Combination of Bloch Oscillations with a Ramsey-Bordé Interferometer: New Determination of the Fine Structure Constant

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We report a new experimental scheme which combines atom interferometry with Bloch oscillations to provide a new measurement of the ratio h/m_{Rb}. By using Bloch oscillations, we impart to the atoms up to 1600 recoil momenta and thus we improve the accuracy on the recoil velocity measurement. The deduced value of h/m_{Rb} leads to a new determination of the fine structure constant $\alpha^{-1} = 137.035\,999\,45\,(62)$ with a relative uncertainty of 4.6×10^{-9}. The comparison of this result with the value deduced from the measurement of the electron anomaly provides the most stringent test of QED.

The fine structure constant α characterizes the strength of the electromagnetic interaction. It is a cornerstone of the adjustment of the fundamental constants [1] and its determination is made in different domains of physics. The most precise values of α are deduced from the measurements of the electron anomaly a_e made in the 1980s by Dehmelt at the University of Washington [2], and, recently, by Gabrielse at Harvard University [3]. This last measurement and an impressive improvement of quantum electrodynamics (QED) calculations [4] gives a value of α with a relative uncertainty of 3.7×10^{-10} which surpasses Dehmelt's result by 1 order of magnitude. Nevertheless, this recent determination of α relies on very difficult QED calculations. To test them, other determinations of α, independent of QED, are required. The most precise are deduced from the measurement of the ratio h/m between the Planck constant and the mass of an atom thanks to the relation deduced from the ionization energy of hydrogen:

$$\alpha^2 = \frac{2R_\infty}{c} \frac{m}{m_e} \frac{h}{m}, \qquad (1)$$

where m_e is the electron mass. The limiting factor is the ratio h/m: the uncertainty of the Rydberg constant R_∞ is 7×10^{-12} [5,6] and that of the mass ratio m/m_e 4.8×10^{-10} [1,7]. This principle has been used on Cs atom by Chu and colleagues with an atom interferometer to measure the recoil velocity $v_r = \hbar k/m$ of the atom when it absorbs a photon of momentum $\hbar k$ and to deduce α with a relative uncertainty of 7.4×10^{-9} [8]. In 2005, our group used the Bloch oscillations (BO) of Rb atoms in an optical lattice to transfer a large number of photon momenta to the atoms: we achieved a precise measurement of atomic recoil velocity and a determination of α with a relative uncertainty of 6.7×10^{-9} [9,10].

In this Letter, we present a combination of the two methods: we use BO to transfer a large number of photon momenta and a Ramsey-Bordé interferometer to precisely measure the induced atomic velocity variation. This leads to a value of α with a relative accuracy of 4.6×10^{-9}. The comparison of this result, which is very slightly dependant on QED, with the value of α deduced from the electron anomaly is the most stringent test of the QED calculations.

Bloch oscillations have been first observed in atomic physics by the groups of Salomon and Raizen [11,12]. The atoms are placed in an optical lattice by shining on them two counterpropagating laser beams whose frequency difference is swept linearly. The atoms undergo a succession of Raman transitions which correspond to the absorption of one photon from a beam and a stimulated emission of a photon to the other beam. The internal state is unchanged while the atomic velocity increases by $2v_r$ per oscillation. The Doppler shift due to this velocity variation is periodically compensated by the frequency sweep and the atoms are accelerated. This method is very efficient to transfer to the atoms a very large number of photon momenta [13]. Another point of view is to consider that the atoms are placed in a standing wave which is accelerated when the frequency difference between the two laser beams is swept. In the atomic frame, the atoms undergo an inertial force. This system is analog to the BO of an electron in a solid submitted to an electric field [11]. There is a similar context when an atom is placed in a vertical standing wave in the gravitational field. This situation has been already investigated for measuring the local acceleration of gravity [14–16]. It has also been suggested to use BO to probe forces near surfaces at a microscopic scale [17,18].

