Is the Spectrum of Highly Excited Mesons Purely Coulombian?

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We show that a static central potential may provide a precise description of highly excited light unflavored mesons. Because of string breaking, this potential becomes of chromoelectric type at sufficiently large quark-antiquark distances giving rise to a Coulombian spectrum. The same conclusion can be inferred for any other meson sector through a straightforward extension of our analysis.

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In the last years, the interest in the highly excited light-quark meson spectrum has been renewed as a consequence of the observation of more than thirty new meson resonances with masses between 1.9 and 2.4 GeV in the exhaustive analyses of Crystal Barrel and PS172 data [1]. Many of the reported new resonances are listed by the Particle Data Group (PDG) [2] in the section "Other Light Unflavoured Mesons" awaiting for confirmation from a separate experiment. Regarding the non-new resonances also found in [1], they are in perfect correspondence (quite similar masses) with the ones listed in the "Light Unflavoured Mesons" section of the PDG. From the theoretical point of view, the extensive spectrum up to 2.4 GeV has made clear an approximate hydrogenlike classification of meson states [3]. In this article, we show through a dynamical study that the physical origin of the hydrogenlike degeneracy may have to do with string breaking once the quark-antiquark average distance in a meson reaches a sufficiently high value. We shall rely on a Constituent Quark Model (CQM) calculation by solving the Schrödinger equation for a static potential. Although the use of a static potential for light-quark systems is in general debatable, we shall justify its applicability for highly excited states. Then, in the spirit of CQM calculations, we shall assume that relativistic corrections to the Schrödinger equation may be to a good extent taken into account through the effective character of the parameters of the potential.

From lattice QCD, the static potential is the ground state energy of a bound state composed of two static color sources and gluons [4]. In the so-called quenched approximation [only valence quark (q) and antiquark ($\bar{q}$)], the static $q\bar{q}$ potential, given by the expectation value of the Wilson loop operator, resembles a funnel potential containing a confining term depending linearly on r, the $q-\bar{q}$ distance, plus a Coulombian term proportional to $(1/r)$. When including sea quarks, i.e., in the so-called unquenched approximation, the static sources q and $\bar{q}$ are screened by light quark-antiquark pairs that pop out of the vacuum. Then, transitions between the static sources state (string) and mesons coming from the recombination of the static sources and the members of the pairs may take place. Two physical effects occur. On the one hand, the confining term becomes a constant (string breaking) at a certain saturation distance, r_s. Although this behavior has not been detected with the Wilson loop technique, the finite temperature potential extracted from Poliakov line correlators at temperatures close to the deconfinement phase transition exhibits a flattening once sea quarks are included into the action [5]. Such a flattening occurs at distances $r_s \simeq 1.15$ fm. The diagonalization of a two-by-two correlation matrix between the string state and a two meson state (each meson formed by one static source and one member of a pair) should confirm it. On the other hand, the Coulomb strength remains higher than in the quenched case. From these indications, we shall assume a static potential of the form

$$
\begin{aligned}
V(r) &= \sigma r - \frac{k}{r} + C \qquad \text{if } r \le r_s \\
V(r) &= \sigma r_s - \frac{k}{r} + C \qquad \text{if } r \ge r_s
\end{aligned}
\tag{1}
$$

where σ stands for the string tension, k for the Coulomb strength, and C for a constant to fix the origin of the potential.

We shall choose this form for the effective $q\bar{q}$ potential in our CQM calculation. Let us recall that in quark models, the effective $q\bar{q}$ potential is found by equating the scattering amplitude of free quark and antiquark with the potential between bound quark and antiquark inside a meson. Thinking of a single exchange diagram the static limit corresponds to no energy transfer, i.e., to $q^0 \equiv (E_q)_{\text{initial}} - (E_q)_{\text{final}} = (E_{\bar{q}})_{\text{final}} - (E_{\bar{q}})_{\text{initial}} = 0$. So the static approximation means $q^0 \simeq 0$. Then, by substituting $E_q = m_q\sqrt{1 + (\vec{p}^2{}_q/m_q^2)}$, we can easily establish as a criterium for the validity of such approximation the requirement

$$
\frac{[(\vec{p}^2{}_q)_{\text{initial}} - (\vec{p}^2{}_q)_{\text{final}}]}{m_q^2} \ll 1. \tag{2}
$$

