Generation of Quantum-Dot Cluster States with a Superconducting Transmission Line Resonator

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We propose an efficient method to generate cluster states in spatially separated double quantum dots with a superconducting transmission line resonator. When the detuning between the double-dot qubit transition frequency and the frequency of the full wave mode in the transmission line resonator satisfies some conditions, an Ising-like operator between an arbitrary two separated qubits can be achieved. Even including the main noise sources, it is shown that the high fidelity cluster states could be generated in this solid system in just one step.

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Introduction.—Quantum entanglement is the root of quantum computation [1], quantum teleportation [2], quantum dense coding [3], and quantum cryptography [4]. However, it is challenging to create multiparticle entangled states through experiment. In 2001, Briegel and Raussendorf introduced highly entangled states, the cluster states [5], which can be used to perform universal one-way quantum computation. Up to now, various schemes have been proposed to generate cluster states in many different types of physical systems. Especially, it has been argued that the cluster states can be generated effectively in a solid state system, such as the superconductor charge qubit [6–8] and the semiconductor quantum dot [9–11].

Electron spins in semiconductor quantum dots are one of the most promising candidates for a quantum bit, due to their potential of long coherence time [12–14]. Producing cluster states in quantum dots has been discussed within the Heisenberg interaction model [9] and the Ising-like interaction model [10], where the long-term interaction inversely ratios to the distance between non-neighboring qubits. Recently, Taylor and Lukin and Childress, Sørensen, and Lukin introduced a technique to electrically couple electron charge states or spin states associated with semiconductor double quantum dots to a transmission line resonator (TLR) via a capacitor [15,16]. The qubit is encoded on the quantum double-dot triplet and singlet states. The interaction Hamiltonian between the qubits and the TLR is a standard Jaynes-Cumming (JC) model [17]. A switchable long-range interaction can be achieved between any two spatially separated qubits with the TLR cavity field. This technique opens a new avenue for quantum information implementation.

In this work, we find that, when the detuning between the qubit transition frequency and the frequency of the full wave mode in the TLR satisfies some conditions, an Ising-like operator between an arbitrary two separated qubits can be achieved from the JC interaction. The highly entangled cluster states can be generated in one step with the auxiliary of an oscillating electric field. Finally, we discuss the feasibility and the cluster state fidelity of our scheme.

Preparation of cluster states.—The system we study includes N identical double-dot qubits capacitively coupling with a TLR by C_c, as shown in Fig. 1. The TLR is coupled to input or output wiring with a capacitor C_e to transmit signals. Two electrons are localized in a double-dot quantum molecule. The two electron charge states, spin states, and the corresponding eigenenergies are controlled by several electrostatic gates.

An external magnetic field is applied along axis z. At a large static magnetic field (100 mT), the spin aligned states $[|(1,1)T_+\rangle = |\uparrow\uparrow\rangle, |(1,1)T_-\rangle = |\downarrow\downarrow\rangle]$ are split from spin antialigned states $[|(1,1)T_0\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}, |(1,1)S\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}]$ due to Zeeman splitting. The notation (n_l, n_r) indicates n_l electrons on the "left" dot and n_r electrons on the "right" dot. In addition to the (1, 1)

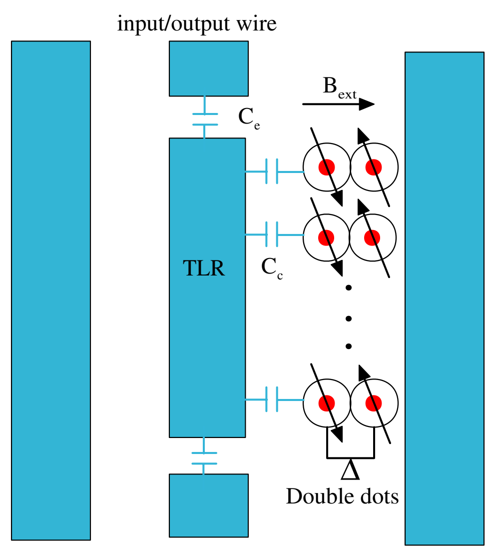

FIG. 1 (color online). Schematic diagram of a TLR and several double quantum-dot coupled systems. A detailed circuit representation of the TLR cavity (blue) can be found in Fig. 1 of Ref. [27]. The double dots are biased with an external potential Δ and capacitive coupling C_c with the TLR. The TLR is connected to the input/output wiring with a capacitor C_e.