In our experiment BO are combined with a Ramsey-Bordé interferometer [19] following the scheme of Fig. 1. BO are inserted between two pairs of $\pi/2$ laser pulses. Each pulse drives a Doppler-sensitive Raman transition between the two hyperfine ground states labeled $|g\rangle$ and $|e\rangle$ ($F = 2$ and $F = 1$ in the case of ^{87}Rb). The first pair of $\pi/2$ pulses creates the two coherent atomic wave packets in internal state $|e\rangle$. We then blow away the atoms remain-
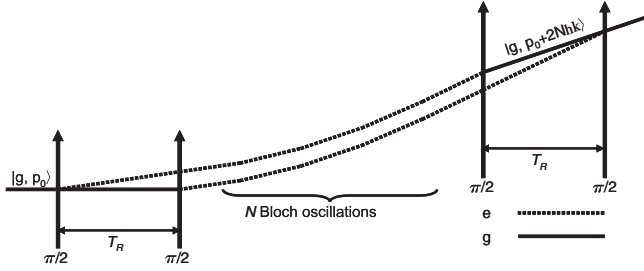
FIG. 1.   Interferometric scheme combining Ramsey-Bordé interferometer and N Bloch oscillations.

ing in the initial state $|g\rangle$ by using a laser beam tuned to a single photon transition. Both wave packets are accelerated by N BO. The second $\pi/2$ pair recombines the two atomic wave packets and reads out the phase difference between the two paths. The interferometer can also be understood in momentum space. After the first pair of $\pi/2$ pulses, the shape of the velocity distribution of the atoms in the level $|e\rangle$ is a Ramsey fringe pattern. The fringe spacing varies as $1/T_R$ (T_R is the delay between the two $\pi/2$ pulses). These atoms are adiabatically loaded into the first Brillouin zone of the optical lattice. Following the Bloch formalism, if the atom has a well-defined momentum $\hbar q_0$ with $|q_0| < k$, the atomic wave function is modified when the optical potential is increased adiabatically (without acceleration) and becomes in the first energy band [10]:

$$|\Psi_{0,q_0}\rangle = \sum_l \phi_0(q_0 + 2lk)|q_0 + 2lk\rangle \qquad (2)$$

with $l \in \mathbb{Z}$. Here $|q_0\rangle$ designs the wave function associated to a plane wave of momentum q_0 and the amplitudes ϕ_0 correspond to the Wannier function [20] in momentum space of the first band. When the potential depth is close to zero, the limit of the Wannier function ϕ_0 is 1 over the first Brillouin zone and zero outside. On the contrary, if the potential depth is large, the Wannier function selects several components in the velocity space. When the optical lattice is accelerated adiabatically, the Wannier function is continuously shifted in the momentum space following the relation:

$$|\Psi(t)\rangle = \sum_l \phi_0(q_0 + 2lk - mv(t)/\hbar)|q_0 + 2lk\rangle \qquad (3)$$

where $v(t)$ is the velocity of the optical lattice. The enveloping Wannier function ϕ_0 is shifted by $mv(t)$ in momentum space. After the acceleration, the potential depth is decreased adiabatically and, in Eq. (4), the Wannier function selects only one component of the velocity distribution. At the end, the wave function is $|\Psi\rangle = |q_0 + 2Nk\rangle$. If Δv is the velocity variation due to the acceleration, the number of Bloch oscillations N is such as $|\hbar q_0 + m\Delta v - 2N\hbar k| < \hbar k$. Consequently, if the initial atomic velocity distribution fits the first Brillouin zone, it is exactly shifted by $2Nv_r$ without deformation. The second pair of $\pi/2$

laser pulses is used to read out the accelerated velocity distribution. By scanning the frequency of the second pair of $\pi/2$ pulses, we realize the convolution of this distribution by a second Ramsey fringe pattern. The shift of the resonance is $2Nkv_r$. The resolution of the recoil measurement is inversely proportional to the spacing between Ramsey fringes and proportional to the number $2N$ of transferred recoils.

