R + \mathcal{L}(g^{ab}, A^b)\right] + S_M$$

where g_{ab} is the metric, R is the Ricci scalar of that metric, S_M is the matter action, and $\mathcal{L}$ is constructed to be generally covariant and local. G is the bare gravitational constant, not necessarily the locally measured value. S_M couples only to the metric g_{ab} and not to A^b and

$$\mathcal{L}(g^{ab}, A^b) \equiv \frac{1}{16\pi G}[K^{ab}{}_{cd}\nabla_a A^c \nabla_b A^d + \lambda(A^b A_b + 1)],$$

where $K^{ab}{}_{cd} \equiv c_1 g^{ab} g_{cd} + c_2 \delta^a{}_c \delta^b{}_d + c_3 \delta^a{}_d \delta^b{}_c - c_4 A^a A^b g_{cd}$ [16]. We will use the notation $c_{12\ldots} \equiv c_1 + c_2 + \ldots$. The gravitational field equations for this model take the form: $G_{ab} = \tilde{T}_{ab} + 8\pi G T_{ab}$ where the stress-energy tensor for the vector field $\tilde{T}_{ab}$ is given in [4] and T_{ab} describes the conventional fluids.

A number of constraints on the c_is have been derived. Most notably a parametrized post-Newtonian (PPN) analysis of the theory leads to a reduction in the dimensionality of parameter space such that c_2 and c_4 can be expressed in terms of the other two parameters: $c_2 = (-2c_1^2 - c_1 c_3 + c_3^2)/3c_1$ and $c_4 = -c_3^2/c_1$ [8]. Additionally, the squared speeds of the gravitational and aether waves with respect to the preferred frame must be greater than one so as to prevent the generation of vacuum Čerenkov radiation by cosmic rays [18]. We shall label this space of models as $\mathcal{C}$. A final constraint arises from considering the effects of the aether on the damping rate of binary pulsars. The rate of energy loss in such systems by gravitational radiation agrees with the prediction of general relativity to one part in 10^3. It has been shown [17] that, for the Einstein-aether theory to agree with general relativity for these systems, we require that $c_+ \equiv c_1 + c_3$ and $c_- \equiv c_1 - c_3$ are related by an algebraic constraint (shown as the dashed line in Fig. 4) [19]. A more exotic, but viable, subset of the parameter space can be considered in which $c_1 = c_3 = 0$. The PPN and pulsar constraints do not apply here and a cosmological analysis is potentially the only way of constraining the values of the coupling constants. We shall label this alternate space of models as $\mathcal{E}$. In what follows we will write down the equations in a general form and then study the two subspaces $\mathcal{C}$ and $\mathcal{E}$ independently.

We now focus on cosmological scales and assume a homogeneous and isotropic background spacetime in which the metric is of the form $g_{ab}dx^a dx^b = -dt^2 + a(t)^2\gamma_{ij}dx^i dx^j$ where t is physical time, γ_{ij} is the identity matrix, and $a(t)$ is the scale factor. Throughout this Letter, subscripts i and j will run 1 to 3. The vector field must respect the spatial homogeneity and isotropy of the system and so will only have a nonvanishing "t" component; this constraint fixes $A^b = (1, 0, 0, 0)$. The energy-momentum tensor of the matter will include the standard menagerie: photons, neutrinos, baryons, dark matter, and the cosmological constant.

In this background, the *t-t* component of the aether stress-energy tensor is equal to $(3/2)\alpha H^2$ where $\alpha \equiv c_1 + 3c_2 + c_3$. For models $\mathcal{C}$ we have $\alpha = -2c_+c_-/(c_+ + c_-)$. The fractional energy densities in the various components are given by $\Omega_i(a) \equiv 8\pi G\rho_i(a)/3H_0^2$ and $\Omega_{AE} = (\alpha/2)\Sigma_i\Omega_i/(1-\alpha/2)$ where $H_0 = 100h\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ is the Hubble constant today and ρ_i is the energy density in the fluid component i. In quasistatic spacetimes the aether exhibits tracking behavior such that the locally measured value of Newton's constant is actually $G_N = G/(1 + c_{14}/2)$ [9]. For models $\mathcal{C}$ we have $c_{14} = 2c_+c_-/(c_+ + c_-)$ [20]. Thus, given a value ρ_i, the actual Ω_i is related to the value Ω_{Ni} inferred using a locally measured value of G_N as: $\Omega_i = (1 + c_{14}/2)\Omega_{Ni}$. Hence, explicitly using our expression for Ω_{AE}, the Friedmann equation becomes:

