# Effects of the $g$ Factor in Semiclassical Kinetic Plasma Theory

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A kinetic theory for spin plasmas is put forward, generalizing those of previous authors. In the model, the ordinary phase space is extended to include the spin degrees of freedom. Together with Maxwell's equations, the system is shown to be energy conserving. Analyzing the linear properties, it is found that new types of wave-particle resonances are possible that depend directly on the anomalous magnetic moment of the electron. As a result, new wave modes, not present in the absence of spin, appear. The implications of our results are discussed.

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In recent years, there has been a rapidly growing interest in the quantum properties of plasmas [1–8]. This has been motivated by applications in, for example, plasmonics [9,10], quantum wells [11], and ultracold plasmas [12]. Common to such applications are rather extreme parameters, compared to most laboratory and space plasmas. In particular, the plasma densities are considered to be very high and/or the temperatures are correspondingly low. For astrophysical plasmas, it is also known that strong magnetic fields [13] may lead to various quantum effects being important. However, a recent work [8] shows that the spin properties of electrons can be important even in high temperature plasmas of modest density and magnetic field strength.

In the present Letter, we will put forward a more elaborate kinetic model, where the electrons are described using a distribution function in an extended phase space, including also variables due to the spin orientation. This model is an extension of a kinetic model used by Refs. [14,15], where we here also include the magnetic dipole force associated with the spin. As a consequence, the magnetic dipole energy also contributes to the energy conservation law which is derived from Maxwell's equations combined with the spin-kinetic model. This system is then used to study linear waves in a homogenous magnetized plasma. The analysis shows that the inclusion of spin gives rise to new phenomena not present within the usual Vlasov model. This also holds for the high temperature regime, where quantum effects are normally suppressed. It turns out that effects of the anomalous magnetic moment are crucial. In particular, new wave modes appear with frequencies $\omega \approx (g/2 - 1)\omega_c$, where $g \simeq 2.002\,319$ is the electron spin $g$ factor and $\omega_c$ is the electron cyclotron frequency. Furthermore, new types of wave-particle interaction can take place that involve the electron spin state. We stress that none of these effects can be seen within quantum fluid models [1–8].

Here we are interested in effects due to the electron spin that may survive even when the macroscopic variations occur on a scale longer than the thermal de Broglie wavelength, which is a scale normally expected to imply classical behavior [16]. In particular, we will by spin here mean the semiclassical properties of the electron due to its magnetic moment and thus will not take into account, e.g., commutation relations. As a starting point, let us consider the evolution equations for momentum and spin resulting from the Pauli Hamiltonian

$$H = -(\hbar^2/2m^2)[\nabla - (iq\mathbf{A}/\hbar)]^2 + \mu_e \boldsymbol{\sigma} \cdot \mathbf{B} + q\Phi, \quad (1)$$

where $q = -e = -|e|$ and $m$ are the electron charge and mass, respectively, $\mathbf{A}$ and $\Phi$ are the vector and scalar potential, respectively, $2\pi\hbar$ is the Planck constant, $\mu_e = -(g/4)e\hbar/m$ is the electron magnetic moment, and $\boldsymbol{\sigma}$ is the vector consisting of the Pauli spin matrices. Using $dF/dt = \partial F/dt + (1/i\hbar)[F, H]$, where $F$ is any operator, we obtain in the Heisenberg picture

$$\dot{\mathbf{v}} = (q/m)(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + (2\mu_e/m\hbar)\nabla(\mathbf{s} \cdot \mathbf{B}), \quad (2a)$$

$$\dot{\mathbf{s}} = (2\mu_e/\hbar)(\mathbf{s} \times \mathbf{B}), \quad (2b)$$

where $\mathbf{s}$ is the spin operator, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, respectively, $\mathbf{v} \equiv \dot{\mathbf{x}}$, and the overdot denotes the total time derivative. Next, letting the number of particles with the expectancy value of the velocity between $\mathbf{v}$ and $\mathbf{v} + d\mathbf{v}$ and spin vector between $\mathbf{s}$ and $\mathbf{s} + d\mathbf{s}$ be given by $dN = f d\mathbf{v} d\mathbf{s}$, we search for an evolution equation for $f$. Noting that $\nabla_s \cdot (\mathbf{s} \times \mathbf{B}) = 0$, particle conservation implies that the phase space density is conserved along fluid elements propagating in the extended phase space, i.e., $\dot{f}(\mathbf{r}, \mathbf{v}, \mathbf{s}, t) = \partial_t f + \dot{\mathbf{r}} \cdot \nabla f + \dot{\mathbf{v}} \cdot \nabla_v f + \dot{\mathbf{s}} \cdot \nabla_s f = 0$. Thus, with Eqs. (2),