By replacing $(\vec{p}_q)_{\text{initial}} = \vec{q} + (\vec{p}_q)_{\text{final}}$, the criterium (2) can be written as

$$\frac{\vec{q}^2 + 2\vec{q}.(\vec{p}_q)_{\rm final}}{m_q^2} \ll 1. \qquad (3)$$

For nonrelativistic as well as for relativistic systems, (3) is satisfied when

$$\frac{|\vec{q}|}{m_q} \ll 1 \quad \text{and} \quad \frac{|\vec{p}_q|}{m_q} \not\gg 1. \qquad (4)$$

Since the main contributions to the interaction come from distances $r \approx 1/|\vec{q}|$, we expect the mesons satisfying (4) to have root mean square radius (rms-radius)

$$\langle r^2\rangle^{1/2} \simeq \frac{1}{|q|} \gg \frac{1}{m_q}. \qquad (5)$$

Notice that for bottomonium ($b\bar{b}$) and charmonium ($c\bar{c}$), the conditions (5) and $|\vec{p}_q|/m_q \not\gg 1$ are well satisfied according to Ref. [6] where a good description of such mesons is attained by means of a quenched potential. The validity of the quenched approximation in this case can be related to the fact that for $b\bar{b}$ and $c\bar{c}$, the saturation distance r_s may be significantly larger than 1.15 fm. The screening effect on $b\bar{b}$ and $c\bar{c}$ has been studied in the literature [7]. From these studies, a value of $(r_s)_{b\bar{b},c\bar{c}}$ up to 1.8 fm can be conjectured. This value may be reflecting the fact that the $b\bar{b}$ and $c\bar{c}$ decays into stable hadrons involve more than two mesons. Then, it can be explained why the predicted $b\bar{b}$ and $c\bar{c}$ spectra involving states with $\langle r^2\rangle^{1/2} \lesssim (r_s)_{b\bar{b},c\bar{c}}$ hardly change from the unquenched to the quenched approximation. The experimental extension of the spectra to states with larger rms-radii is essential to confirm or refute this conjecture.

Here we center on light-quark mesons. We shall restrict for simplicity to isospin $I = 1$ mesons [they contain only u ($\bar{u}$) and/or d ($\bar{d}$)]: π_J, $\mathbf{b}_J$, ρ_J, and $\mathbf{a}_J$ (J: total angular momentum). Thus, we avoid all possible complications coming from $q\bar{q}$ annihilation and from $s\bar{s}$ components. The mass of the constituent quarks $m_{u(\bar{u}),d(\bar{d})}$, named henceforth m_u, is a parameter of our model. We shall fix its value from the average dynamical quark mass generated by Spontaneous Chiral Symmetry Breaking (SCSB), $m_u(|\vec{q}|)$, in the energy region under consideration. From instanton model calculations [8] confirmed by lattice QCD [9], we know the explicit $m_u(|\vec{q}|)$ dependence so that it has its maximum value at $|\vec{q}| = 0$ [$m_u(0) \simeq 0.350$ GeV] and decreases when increasing $|\vec{q}|$. For $|\vec{q}| = 0.1$ GeV, for instance, one has $m_u(0.1 \text{ GeV}) = 0.332$ GeV. Therefore, (4) tells us that the static approximation might only be applied for $|\vec{q}| < 0.1$ GeV. Then, we shall use $m_u = 340$ MeV as an average mass in this interval for which we expect from (5) to have mesons with $\langle r^2\rangle^{1/2} \gg 0.6$ fm. It should be added that SCSB has another important effect: the appearance of Goldstone bosons. This effect is not explicitly reflected in (1). We shall comment on this later.