subspace, the doubly occupied state $|(0,2)S\rangle$ is coupled to $|(1,1)S\rangle$ via tunneling T_c. $|(0,2)S\rangle$ and $|(1,1)S\rangle$ have a potential energy difference of Δ. Because of the tunneling between the two adjacent dots, the $|(1,1)S\rangle$ and $|(0,2)S\rangle$ hybridize. We can get the double-dot eigenstates:

$$|+\rangle = -\sin\theta|(1,1)S\rangle + \cos\theta|(0,2)S\rangle, \quad (1)$$

$$|-\rangle = \cos\theta|(1,1)S\rangle + \sin\theta|(0,2)S\rangle, \quad (2)$$

where $\tan\theta = -2T_c/(\Omega + \Delta)$ and $\Omega = \sqrt{4T_c^2 + \Delta^2}$. Ω is the energy gap between the eigenstates $|+\rangle$ and $|-\rangle$. We can choose $\Delta = 0$ in order to suppress the fluctuations in control electronics; then $|+\rangle = [|(1,1)S\rangle + |(0,2)S\rangle]/\sqrt{2}$ and $|-\rangle = [|(1,1)S\rangle - |(0,2)S\rangle]/\sqrt{2}$. The qubit is encoded on the states $|+\rangle$ and $|-\rangle$.

An oscillating electric field, whose frequency is coincidental with the qubit energy gap, is applied to the left-side gate of the double quantum dots. The single-qubit operation Hamiltonian in the interaction picture can be expressed as

$$H_i = \eta|(1,1)S\rangle\langle(1,1)S| = \eta(I + \sigma_i^x) = \eta\sigma_i^x, \quad (3)$$

where $\eta = \vec{d}\cdot\vec{E}$, $\vec{d}$ is the dipole moment of the left quantum dot, and $\vec{E}$ is the oscillating electric field.

We consider a TLR of length L, with capacitance per unit length C_0, and characteristic impedance Z_0. Neglecting the higher energy modes, we can consider only the full wave mode, with the wave vector $k = \frac{\pi}{L}$ and frequency $\omega = \frac{k}{C_0 Z_0}$ [16]. The TLR can be described by the Hamiltonian $H_{\rm cavity} = \omega a^\dagger a$, where $\hbar = 1$ and $a^\dagger$ and a are the creation and annihilation operators, respectively, for the full wave mode of the TLR. In the interaction picture, the interaction between N qubits and the TLR can be described by the Hamiltonian [17]

$$H_{\rm int} = g_0 \sum_{i=1}^{N} (e^{i\delta t} a^\dagger \sigma_i^- + e^{-i\delta t} a \sigma_i^+), \quad (4)$$

where $\sigma_i^+ = |+\rangle\langle-|$, $\sigma_i^- = |-\rangle\langle+|$, $\sigma_i^x = \sigma_i^+ + \sigma_i^-$, and $\delta = \omega - \Omega$. Here we can presently assume the coupling strength is homogenous. The overall coupling coefficient can be described by [16]

$$g_0 = \omega \frac{C_c}{2C_{\rm tot}} \sqrt{\frac{2z_0}{R_Q}}, \quad (5)$$

where $R_Q = h/e^2 \approx 26$ kΩ. $C_{\rm tot}$ is the total capacitance of the double dot.

If the interaction time τ satisfies

$$\delta\tau = 2k\pi, \quad (6)$$

the evolution operator for the interaction Hamiltonian (4) can be expressed as [18]

$$U(\tau) = \exp\left[-i\frac{\lambda}{2}\tau\left(\sum_{i=1}^{N}\sigma_i^x\right)^2\right] = \exp\left(-i\lambda\tau \sum_{j>i=1}^{N}\sigma_i^x\sigma_j^x\right), \quad (7)$$

where $\lambda = g_0^2/2\delta$.