$$H^2 = H_0^2\frac{2 + c_{14}}{2-\alpha}\sum_i\Omega_{Ni}.$$

To fully explore the cosmological consequences of the aether, we must consider linear perturbations around the background. We will do so in the synchronous gauge and use conformal time coordinates: given $g_{ab}dx^adx^b = -a^2d\tau^2 + a^2[\gamma_{ij} + h_{ij}]dx^idx^j$, the two scalar potentials η and h are defined by: $h_{ij}(\mathbf{x},\tau) = \int d^3ke^{i\mathbf{k}\cdot\mathbf{x}}[\hat{k}_i\hat{k}_jh(\mathbf{k},\tau) + (\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij})6\eta(\mathbf{k},\tau)]$. The aether field can be written as $A^d = \frac{1}{a}(1, \partial_iV)$ for a scalar V; the zeroth component is fixed equal to a^{-1} by the gauge choice and the fixed-norm constraint—see [10]. Instead of V itself, we choose to use the variable: $\xi \equiv V - \frac{1}{2k^2}(h + 6\eta)'$ with which the evolution equations take a particularly instructive form.

The gravitational field equations are

$$\left(1 - \tfrac{1}{2}\alpha\right)k^2\eta' = 4\pi Ga^2ik^j\delta T^0_j + \tfrac{1}{2}k^4c_{123}\xi$$

and

$$\begin{aligned}\left(1 + \tfrac{1}{2}c_{14}\right)(\mathcal{H}h' - 2k^2\eta) = {}& -8\pi Ga^2\delta T^0_0 \\ & - \tfrac{1}{2}(c_{14} + \alpha)6\mathcal{H}\eta' - \tfrac{3}{2}c_{14}\Sigma_f \\ & + c_{14}(1 + c_+)k^2(\xi' + 2\mathcal{H}\xi).\end{aligned}$$

For models $\mathcal{C}$ we have $c_{123} = 2c_+^2/3(c_+ + c_-)$.

The aether equation of motion is

$$\begin{aligned}0 = {}& c_{14}(1+c_+)\xi'' + 2\mathcal{H}c_{14}(1+c_+)\xi' + \Bigg[2c_{14}(1+c_+) \\ & \times\left(\frac{a''}{a} - \mathcal{H}^2\right) - (c_{14}+\alpha)\left(\frac{a''}{a} - 2\mathcal{H}^2\right) + c_{123}k^2\Bigg]\xi \\ & + (c_{14}+\alpha)\eta' + (c_{14}+\alpha)\frac{1}{k^2}\left(\mathcal{H}^2 - \frac{1}{2}\frac{a''}{a}\right)(h' + 6\eta') \\ & - \frac{3}{2}\frac{c_{14}}{k^2}\Sigma_f',\end{aligned}$$

where $\mathcal{H}$ is the conformal Hubble parameter, primes are derivatives with respect to τ and $\Sigma_f \equiv -8\pi Ga^2(\hat{k}_i\hat{k}^j - \frac{1}{3}\delta^j_i)\Sigma^i_j$, where Σ^i_j is the traceless component of the fluid stress-energy tensor. The homogeneous "sourceless" solution to the above equation during an era where $a \propto \tau^n$ is $\xi(k,\tau) = \tau^{1-2n}[f_1(k)J(\beta, c_sk\tau) + f_2(k)Y(\beta, c_sk\tau)]$ where f_i are functions to be fixed by boundary conditions, J and Y are Bessel functions and the various constants are defined through: $b_1 \equiv -2n - (c_{14} + \alpha)\times(n^2 + n)/[c_{14}(1 + c_+)]$, $\beta \equiv (1 - 8n + b_1 + b_1^2)^{1/2}$ and $c_s^2 \equiv c_{123}/(c_{14}(1 + c_+))$. With c_s^2 positive, the solutions are damped and oscillatory solutions when $c_sk\tau \gg 1$ and power law when $c_sk\tau \ll 1$ [13].