The experimental setup has been described previously [9,10]. A cold atomic sample of 3×10^7 atoms (^{87}Rb) is produced in a σ^+-σ^- optical molasses loaded from a magneto-optical trap (MOT). The atoms are then optically pumped to the $F = 2$, $m_F = 0$ sublevel. The optical lattice is derived from the interference of two counterpropagating vertical beams generated by a Ti:sapphire laser. Its frequency is blue detuned by ~ 40 GHz from the $D2$ line. The lattice is adiabatically raised during 500 μs, then accelerated during 3 ms and adiabatically lowered (500 μs). The lattice depth is about $100E_r$ ($E_r = \hbar^2k^2/2m$ is the recoil energy). The $\pi/2$ laser pulses are derived from two external cavity phase-locked diodes lasers. The power of these Raman beams is 13 mW (per beam), their radius at $1/e^2$ is 2 mm. Their frequencies are blue shifted by 310 GHz from the $D2$ line in order to reduce photon scattering and light shifts. Their beat frequency is precisely controlled: a frequency sweep is used during each pair of $\pi/2$ pulses in order to balance the acceleration of gravity and a frequency jump allows to compensate the Doppler shift due to any velocity variation Δv between the two pairs of $\pi/2$ pulses. We use a symmetric Ramsey-Bordé interferometer, i.e., the laser beams directions are the same for the four pulses. In this case, we are only sensitive to the velocity variation induced by the BO and the acceleration of gravity.

When we increase the number of BO up to 50, the atoms collide with the window of the vacuum chamber. To overcome this problem, we load the atom interferometer with atoms initially accelerated using also BO. In such a way, the atomic velocity at the end of the interferometric trajectory is close to zero [9,10].

The ratio of h/m_{Rb} is deduced from four spectra obtained by using a first experimental protocol P1. It consists of the following: (i) accelerating the atoms either upwards or downwards keeping the same spacing time T between the two $\pi/2$ pulses pairs (the numbers of BO are, respectively, labeled N^{up} and N^{down}). In both cases the velocity variation due to the gravity acceleration g is gT, and can be canceled by making the difference between the two measurements. (ii) For each initial acceleration, we record two spectra by exchanging the directions of the Raman beams. In this case the contribution of some systematics arising from parasitic level shifts (ac stark or second-order Zeeman effect) changes sign and is then substantially reduced by averaging these two measurements. Two typical interference fringes patterns are shown in Fig. 2. The effective number of BO is $N^{up} + N^{down} = 1000$, corresponding to 2000 recoil velocities between the upper and
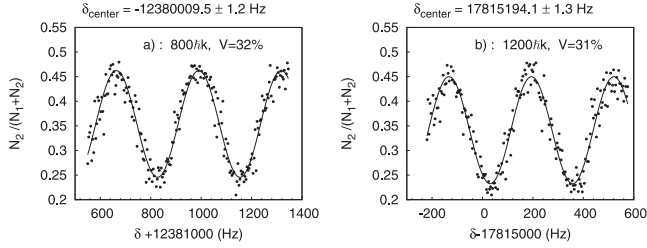
FIG. 2.   Typical set of two spectra used for a determination of α corresponding to the up (a) and down (b) trajectories. Here N_1 and N_2 are, respectively, the number of atoms in $F = 1$ and $F = 2$ after the second pair of $\pi/2$ laser pulses.

lower trajectories. The spacing time T_R is 2.6 ms and the duration of each $\pi/2$ pulse is 400 μs. The central fringe is determined with an uncertainty lower than 1.4 Hz($\sim \nu_r/10\,000$). Each spectrum is plotted with 200 points and is obtained in 6 min. We achieve an excellent fringes visibility of about 30% for 600 BO.