$$\partial_t f + \mathbf{v} \cdot \nabla f + \left[\frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \frac{2\mu_e}{m\hbar}\nabla(\mathbf{s} \cdot \mathbf{B})\right] \cdot \nabla_v f + \frac{2\mu_e}{\hbar}(\mathbf{s} \times \mathbf{B}) \cdot \nabla_s f = 0. \quad (3)$$

For the concept of a distribution function in phase space to be well defined, the delocalization of the wave function of the particles cannot be too large. To establish the limit of validity, we therefore would like to connect (3) directly to the Pauli equation. Studying the one-particle equation $i\hbar\partial_t \Psi_\alpha = H\Psi_\alpha$, where $\Psi_\alpha$ is the electron wave function and the subscript $\alpha$ is a particle index, the Pauli equation can be transformed into fluidlike variables by the transformation $\Psi_\alpha = \sqrt{n_\alpha}\exp(iS_\alpha/\hbar)\varphi_\alpha$, where $\varphi_\alpha$ is a unit normalized two-spinor. Defining $\mathbf{v}_\alpha = (1/m)\times(\nabla S_\alpha - i\hbar\varphi_\alpha^\dagger\nabla\varphi_\alpha) - (q/m)\mathbf{A}$ and $\mathbf{s}_\alpha = (\hbar/2)\varphi_\alpha^\dagger\boldsymbol{\sigma}\varphi_\alpha$, the Pauli equation can be rewritten as evolution equations for $n_\alpha$, $\mathbf{v}_\alpha$, and $\mathbf{s}_\alpha$ [5], which have a fluidlike form, although we are still dealing with single particle equations. These equations resemble Eqs. (2a) and (2b) above but contain several other terms due to the difference of a two-spinor from a classical particle with a magnetic moment. In particular, a forcelike term that arises in this way, even in the absence of spin, is the so-called Bohm potential which results in a force $\mathbf{F}_B = (\hbar^2/2m)\nabla[(\nabla^2\sqrt{n_\alpha})/\sqrt{n_\alpha}]$. Assuming a characteristic (thermal) energy of the particles, the effect of the Bohm potential is seen to be small, provided the gradient scale length is longer than the thermal de Broglie wavelength [16]. Similarly, quantum corrections to the Vlasov equation due to wave function dispersion [16] have been shown to be small whenever spatial gradients are long compared to this quantum scale. Furthermore, the spin properties induces higher order force terms $F_j = (1/m)\partial^i[(\partial_j s_k)(\partial_i s^k)]$ [17] that are small (compared to the convective derivative), provided the gradient scale length is longer than the thermal de Broglie wavelength. Finally, higher order terms in the spin evolution equation [17] resulting from the Pauli equation can be neglected when the same condition is fulfilled. The kinetic equation put forward here agrees with the one used by Refs. [14,15], except that their equation did not contain the magnetic dipole force. This term is often small as compared to the Lorentz force, but we will show that it can still be important.