It was shown in [10] that the primordial scalar power spectrum $\mathcal{P}_\Phi$ (where Φ is the trace perturbation to the metric in the conformal Newtonian gauge) is modified relative to that of a universe with no aether, $\tilde{\mathcal{P}}_\Phi$, through $\mathcal{P}_\Phi = \tilde{\mathcal{P}}_\Phi[\frac{1-\frac{\alpha}{c_{14}}c_+}{1+c_+}]^2$ while ξ and ξ' are driven to a vanishingly small value compared to their values at the onset of inflation. We will work with the equivalent initial conditions in the synchronous gauge.

To study these effects in detail, we have modified the Boltzmann code CMBEASY [21] by adding a Newton-Raphson solver for the Hubble parameter, and including the aether components in the density and pressure; the perturbation evolution has been modified by adding ξ and ξ' as the integrated components. In Fig. 1 we show the effect of the aether on the angular power spectrum of anisotropies in the CMB and the power spectrum of galaxies [we superpose the Wilkinson Microwave Anisotropy

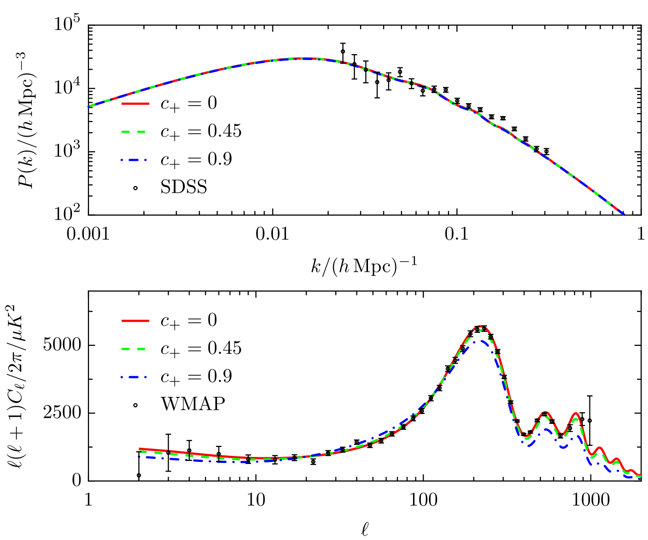

FIG. 1 (color online). The angular power spectrum of the CMB (bottom) and the power spectrum of galaxies (top) for a sample of class $\mathcal{C}$ Einstein-aether models, with different c_+ (with c_- chosen to satisfy the weak-field binary pulsar constraint—the dashed line of Fig. 4) The other parameters have their ΛCDM best-fit values, with the Ω_i rescaled as described in the text. Superposed are the WMAP and SDSS data sets.

TABLE I. Mean and 1σ error values of the marginalized likelihoods for a range of cosmological parameters. The left most column is for ΛCDM with no aether, the central column for general class $\mathcal{C}$ models, and the right hand column for class $\mathcal{C}$ models with the weak pulsar constraint (on the dashed line of Fig. 1).

Parameter	ΛCDM	General	Weak pulsar
$\Omega_c h^2$	0.137 ± 0.004	0.097 ± 0.01	0.098 ± 0.01
$\Omega_b h^2$	0.022 ± 0.001	0.019 ± 0.002	0.019 ± 0.002
H_0	69.7 ± 1.6	71.7 ± 2.0	72.5 ± 2.4
τ_D	0.08 ± 0.029	0.077 ± 0.027	0.078 ± 0.028
n_s	0.955 ± 0.015	0.976 ± 0.02	0.984 ± 0.024
Ω_Λ	0.671 ± 0.019	0.61 ± 0.05	0.67 ± 0.028
c_1	$\cdots$	-0.46 ± 0.14	-0.26 ± 0.12
c_2	$\cdots$	0.34 ± 0.1	0.20 ± 0.09
c_3	$\cdots$	-0.23 ± 0.1	-0.12 ± 0.05
c_4	$\cdots$	0.13 ± 0.09	0.05 ± 0.02

Probe (WMAP) and Sloan Digital Sky Survey (SDSS) data] for a selection of parameters in class $\mathcal{C}$.