When Δ is changed to zero, the coupling coefficient between the qubits and TLR is maximal. An oscillating electric field is applied to all of the qubits at the same time. In the interaction picture, the total Hamiltonian of the system can be written as

$$H_{\rm tot} = g_0 \sum_{i=1}^{N} (e^{i\delta t} a^\dagger \sigma_i^- + e^{-i\delta t} a \sigma_i^+) + \eta \sum_{i=1}^{N} \sigma_i^x. \quad (8)$$

When the operation time τ satisfies the condition (6), we can obtain the total evolution operator

$$U(\tau) = \exp\left(-i\eta\tau \sum_{i=1}^{N}\sigma_i^x - i\lambda\tau \sum_{j>i=1}^{N}\sigma_i^x\sigma_j^x\right). \quad (9)$$

When $\eta = (N-1)\lambda$, the total evolution operator is given by

$$U(\tau) = \exp\left(-4i\lambda\tau \sum_{j>i=1}^{N} \frac{1+\sigma_i^x}{2}\frac{1+\sigma_j^x}{2}\right). \quad (10)$$

In order to generate the cluster states, the initial state of N qubits should be prepared in the state $\bigotimes_{i=1}^{N}|-\rangle_i = (1/\sqrt{2})\bigotimes_{i=1}^{N}(|0\rangle_i + |1\rangle_i)$, where $|0(1)\rangle_i = (1/\sqrt{2})(|-\rangle_i \pm |+\rangle_i)$ are the eigenstates of σ_i^x with the eigenvalues ± 1. Next, we discuss how to prepare the initial state in experiment. First, we can prepare the two electrons in double quantum dots to the state $|(0,2)S\rangle$ at a large positive potential energy difference Δ [19]. Then the $|(0,2)S\rangle$ can be changed to the state $|-\rangle = [|(1,1)S\rangle - |(0,2)S\rangle]/\sqrt{2}$ by rapid adiabatic passage [12]. After the initial state has been prepared, the qubits would be resonantly coupled with the TLR. We apply an oscillating electric field to each qubit at the same time. Thus the initial state would evolve under the total operator (10). If the evaluation interaction time satisfies

$$4\lambda\tau = (2n+1)\pi, \quad (11)$$

with n being integer, the spatially separated double quantum dots can be generated to the cluster states

$$|\Psi\rangle_N = \frac{1}{2^{N/2}} \bigotimes_{i=1}^{N}\left(|0\rangle_i(-1)^{N-i}\prod_{j=i+1}^{N}\sigma_i^x + |1\rangle_i\right). \quad (12)$$

The effective coupling coefficient $g_0\frac{2T_c}{\Omega}$ can be tuned by external potential Δ. When Δ is changed, the states $|0(1)\rangle$ would change according to Eqs. (1) and (2), but the expression (12) of the cluster states is unchanged. When the cluster states are generated at the time of τ, we can remove the oscillating electric field and change Δ to decouple all of the qubits to the TLR. Then the cluster states can be preserved.

Feasibility of the scheme.—The sample of the TLR and quantum-dot coupled system can be obtained in a two-step fabrication process on a GaAs/AlGaAs heterostructure. First, quantum dots are formed in the two-dimensional electron gas below the surface, using electron beam lithography and Cr-Au metallization. Then the TLR is fabricated by conventional optical lithography [20]. The main technical challenges for experimental implementation of our proposal are the design and nanofabrication of the sample [21]. The diameter of the quantum dot is about 400 nm, and the corresponding capacitance of the double-dot C_{tot} is about 200 aF. The distance between the two double-dot molecules should be 4 μm, which is tenfold the distance between two quantum dots within a double dot. Thus we can neglect the interaction between the double-dot molecules safely [22]. Since the energy gap between $|+\rangle$ and $|-\rangle$ is about 10 μeV at the operation point, the experimental manipulation should be implemented in a dilution refrigerator with a temperature below 100 mK. Both of the conditions (6) and (11) are satisfied whenever $\delta = g_0\sqrt{4k/(2n+1)}$. From Eq. (5), the coupling coefficient g_0 can be up to $\omega/16$ with a large coupling capacitor $C_c \approx 2C_{\mathrm{tot}} \approx 400$ aF. For $k = 1$, $n = 0$, $\omega/2\pi = 2$ GHz, and $g_0/2\pi = 125$ MHz, the operation time of the generation of cluster states is $\tau = \frac{2\pi}{\delta} = 4$ ns.

Decoherence.—In our system, the main decoherence processes are the dissipation of the TLR and the spin dephasing, charge relaxation, and additional dephasing of the double-dot molecules. The dissipation of the TLR that occurred through coupling to the external leads can be described by the photon decay rate $\kappa = \omega/Q$, where Q is the quality factor. For $Q = 1 \times 10^5$, $\omega = 2\pi \times 2$ GHz in our situation, the photon decay time $1/\kappa \approx 50$ μs is 4 orders longer than the operation time τ. Therefore the cavity loss can be neglected in our situation.