The number of BO and the spacing time between the two $\pi/2$ laser pulses can be potentially increased. They are still limited, respectively, by the displacement of the atoms in the vacuum chamber and vibrations. To overcome the first limit, we have built an "atomic elevator." The idea consists in displacing the atoms to the bottom or the top of the cell after the molasses phase. For that we take advantages of the high efficiency and the precise acceleration control of BO (through the control of the frequency difference between the two laser beams). We then start the measurement protocol at this position instead of the center of the cell.
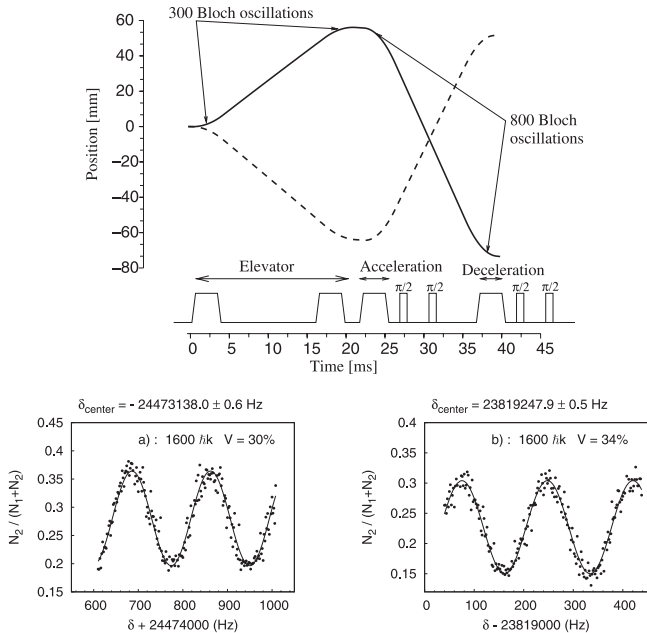

FIG. 3.   Trajectories of the atom using the "atomic elevator" technique and the corresponding pulses sequence. The spectra (a) and (b) correspond to an up and a down trajectory.

Figure 3 illustrates the new experimental sequence (protocol P2) and the corresponding atomic trajectories: after the cooling process all the atoms are accelerated for instance in the upward direction by using $N_{\rm elv}$ BO, thereafter when they come near the upper window they are stopped by using $-N_{\rm elv}$ BO. We then perform a measurement following the protocol P1 on a longer interaction distance. In such a way, we transfer 1600 photon momenta and obtain a fringes pattern with a typical visibility of 30% (see Fig. 3). Moreover, the uncertainty on the fringes center is reduced to less than 0.7 Hz by increasing the delay T_R to 5.4 ms and by implementing a vibration isolation platform.

We have used both protocols P1 and P2 to obtain 221 determinations of α. For these measurements, the effective number of BO $N^{\rm up} + N^{\rm down}$ varies between 200 and 1600. The dispersion of the values is $\chi^2/(n-1) = 1.85$ and the resulting statistical uncertainty on α is 3×10^{-9}.

The systematics effects are already detailed in [9,10]. For instance, in the case of the P2 protocol, they are listed in Table I. These corrections and uncertainties are very similar to those of the 2005 measurement. We have reduced the uncertainties of the two main corrections, due to the second-order Zeemann effect and to the geometry of the laser beam, thanks to a careful mapping of the magnetic field and a precise measurement of the wave-front curvature. Finally the global correction on α^{-1} is $(-9.54 \pm 3.4) \times 10^{-9}$. Taking into account the statistical uncertainty (3×10^{-9}), the total relative uncertainty on α is then 4.6×10^{-9}. We obtain for $h/m_{\rm Rb}$ the value $4.591\,359\,246\,(42) \times 10^{-9}$ m^2 s^{-1} and for α:

$$\alpha^{-1}({\rm LKB\,08}) = 137.035\,999\,45\,(62)\ \ [4.6 \times 10^{-9}], \quad (4)$$

where the number in the square bracket is the relative uncertainty and LKB08 (Laboratoire Kastler Brossel) denotes this work. This value is reported on the Fig. 4 (dot labeled LKB 08) which summarizes the most relevant