A complete model is formed by combining Eq. (3) with Maxwell's equations, where the current density is

$$\mathbf{j} = \mathbf{j}_{\mathrm{free}} + \nabla\times\mathbf{M} = \sum_i\left[q_i\int\mathbf{v}f_i d\Omega + \frac{2\mu_i}{\hbar}\nabla\times\int\mathbf{s}f_i d\Omega\right]. \quad (4)$$

The sum is over particle species $i$ with charge $q_i$, and the last term is the magnetization current due to the spin. Normally, the spin contribution from the ions can be neglected compared to that of the electrons, due to their smaller magnetic moment. In what follows, we will therefore include only the electron physics and drop the sum over species. The integration $d\Omega$ is made over three velocity variables and two spin degrees of freedom. Since the spin vector is constructed from the expectancy value of the spin operator, it has a fixed length $|\mathbf{s}| = \hbar/2$. The spin orientation will be described using spherical coordinates. From this model it is straightforward to show that the energy conservation law $\partial_t W + \nabla\cdot\mathbf{P} = \mathbf{0}$ is fulfilled, with the energy density $W$ and energy flux $\mathbf{P}$ given by $W = (\varepsilon_0 E^2 + \mu_0^{-1}B^2)/2 - \mathbf{B}\cdot\mathbf{M} + \int(mv^2/2)f d\Omega$ and $\mathbf{P} = \mathbf{E}\times(\mu_0^{-1}\mathbf{B} - \mathbf{M}) + \int[(mv^2/2)\mathbf{v} - (2\mu_e/\hbar)\mathbf{v}(\mathbf{B}\cdot\mathbf{s})]f d\Omega$, respectively. The last term in the energy flux expression represents the convection of magnetic dipole energy.

Next, we will use Eq. (3) to study linear waves in a magnetized plasma. Dividing the variables as $f = f_0(\mathbf{v},\mathbf{s}) + f_1(\mathbf{r},t,\mathbf{v},\mathbf{s})$, and $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1(\mathbf{r},t)$, the linearized Vlasov equation can be written

$$\left[\partial_t + \mathbf{v}\cdot\nabla + \frac{q}{m}(\mathbf{v}\times\mathbf{B}_0)\cdot\nabla_v + \frac{2\mu_e}{\hbar}(\mathbf{s}\times\mathbf{B}_0)\cdot\nabla_s\right]f_1 = -\frac{q}{m}(\mathbf{E} + \mathbf{v}\times\mathbf{B}_1)\cdot\nabla_v f_0 - \frac{2\mu_e}{\hbar}\left[\frac{\nabla(\mathbf{s}\cdot\mathbf{B}_1)}{m}\cdot\nabla_v + (\mathbf{s}\times\mathbf{B}_1)\cdot\nabla_s\right]f_0. \quad (5)$$

Before proceeding further, we will for definiteness specify the unperturbed equilibrium distribution $f_0$. In thermodynamic equilibrium and for a large chemical potential (which applies for $n\lambda_{dB}^3 \gg 1$, where $\lambda_{dB}$ is the thermal de Broglie wavelength and $n$ is the electron number density), the Fermi-Dirac equilibrium distribution reduces to

$$f_0 = \frac{n_0}{4\pi}\left(\frac{m}{2\pi k_B T}\right)^{3/2}\exp\left[-\frac{(mv^2/2 + 2\mu_e\mathbf{s}\cdot\mathbf{B}_0/\hbar)}{k_B T}\right]. \quad (6)$$

This results in a zero order magnetization $\mathbf{M}_0 = n_0\mu_e\eta(\mu_e B_0/k_B T)$, where $\eta$ is the Langevin function, $T$ is the temperature, $n_0$ is the equilibrium density, and $k_B$ is the Boltzmann constant. Next, letting $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$, introducing cylindrical coordinates $(v_\perp, \varphi_v, v_z)$ in velocity space and spherical coordinates $(\varphi_s, \theta_s)$ in spin space, noting that the term $(\mathbf{v}\times\mathbf{B}_1)\cdot\nabla_v f_0$ in (5) can be dropped when the unperturbed distribution function is Maxwellian, and Fourier analyzing, Eq. (5) is written

$$\left[i(\omega - \mathbf{k}\cdot\mathbf{v}) + \omega_c\frac{\partial}{\partial\varphi_v} + \omega_{cg}\frac{\partial}{\partial\varphi_s}\right]\tilde{f}_1 = \left[\frac{q}{m}\tilde{\mathbf{E}} + \frac{2\mu_e}{m\hbar}\nabla(\mathbf{s}\cdot\tilde{\mathbf{B}}_1)\right]\cdot\nabla_v f_0 + \frac{2\mu_e}{\hbar}(\mathbf{s}\times\tilde{\mathbf{B}}_1)\cdot\nabla_s f_0, \quad (7)$$