Nuclear spins are one of the main noise sources in semiconductor quantum dots via hyperfine interaction. The hyperfine field can be treated as a static quantity, because the evolution of the random hyperfine field is several orders slower (>10 μs) than the electron spin dephasing. In the operating point, the most important decoherence due to the hyperfine field is the spin dephasing between the states $|(1,1)S\rangle$ and $|(1,1)T_0\rangle$. By suppressing nuclear spin fluctuations [23], the spin dephasing time obtained by quasistatic approximation can be $T_2^* = \hbar/(g\mu_B\langle\Delta B_n^z\rangle_{\mathrm{rms}}) \approx 1$ μs, where ΔB_n^z is the nuclear hyperfine gradient field between two dots and rms denotes a root-mean-square time-ensemble average. The coupling to the phonon bath will lead to the relaxation of the charge freedom. Using the spin-boson model, the relaxation time of the qubits can be obtained by the Fermi golden rule [15]. The charge relaxation time T_1 is about 1 μs at the operation point. Additional dephasing is assumed to arise from the low-frequency fluctuations of the control electronics, which typically have the $1/f$ spectrum. In our system, it is assumed that the origin of $1/f$ noise is the random drift of the gate bias when an electron tunnels in or out of the metallic electrode. Assuming $1/f$ noise subject to Gaussian statistics, we found the addition dephasing time $T_{2,\alpha} \sim \Omega T_{2,\mathrm{bare}}^2$ is about 100 ns at the optimal working point ($\Delta \approx 0$), where $T_{2,\mathrm{bare}}$ will be discussed below in detail and can be up to 10 ns. Thus the total operation time of the present proposal $\tau \approx 4$ ns is much shorter than the spin dephasing time, charge relaxation, and additional dephasing time of the qubits.

Fidelity of the cluster states.—For simplicity, we assume the control electronics fluctuations are Gaussian. These noises would lead to the fluctuations of the parameter λ via the electric potential difference Δ. Suppose $\Delta_i(t) = \Delta + \epsilon_i(t)$, $\langle\epsilon_i(t)\rangle = 0$, $\langle\epsilon_i(t)\epsilon_j(t')\rangle = \int S_{ij}(\omega)e^{i\omega(t-t')}d\omega$ (i labeling the ith qubit). The fluctuations of the oscillating electric field would result in the fluctuations of the parameter η. The fluctuations of λ and η would add an unwanted phase θ_i to the desired value π [24]. Including the fluctuations, the evolution operator (10) should be rewritten in the form of

$$U(\tau) = \exp\left(-i\pi\sum_{j>i=1}^{N}\frac{1+\sigma_i^x}{2}\frac{1+\sigma_j^x}{2}\right) \times \exp\left(-i\int_0^\tau \delta\eta(t)dt\sum_{i=1}^{N}\sigma_i^x\right) \times \exp\left(-i\int_0^\tau \delta\lambda(t)dt\sum_{j>i=1}^{N}\sigma_i^x\sigma_j^x\right), \tag{13}$$

where $\lambda(t) = \lambda + \delta\lambda(t)$ and $\eta(t) = \eta + \delta\eta(t)$. Since $\delta\lambda + \frac{\delta\eta}{N-1}$ and $\delta\eta + (N-1)\delta\lambda$ are larger than $\delta\lambda$ and $\delta\eta$, we can write the unwanted phase $\theta_i = \int_0^\tau 4[\delta\lambda(t) + \frac{\delta\eta(t)}{N-1}]dt = \theta_{1,i} + \theta_{2,i}$, where $\theta_{1,i} = \int_0^\tau 4\delta\lambda(t)dt$ and $\theta_{2,i} = \int_0^\tau 4\frac{\delta\eta(t)}{N-1}dt$.