To check the applicability of the static approximation, we proceed to calculate the spectrum of $I = 1$ mesons with an effective potential of the form (1). For this purpose, we have to fix the parameters of the potential. For $(r_s)_u$, we take 1.15 fm. For the string tension σ_u, we shall use the value $\sigma_u = 932.7$ MeV/fm obtained from the Regge trajectory for ρ_J and $\mathbf{a}_J$ (see for instance [4]) in accord with lattice evaluations. Regarding k_u and C_u, we shall fix them by fitting the average masses of the experimental states with higher orbital angular momentum L since we expect these states to have large rms-radii due to the centrifugal barrier. Let us realize that the calculated masses coming out from the Schrödinger equation will only depend on L ($L = 0, 1, 2, 3\ldots$) and on the radial quantum number n_r ($n_r = 1, 2, 3\ldots$). We shall denote them as M_{L,n_r}. So we should compare them to the average masses of experimental (L, n_r) multiplets, $[(M)_{L,n_r}]_{\rm Exp}$. The existence of such multiplets has been suggested elsewhere [3]. Actually, the assumption of a long distance interaction depending only on r drives naturally to a $SU(4)_{\text{Spin-Isospin}} \times O(3)$ group of symmetry so that the $I = 1$ light unflavored mesons belong to 15-plets [note that the product of $SU(4)$ quark and antiquark representations is $4 \times \bar{4} = 15 + 1$; the singlet representation 1 contains only $I = 0$ mesons that we do not consider]. Indeed, we should better talk about super 15-plets since for each member of the multiplet, there are so many experimental states as possible different J values (J degeneracy). As we consider only $I = 1$ mesons, we define their average mass in the corresponding 15-plet (L, n_r) as

$$(M_{L,n_r})_{\rm Exp} = \left(\sum_J (2J+1)\right)^{-1} \sum_{X,J} (2J+1) X_J \qquad (6)$$

being X_J the experimental masses assigned to the multiplet ($X_J \equiv M_{\pi_J}$, M_{ρ_J} or $M_{\mathbf{b}_J}$, $M_{\mathbf{a}_J}$).

The maximum value of L for which we have some candidate from the PDG catalog (the section "Light Unflavoured Mesons") or from Ref. [1], named henceforth CBC (for Cristal Barrel Collaboration), is $L = 5$. In fact, there is only one candidate, the PDG resonance $\mathbf{a}_6(2450 \pm 130)$. Although the error bar is big, the existence of a nonconsidered $I = 0$ PDG resonance $\mathbf{f}_6(2465 \pm 50)$ that can be assigned to the same 15-plet ($L = 5$, $n_r = 1$) makes us confident about the average PDG mass. For ($L = 4$, $n_r = 1$), we also have only one PDG resonance, $\rho_5(2330 \pm 35)$ but a complete set of CBC candidates [$\pi_4(2250 \pm 15)$, $\rho_3(2260 \pm 20)$, $\rho_4(2230 \pm 25)$, $\rho_5(2300 \pm 45)$] (an explanation for the PDG-CBC difference in mass for ρ_5 is given in Ref. [1]). Let us also notice the existence of $I = 0$ CBC resonances, ω_3, ω_4, and ω_5 at about the same mass.

By choosing $k_u = 2480$ MeV $\cdot$ fm and $C_u = 1070$ MeV, we reproduce correctly their average masses [for ($L = 4$, $n_r = 1$) a value in between the PDG and CBC averages is chosen]. It is noteworthy that the calculated rms-radii for the fitting states (3.7 and 2.6 fm) consistently satisfy $\langle r^2\rangle^{1/2} \gg 0.6$ fm. Moreover, the calculated values of $|\vec{p}_q|/m_q$ (1.05 and 1.3), although indicating the relativistic character of quark and antiquark, are not much greater than 1, as required from (4).

The results for the spectrum of states with $\langle r^2\rangle^{1/2} >$ 2.0 fm, for which we expect the static approximation may work (all the states have $|\vec{p}_q|/m_q < 1.7$), are shown in Table I (we include for completeness multiplets with rms-radii below 2.0 fm as (2, 2) and (3, 1). Our results are compared to CBC and PDG average masses. The states entering in the calculation of the averages are specified. The multiplets (1, 4) and (1, 3) lack an $\mathbf{a}_0$ despite having an available candidate $\mathbf{a}_0(2025)$. The reason for not including this state is, apart from the ambiguity in its assignment, the general deficient description of a_0 states provided by quark models pointing out the need to incorporate more than two valence components. It should be noted additionally that $\rho_3(2260 \pm 20)$ has been assigned to two multiplets (4,1) and (2,3). The reason for this double assignment is the assumption that the ρ_3 resonances belonging to such multiplets would be, as indicated by their partners in the multiplets, almost degenerate. All the multiplets considered have at least one PDG cataloged state or one resonance rated at least three stars in Ref. [1]. The multiplet (0,4) has not been considered since the only trustable assignment to it, the CBC $\pi(2070)$, has only a two-star rating (let us point out that if we assigned the PDG $\rho(1900)$ to this multiplet, our result would be in perfect accord with data). As can be checked, our results seem to agree with data for $\langle r^2\rangle^{1/2} > 2.0$ fm. To be more precise, we can rely on the approximate linearity and equidistance of Regge trajectories satisfied by data up to a mass of 2.3 GeV [1,3,10] with a standard Regge slope of about 1.1 GeV^2. From our results for (1, 3), (2, 3), a Regge slope of 1.4 GeV^2, far above the standard value, would be obtained for the corresponding $(L, 3)$ trajectories. We interpret this as an indication that the static approximation is doubtful for the (1, 3) state. Let us also note that a Regge slope of 0.79 GeV^2, quite below the standard value, would be obtained from our chosen masses for (4, 1) and (5, 1) in the $(L, 1)$ trajectories indicating that the calculated spectrum tends to a Coulombian one when increasing the energy. This is clearly indicated by the almost mass degeneracy for states with the same values of $(L + n_r)$ for $(L + n_r) > 6$ ($\langle r^2\rangle^{1/2} > 4.0$ fm). Then, we can give a closed formula for the mass of highly excited mesons