TABLE I.   Error budget on the determination of $1/\alpha$ (systematic effect and relative uncertainty in ppb).

| Source	Correction	Relative uncertainty
Laser frequencies		0.4
Beams alignment	-2	2
Wave-front curvature and Gouy phase	-11.9	2.5
Second-order Zeeman effect	4.9	1
Quadratic magnetic force	-0.59	0.2
Gravity gradient	-0.07	0.02
Light shift (one photon transition)		0.1
Light shift (two photon transition)		0.01
Light shift (Bloch oscillation)	0.48	0.2
Index of refraction atomic cloud		0.3
Index of refraction background vapor	-0.36	0.3
Rydberg constant and mass ratio [1]		0.23
Global systematic effects	-9.54	3.4
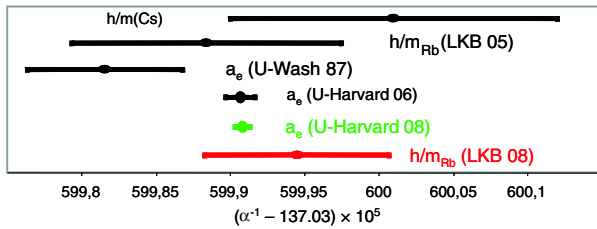
FIG. 4 (color online).   The most accurate determinations of the fine structure constant, in green (or light gray) the recent determination deduced from the g-2 measurement (U-Harvard 08) and in red (or gray) this work (LKB 08).

determinations of the fine structure constant [1–4,8–10]. This new determination is in agreement with our previous value (dot labeled LKB 05) and with the last result deduced from the electron anomaly [3]:

$$\alpha^{-1}(a_e) = 137.035\,999\,084\,(51)\quad[0.37\times10^{-9}].\quad(5)$$

Although the uncertainty of this value is more than 10 times smaller than our result, the comparison of these two results provides the most stringent test of the QED. To improve it, we are investigating deeply the limits of our experiment. We are building a larger vacuum chamber to increase the number of BO. A relative uncertainty lower than 10^{-9} can be reasonably expected with a better control of the magnetic field and of the geometric parameters of the laser beams. This performance would provide the possibility to reach an unprecedented test of the QED theory, or, if we consider the QED theory exact, a test of a possible substructure of the electron [21,22].

In conclusion, we have demonstrated the possibility to combine the BO with an atom interferometer. Compared with our previous work, this combination allows us to increase the resolution without decreasing the signal-to-noise ratio. We have obtained a new determination of the fine structure constant in agreement with our last determination based on a noninterferometric method. The comparison of this result with the value of α deduced from the measurement of the electron anomaly is the best test of QED. This Letter also opens a new way for the design of atom interferometers based on BO. The combination of atom interferometry with BO can be used for the measurement of the gravity constant g as proposed in [16]. It could significantly improve the sensitivity of measurements of g done with a very large number of BO [23,24]. Thanks to a larger vacuum chamber, we plan to increase the number of BO in order to reduce the uncertainty of the measurement of α to 1 ppb. In this Letter, we have studied a simple scheme where the BO are inserted between the two pairs of $\pi/2$ pulses. Another way to improve the precision of atom interferometers consists in using n-photon momentum beam splitter [25]. As described in [26], BO can be used to realize a large momentum beam splitter. The idea con-

sists to initially create two coherent wave packets by using a $\pi/2$ Raman pulse and then load those wave packets in the first and third band of an accelerated optical lattice. By choosing suitable lattice parameters, only atoms in the first band will be accelerated, increasing the relative momentum between the two interfering paths. A preliminary calculation of the phase shifts due to the BO shows the possibility of increase the sensitivity by a factor n of the order of 10.

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