Since δ and λ satisfy Gaussian distribution, $\theta_{1,i}$, $\theta_{2,i}$, and θ_i have Gaussian distribution $G(0,\sigma_{1,i}^2)$, $G(0,\sigma_{2,i}^2)$, and $G(0,\sigma_i^2)$, respectively. Ignoring the correlative fluctuations, the variance of $\theta_{1,i}$ at the optimal working point is

$$\sigma_{1,i}^2 = \left(\frac{2g_0^2}{\Omega\delta^2}\right)^2\left\langle\left(\int_0^\tau \epsilon^2(t)dt\right)^2\right\rangle, \tag{14}$$

where

$$\left\langle\left(\int_0^\tau \epsilon^2(t)dt\right)^2\right\rangle = \left(\int S_i(\omega)d\omega\right)^2\tau^2 + 2\left(\int S_i(\omega) \times \frac{\sin\omega\tau}{\omega\tau}d\omega\right)^2\tau^2. \tag{15}$$

For the low-frequency noise, $S_i(\omega)$ has a high frequency cutoff $\gamma \ll \frac{1}{\tau}$. Therefore we can get $\langle[\int_0^\tau \epsilon^2(t)dt]^2\rangle = 3[\int S_i(\omega)d\omega]^2\tau^2$. Assuming $\frac{1}{T_{2,\mathrm{bare}}^2} = \int S_i(\omega)d\omega$, we can obtain the variance $\sigma_{1,i}^2 = \frac{12g_0^4}{\delta^4}(\frac{\tau}{\Omega T_{2,\mathrm{bare}}^2})^2$. Taking $T_{2,\mathrm{bare}} \approx 10$ ns from Refs. [19,25,26], the variance of $\theta_{1,i}$ is $\sigma_{1,i} = 0.022\pi$. The fluctuations of the oscillating electric field

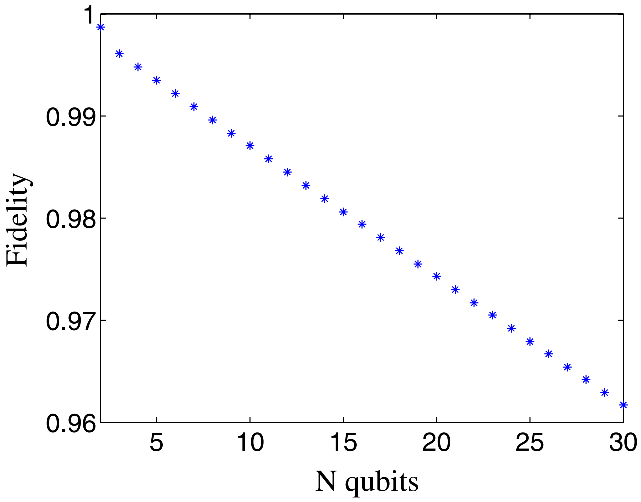

FIG. 2 (color online). The fidelity of N-qubit cluster states.

root in the electronics noise. The fluctuations can be reduced in a small value with a better high- and low-frequency filtering technique. Supposing $\sigma_{\Delta\eta}/\Delta\eta \approx 2\%$, the variance of $\theta_{2,i}$ is $\sigma_{2,i} = 0.006\pi$. So θ_i has a Gaussian distribution $G[0, (0.023\pi)^2]$. The fidelity of N qubit cluster states is calculated according to the formula $F = |2^{-N}\sum_{z_i}\prod_{j=1}^{N-1}(\int \frac{1}{\sqrt{2\pi}\sigma_i} e^{-(\theta_j^2/2\sigma_i^2)} e^{i\theta_j} d\theta_j)^{z_j z_{j+1}}|^2$ from Ref. [24], as shown in Fig. 2. The fidelity of a 30-qubit cluster state can be 96.2%.

Conclusion and discussions.—Distinguished from cavity quantum electrodynamics in atomic quantum information processing, our scheme can realize the long-range interaction among the double-dot molecules with the TLR in a solid microchip device. This technique can couple the static qubit in the solid state system to the flying qubit (the cavity photon) [15]. Compared with other schemes, the present proposal based on quantum-dot molecules has four potential advantages: integration and scaling in a chip, easy addressing, high controllability, and a long coherence time associated with the electron spin. As discussed above, the preparation of the initial state can be easily implemented without interqubit coupling in our scheme. When the initial state has been prepared, the quantum-dot cluster states can be produced with only one step. The cluster states can be preserved easily by switching off the coupling between the qubits and TLR cavity field.

In conclusion, we proposed a realizable scheme to generate cluster states in only one step in a new scalable solid state system, where the spatially separated semiconductor double-dot molecules are capacitive coupling with a TLR. An effective, switchable long-range interaction can be achieved between any two double-dot qubits with the assistance of a TLR cavity field. The experimental related parameters and the possible fidelity of generated cluster states have been analyzed. Because of the long relaxation and dephasing time at the optimal working point, the present scheme seems implementable within today's techniques.

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