where we have introduced the frequencies $\omega_c = qB_0/m$ and $\omega_{cg} = 2\mu_e B_0/\hbar$. Note that $\omega_c < 0$ and that $\omega_{cg} = (g/2)\omega_c$. Equation (7) can be solved by an expansion of $\tilde{f}_1$ in the eigenfunctions $\psi_a(\varphi_v, v_\perp) = (2\pi)^{-1/2}\times\exp[-i(a\varphi_v - k_\perp v_\perp\sin\varphi_v/\omega_c)]$. Thus, we let

$$\tilde{f}_1 = \sum_{a,b} g_{ab}(v_\perp, v_z, \theta_s)\psi_a(\varphi_v, v_\perp)\exp(-ib\varphi_s), \quad (8)$$

where $a = 0, \pm1, \pm2, \ldots$ and $b = -1, 0, 1$. Using the orthogonality properties $\int_0^{2\pi}\psi_a\psi_b^* d\varphi_v = \delta_{ab}$, we find

$$i(\omega - k_z v_z - a\omega_c - b\omega_{cg})g_{ab} = I_{ab}(v_\perp, v_z, \theta_s), \quad (9)$$

with

$$I_{ab} = \int_0^{2\pi}\int_0^{2\pi}\left\{\left[\frac{q}{m}\tilde{\mathbf{E}} + \frac{2\mu_e}{m\hbar}\nabla(\mathbf{s}\cdot\tilde{\mathbf{B}}_1)\right]\cdot\nabla_v f_0 + \frac{2\mu_e}{\hbar}(\mathbf{s}\times\tilde{\mathbf{B}}_1)\cdot\nabla_s f_0\right\}\psi_a^*\exp(ib\varphi_s)d\varphi_v d\varphi_s. \quad (10)$$

A useful relation when trying to write results in a more explicit form is the Bessel expansion

$$\psi_a(\varphi_v, v_\perp) = \frac{1}{\sqrt{2\pi}}\sum_b J_b\left(\frac{k_\perp v_\perp}{\omega_c}\right)\exp[i(b-a)\varphi_v]. \quad (11)$$

Here it is seen that the results are much simplified in the limit where $k_\perp v_{\rm th}/\omega_c$ is small (where we can estimate $v_\perp$ with the thermal velocity $v_{\rm th}$), but as is well known (see, e.g., Ref. [18]), in general, the conductivity tensor components turns into sums over Bessel functions. The conductivity tensor $\sigma^{ij}$, defined by $j^i = \sigma^{ij}E_j$, is found from Eq. (4) by expressing the magnetic field in terms of $\tilde{\mathbf{E}}$ and then solving for $\tilde{f}_1$ in terms of $\tilde{\mathbf{E}}$ using the eigenfunctions as outlined above. It is illustrative to divide the conductivity tensor into two contributions $\sigma^{ij} = \sigma^{ij}_{\rm free} + \sigma^{ij}_{\rm magn}$ due to the free current and the magnetization current. Furthermore, $\sigma^{ij}_{\rm free}$ could be divided further into contributions due to the Lorentz force and contributions in $I_{ab}$ that contain spin which give the classical and spin parts of $\sigma^{ij}_{\rm free}$. However, we will not present the general expressions for $\sigma^{ij}$ here, as these results are complicated and need extensive analysis for a useful interpretation. Instead, we focus on special cases in order to point out two of the main new features resulting from the spin.

First, the factor $(\omega - k_z v_z - a\omega_c - b\omega_{cg})$ in (9) reveals that the standard wave-particle resonances are extended to involve the spin forces. The spectrum of resonances is thereby much extended. In particular, the combination $a = 1$, $b = -1$ gives a resonant velocity at $v_z = (\omega - \Delta\omega_c)/k_z$, where we have introduced $\Delta\omega_c = (g/2 - 1)eB_0/m$. The physics of this resonance is slightly more complicated than the well-known resonances with $b = 0$. In particular, it does not occur for strictly parallel propagation to $\mathbf{B}_0$, since the coefficient $I_{1-1}$ in (10) becomes zero in that limit. Furthermore, inclusion of the spatial variations of the wave perpendicular to the magnetic field must include a finite argument of the Bessel functions in (11). This means that the spin resonances depend on finite Larmor radius effects. Moreover, integration over $\varphi_s$ destroys any contribution from this resonance in the free current density, and thus only the magnetization current survives. Still it is clear that such resonances can be the dominating wave-particle damping mechanism, in case the wave frequency is of the order of a few thousands of the cyclotron frequency.