The dominant effect for smaller values of c_+ is on the large-scale CMB, through the integrated Sachs-Wolfe effect; it leads to a suppression on large scales (which curiously enough is favored by large-scale CMB data). As expected, the overall growing mode of matter perturbations is very weakly affected and the change on the power spectrum of galaxies is marginal.

As is usual in cosmological model testing, we compute parameter constraints using a Monte Carlo Markov Chain (MCMC) [22]. We explore a six-dimensional parameter space consisting of the fractional baryon density Ω_b, the fractional matter density Ω_M, the Hubble constant H_0, the scalar spectral index n_S, the optical depth τ_D, the overall amplitude of fluctuations, the bias factor of SDSS galaxies and the two aether parameters c_+ and c_-. We constrain parameters using the WMAP 3-year release, the Boomerang 03 release, and data from ACBAR and VSA [23], as well as the SDSS and 2DF surveys [24,25]. We also use measurements of the luminosity distance as a function of redshift from supernovae Ia measurements [26] but have found that these data sets have very little ability to constrain this class of models.

The marginalized constraints from the CMB and large-scale structure on the two aethereal parameters in model $\mathcal{C}$ are shown in Fig. 4 [16]. The best-fit aether model is mildly superior to standard ΛCDM cosmology, at about 2σ, at a cost of two extra parameters.

The soft lower limit at $c_- > -0.5$ comes from a prior on the baryon fraction. This signals an important characteristic of these models: the strong correlation between the fractional energy density in the aether, Ω_{AE}, and the other energy components. This is perhaps the primary result of our analysis and is illustrated in Fig. 3: the CMB and large scale structure (LSS) data restrict the background to evolve as in the ΛCDM case, which in turn leads to a rescaling of the different energy components in the presence of the aether. Naturally this also affects the constraints on the other cosmological parameters. These constraints, under the ΛCDM and aether models with and without the weak binary pulsar constraint, are shown in Table I. As expected, the largest shift is seen in the Ω_i.

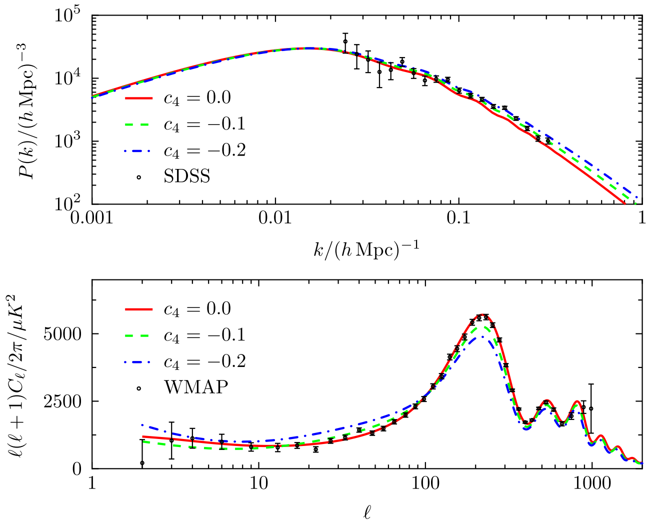

FIG. 2 (color online). The angular power spectrum of the CMB (bottom) and the power spectrum of galaxies (top) for a sample of exotic class $\mathcal{E}$ Einstein-aether models where $c_1 = c_2 = c_3 = 0$. The other parameters have their ΛCDM best-fit values, with the radiation density modified to account for the change in the gravitational constant. Superposed are the WMAP and SDSS datasets.

As stated above, the CMB and LSS play the dominant role in generating these constraints, and interestingly enough this is through the change in the background evolution and its effect on the metric perturbations, and not necessarily through the presence of perturbations in the vector field. Indeed, artificially switching off the perturbations in the aether field has essentially no effect on the power spectrum of LSS and a small effect (of approximately 10%) on the angular power spectrum of the CMB.