$$(M_{L,n_r})_{(L+n_r)\geq 6} \simeq m_u + m_{\bar{u}} + \sigma_u(r_s)_u - \frac{\mu}{2}\frac{k_u^2}{(L+n_r)^2} + C_u \tag{7}$$

where $\mu = m_u/2$ is the reduced mass of the system. The predicted masses for multiplets with the same value of $(L + n_r)$ are listed in Table II up to $(L + n_r) = 9$. Let us also remark the accidental degeneracy between positive $(+)$ and negative $(-)$ parity states with $L(+) - L(-) =$ odd $= n_r(-) - n_r(+)$.

Moreover, as the Coulombian energy [fourth term on the right hand side of (7)] has 0 as an upper bound, we predict that the $I = 1$ light unflavored meson spectrum has an upper bound or limiting mass given by

$$M_{\text{Limit}} \simeq m_u + m_{\bar{u}} + \sigma_u(r_s)_u + C_u = 2823 \text{ MeV}. \tag{8}$$

TABLE I. Calculated masses (column III) and rms-radii (column II) for (L, n_r) multiplets (column I) from the set of parameters $m_u = 340$ MeV, $\sigma_u = 932.7$ MeV/fm, $k_u =$ 2480 MeV · fm, and $C_u = 1070$ MeV. Experimental CBC and PDG average masses (columns IV and V) are shown for comparison. In both cases, the candidates to be members of the multiplets are indicated. The superindex † in the (5,1) and (4,1) calculated masses indicates the average mass values chosen in the corresponding multiplets to fix the parameters.

(L, n_r)	$\langle r^2\rangle^{1/2}$ fm	M_{L,n_r} MeV	$[(M)_{L,n_r}]_{\text{CBC}}$ MeV	$[(M)_{L,n_r}]_{\text{PDG}}$ MeV
(5,1)	3.7	2450†		2450 ± 130 $a_6(2450)$
(1,4)	3.4	2255	2219 ± 43 $b_1(2240)$ $a_1(2270)$, $a_2(2175)$	
(2,3)	3.2	2254	2248 ± 37 $\pi_2(2245)$, $\rho(2265)$ $\rho_2(2225)$, $\rho_3(2260)$	$\rho_3(2250)$
(3,2)	2.9	2258	2258 ± 38 $b_3(2245)$, $a_2(2255)$ $a_3(2275)$, $a_4(2255)$	
(4,1)	2.6	2283†	2262 ± 28 $\pi_4(2250)$, $\rho_3(2260)$ $\rho_4(2230)$, $\rho_5(2300)$	2330 ± 35 $\rho_5(2350)$
(1,3)	2.1	1919	1947 ± 47 $b_1(1960)$ $a_1(1930)$, $a_2(1950)$	
(2,2)	1.9	1913	1980 ± 23 $\pi_2(2005)$, $\rho(2000)$ $\rho_2(1940)$, $\rho_3(1982)$	$\rho_3(1990)$
(3,1)	1.6	1937	2023 ± 24 $b_3(2032)$, $a_2(2030)$ $a_3(2031)$, $a_4(2005)$	2010 ± 12 $a_4(2040)$

TABLE II. Predicted masses for some (L, n_r) multiplets with $L + n_r > 6$. Parameters as in Table I.