The second point to be pursued in more detail is that the spin gives rise to new wave modes not present in the standard Vlasov picture. First, we assume that the ions are immobile, constituting a neutralizing background. Next, we consider the limit of exactly perpendicular propagation and thus let $\mathbf{k} = k_\perp\hat{\mathbf{x}}$. In this case, modes with the approximate polarization $\tilde{\mathbf{E}} = \tilde{E}_z\hat{\mathbf{z}}$ and $\tilde{\mathbf{B}} = \tilde{B}_y\hat{\mathbf{y}} = (k_\perp\tilde{E}_z/\omega)\hat{\mathbf{y}}$ are possible, provided $\sigma^{xy}$ and $\sigma^{xz}$ are sufficiently small, which can be verified *a posteriori* (see, e.g., Ref. [18]). From Eqs. (8)–(10) the distribution function is expressed in terms of the electric field that together with (4) determines the conductivity tensor components. For the given polarization, only the $\sigma^{zz}$ component is needed. Furthermore, for $\omega \ll |\omega_c|$, terms with coefficients $a \neq b$ in (8) will be smaller than those with $a = b$, and thus only the latter are kept. Carrying out the $\varphi_s$ and $\varphi_v$ integrations [using (11)], the $z$ component of Ampere's law gives

$$\omega^2 = k^2c^2 + \omega_p^2\int\left\{J_0^2(k_\perp v_\perp/\omega_c) + \frac{k^2\hbar^2\Delta\omega_c\sin^2\theta_s}{4m(\omega - \Delta\omega_c)k_BT}\times[J_1^2(k_\perp v_\perp/\omega_c)]\right\}f_0 d\Omega, \quad (12)$$

where $\omega_p = (n_0e^2/\epsilon_0 m)^{1/2}$ is the plasma frequency. If we drop the term proportional to $J_1^2$ due to the spin in (12), we have the usual ordinary mode, where higher terms in the sum over Bessel functions have been dropped, due to the condition $\omega \ll |\omega_c|$. Typically, the second term due to the spin is a small correction when the wavelength is longer than the thermal de Broglie wavelength. However, it can clearly be dominant for frequencies close to the resonance $\omega \approx \Delta\omega_c$. In Fig. 1, numerical solutions of the dispersion relation is plotted for different parameters $\zeta$ and $\eta$ close to this resonance, where $\zeta = \hbar^2\omega_c^2/m^2v_t^4$, $\eta = \omega_c^2c^2/\omega_p^2v_t^2$, and $v_t = (k_BT/m)^{1/2}$. To some extent these new solutions resembles the well-known Bernstein modes (see, e.g., Ref. [18]). However, unless the ratio $\zeta/\eta$ is larger than