So far we have focused on models in class $\mathcal{C}$, where we found that the coupling constants are allowed to vary quite widely. In the case of models in class $\mathcal{E}$, the cosmological data are far more restrictive. For example, fixing $c_1 = c_2 = c_3 = 0$ we find that $-0.05 < c_4 < 0$ (note that in

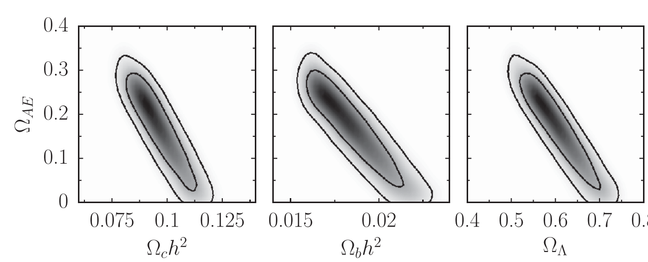

FIG. 3. Joint constraints on the fractional aether density Ω_{AE}, with the physical dark matter density $\Omega_c h^2$, the physical baryon density $\Omega_b h^2$, and the fractional Λ density Ω_Λ. The contours are 1 and 2σ.

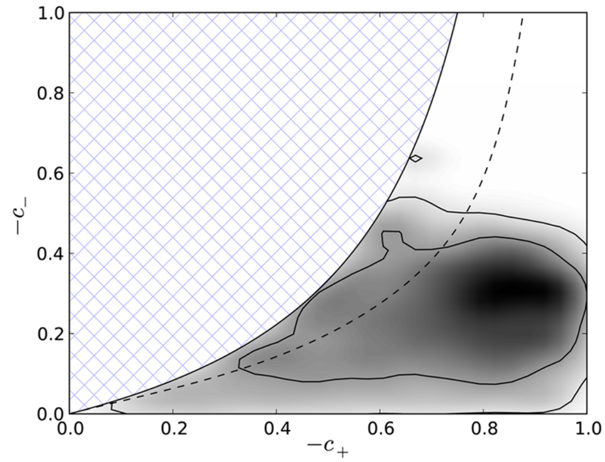

FIG. 4 (color online). Joint constraints on the parameters $-c_+$ and $-c_-$. The black lines are the 1 and 2σ contours, where we have marginalized over the values of the parameters. The hatched region is excluded by Čerenkov constraints; the dashed line indicates where weak-field constraints from binary pulsars are met. Both are taken from [4].

this case $\Omega_{AE} = 0$). If we allow c_2 to be nonzero we find that both c_2 and c_4 must be in $[-0.01, 0]$. The reason for this constraint is illustrated in Fig. 2; at low ℓ the integrated Sachs-Wolfe effect induced by the modified potentials is large enough to disrupt the C_ℓ. These are the strongest constraints on these parameters that currently exist.

In this Letter we have studied the effect of Lorentz violation on cosmology as parametrized by the Einstein-aether model. We have found that Lorentz violation in this form is compatible with current cosmological data and, combined with other noncosmological probes we have found constraints on c_+ and c_-. The data also require the rescaled combination of density parameters, in which the background evolution is unchanged from a ΛCDM universe. We have also found tight constraints on the other allowed range of parameter space $\mathcal{E}$, which has, until now, been relatively unconstrained by other methods. Collectively these constraints arise from tests on distance scales spanning more than 15 orders of magnitude.

There are of, course, other possible ways of parametrizing Lorentz violation which are not encompassed by the Einstein-aether model. In particular, one may relax the fixed-norm constraint on the aether field [27] or couple it directly to the matter content of the Universe [3]. Such theories tend to have a much stronger effect on the evolution of the background cosmology [28] or lead to distinct experimental signatures [29]. Hence the results presented here are currently the most comprehensive (though conservative) constraints on the c_is, and thus on Lorentz-violating vector theories.

We thank T. Jacobson, B. Foster, C. Skordis, and G. Starkman for useful discussions. We are extremely grateful to D. Mota for discussions and for access to computer codes relating to earlier work. T. G. Z. and J. A. Z. both received separate support from STFC.

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