$(L + n_r)$ (L, n_r)	m_{L,n_r}(MeV)
6 (0,6), (1,5), (2,4), (3,3), (4,2)	2450
7 (0,7), (1,6), (2,5), (3,4) (4,3), (5,2), (6,1)	2548
8 (0,8), (1,7), (2,6), (3,5) (4,4), (5,3), (6,2), (7,1)	2613
9 (0,9), (1,8), (2,7), (3,6), (4,5) (5,4), (6,3), (7,2), (8,1)	2657

This limit is compatible with current data since reported resonances with higher mass listed in the PDG section "Other Light Unflavored Mesons" may be assigned to mesons containing $s\bar{s}$. Although a specific analysis parallel to the one just performed would be required for these states, we can expect their limiting mass to increase with respect to the value in (8) by at least an amount $[(m_s - m_u) + (m_{\bar{s}} - m_{\bar{u}})] \sim 300$–$500$ MeV. Then, the resulting limit ($\lesssim 3300$ MeV) would be compatible with all existing light unflavored meson candidates. Beyond this limit, one meson states cannot exist. Instead, the system fragments into several mesons.

Furthermore, a mass limit for baryons containing only quarks u and d may be derived through the simple prescription $(M_B)_{\rm Limit} \simeq 3m_u + (3/2)[\sigma_u (r_s)_u + C_u] = 4213$ MeV. This value is consistently above the most massive reported nucleon and delta resonances in the PDG sections $N(\sim 3000$ Region) and $\Delta(\sim 3000$ Region).

With respect to the fitted values of the parameters, some comment is in order. Since we are dealing with $q - \bar{q}$ distances much larger than M_{π}^{-1}, we do not expect significant Goldstone boson contributions. Then, the parameters may be incorporating mainly gluon contributions (apart from possible relativistic quark kinetic corrections). So the Coulomb strength, k, can be tentatively related to an effective quark-antiquark-gluon coupling, α_s, through the color relation $k = (4/3)\alpha_s$. Then, from the fitted value of $k = 2480$ MeV $\cdot$ fm, we get $\alpha_s(|\vec{q}^2| < 0.01 \text{ GeV}^2) = 9.4$. It is interesting to realize that this value for α_s is precisely the same reported in Ref. [11] from a solution of the truncated Schwinger Dyson equations. However, this "coincidence" should be taken with caution since a smaller value has been obtained in other calculations [12]. As a matter of fact, the only general conclusion we may extract from the work of all the groups is the almost constancy of α_s for sufficiently low $|\vec{q}^2|$. Undoubtedly, the validity of (1) for highly excited light-quark mesons has much to do with this constancy of α_s for $|\vec{q}^2| < 0.01$ GeV^2.

To summarize, our study of highly excited $I = 1$ light unflavored mesons shows that their spectrum could be purely Coulombian. As a consequence, it would have a limiting mass. In fact, the idea that the meson spectrum might have an upper bound is not new. A mass limit of 3.2 GeV for light-quark mesons involving $u\bar{u}$, $d\bar{d}$, as well as $s\bar{s}$ components was suggested some years ago from a study of Regge trajectories [13]. Our dynamical analysis supports such suggestion confirming string breaking as the underlying physical mechanism. However, the lack of trustable experimental data beyond 2.5 GeV leaves opened the door to other interpretations. For instance, one could think that the observed flattening of the confining potential corresponded indeed to a severe softening of the confining interaction in the region under study. Then, the good effective description achieved would be compatible with a very slight increase of the interaction with the distance and consequently with an unbound meson spectrum. Moreover, the nonrelativistic hydrogenlike symmetry we have made dynamically evident may be only the effective face of a broader relativistic symmetry in the energy region under consideration as discussed in [3]. In our model, the higher the meson mass, the lesser the $|\vec{p}_q|/m_q$ value and the less relativistic the system (when approaching the limiting mass $|\vec{p}_q|/m_q \to 0$) implying a nonrelativistic Coulombian symmetry at the long range. Consistent relativistic calculations beyond our effective CQM treatment could shed more light on this point.

Let us also emphasize that the analysis we have performed may be extended to any other meson sector although the current lack of data may make it not feasible. Then, a definite answer to the general question about the Coulombian nature of the highly excited meson spectrum has to be postponed until more complete data are available. We encourage an experimental effort along this line. In the meantime, we hope our results may be suggestive and motivate further studies in the field.

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