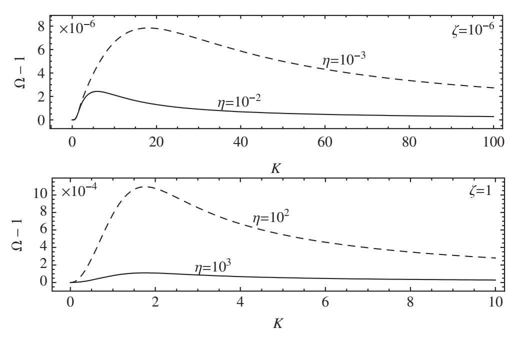


FIG. 1. The normalized frequency $\Omega = \omega/\Delta\omega_c$ plotted around $\Delta\omega_c$ as a function of the normalized wave number $K = kv_t/\omega_c$ for different parameter values of the parameters $\zeta = \hbar^2\omega_c^2/m^2v_t^4$ and $\eta = \omega_c^2c^2/\omega_p^2v_t^2$. The upper panel represents possible laboratory values (for example, $B_0 = 6.0$ T, $T = 10^4$ K, and $n = 10^{22}$ cm$^{-3}$ correspond to $\zeta = 10^{-6}$ and $\eta = 10^{-2}$, relevant for laser-plasma interaction experiments), while the lower are representative for extreme astrophysical environments (for example, $B_0 = 6.0\times10^6$ T, $T = 10^{10}$ K, and $n = 3\times10^{21}$ cm$^{-3}$ correspond to $\zeta = 1$ and $\eta = 10^{-3}$, relevant for a thick hot accretion disk surrounding a pulsar).

unity (corresponding to a high density low temperature plasma), the deviation from the resonant frequency will be small. For an unperturbed distribution different from the thermodynamic equilibrium expression, the shift from the precise resonance can be much enhanced, since the two last terms in (10) contributing to $I_{1-1}$ almost cancel (the terms match as $g/2 - 1$) for $f_0$ given by (6) as considered here but not for a general equilibrium distribution. Furthermore, it should be noted that this approximate cancellation in (10) does not occur for general angles of propagation. For arbitrary angles of propagation, the denominator $\omega - \Delta\omega_c$ of our spin term is replaced by $\omega - k_z v_z - \Delta\omega_c$. It is interesting to note that the sign of this contribution is determined by the last term in (10). In particular, in case the unperturbed distribution $f_0$ has a larger fraction of particles in the higher energy spin state, $\partial f_0/\partial s_z$ changes sign, which should consequently change the sign of the imaginary part of the dispersion relation, leading to instability rather than wave-particle-spin damping. However, more analysis is needed to definitely confirm this conjecture.

In the present Letter, we have put forward a Vlasov-type equation, in a phase space extended to include the spin degrees of freedom. This equation extends a previous equation due to Refs. [14,15], by including the magnetic dipole force associated with the spin, and can be viewed as a semiclassical limit of the Pauli equation. Including the spin-magnetization current in Maxwell equations, the resulting system is shown to be energy conserving. To gain some understanding of the model, we have outlined the theory for linear wave propagation in magnetized plasmas and demonstrated the appearance of new wave modes due to the spin. These wave modes depend on resonances associated with both the orbital and the spin gyration and are much different from the well-known spin waves in ferromagnetic materials [19].

It is of interest to discuss the connection between our spin-kinetic model and the ordinary Vlasov equation. Relating $\mathbf{E}$ and $\mathbf{B}$ through Faraday's law, we see that the relative strength of the spin force is $k^2\hbar/m\omega$. This parameter is often small, which explains why the classical Vlasov equation in many cases is a very good approximation. On the other hand, using this parameter, it is easy to underestimate the significance of the spin force. First, wave-particle resonances and/or new wave modes associated with resonances, as the one described above, may enhance the significance of spin effects. Second, when the electric field is perpendicular to $\mathbf{B}_0$ and the wave frequencies are low, the $\mathbf{E} \times \mathbf{B}$ motion of electrons and ions gives a zero current to leading order in $\omega/\omega_c$, whereas similar cancellations do not occur for the magnetic dipole force. Furthermore, even if the electric field is perpendicular to $\mathbf{B}_0$, the dipole force may have a parallel component leading to a large current in this direction.

The exploration of Eq. (3) is a very rich problem. Besides the more obvious generalization to a complete linear theory, including arbitrary angles of propagation, there is a large set of nonlinear problems that should be studied. The results in Ref. [8] indicate that the significance of the spin force can be increased in the nonlinear regime as compared to the linear one, when the spin dependence of the ponderomotive force can separate particles of different spin. Finally, it should be stressed that the current model is intended for the ''weak quantum'' regime where the characteristic length scale is longer than the thermal de Broglie wavelength and the Zeeman energy is smaller than the thermal energy. A very interesting problem is generalizations to the regime where the Zeeman energy is comparable or larger than the thermal energy. For such parameters, found in astrophysical settings, effects such as Landau quantization [20] will enter the picture.

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