Quantum Control of a Trapped Electron Spin in a Quantum Dot Using Photon Polarization

François Dubin,[1,2] Monique Combescot,[3] Gavin K. Brennen,[4,5] and Romain Melet[3]

[1]*ICFO–Institut de Ciències Fotòniques, Mediterranean Technology Park, E-08860 Casteldefells, Spain*

[2]*Institute for Experimental Physics, University of Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria*

[3]*Institut des NanoSciences de Paris, Université Pierre et Marie Curie–CNRS, 140 rue de Lourmel, 75015 Paris*

[4]*Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, 6020 Innsbruck, Austria*

[5]*Center for Quantum Information Science and Security, Macquarie University, 2109, NSW Australia*

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We present an original scheme to rotate at will one electron spin trapped in a quantum dot by just acting on pump-laser polarization: The quantum control is based on the virtual excitation of electron light-hole pairs with π symmetry, as possibly done by using a single laser beam with a propagation axis slightly tilted with respect to a weak magnetic field. This allows us to fully control the effective axis of the electron spin rotation through the pump polarization. Our analysis shows that quantum dots with inverted valence states are ideal candidates for ultrafast, high-fidelity, all optical control.

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The first milestones towards quantum information processing with semiconductor quantum dots have been reported recently [1–5]. Like trapped ions or atoms, individual quantum dots offer a resource for quantum memories that can be ideally written and readout with single photons. These nanostructures are now well controlled and can be actively positioned, for instance, inside an integrated optical cavity [3].

Although a single quantum dot usually contains $\sim 10^4$–10^6 atoms, it is often referred to as a single macroscopic atom: Its optical excitation, a confined electron-hole pair, exhibits discrete energy levels, similar to the ones of a single atom. Various experiments have now shown that not only an electron-hole pair but also a single electron can be injected in a neutral dot. This opens important perspectives for the study of a semiconductor matrix at the single atom level, by using a trapped electron as a probe for its environment [6,7]. On the other hand, the projections of the electron spin along an external magnetic field can be used as qubit states for quantum memory.

Reliable information storage encoded in electron spin states requires long coherence times, i.e., efficient protection against spin flip. Extensive studies of decoherence channels for charged dots have shown that the main source for decoherence is the hyperfine interaction between the trapped electron and the nuclear spins of the dot. Several groups have reported coherence times $T_2^* \approx 10$ ns [5,8], which should be largely increased by implementing spin-echo techniques [9].

Up to now, most works have treated single charged quantum dots when a *strong* magnetic field ($\simeq$5–10 T) is applied *perpendicular* to the crystal growth axis [4,5,8,10–12]. In this Voigt configuration, the coherent manipulations are based on a resonant Raman coupling between the two spin projections along the field. This coupling can be achieved directly through *two* frequency detuned laser beams [10,11]. Alternatively, coherent coupling can be stimulated by the mode of an optical cavity to perform single [13] and two qubit gates [12]. Stimulated Raman adiabatic passage has also been proposed to achieve single qubit rotations and quantum gates [14]. Moreover, rotations of a trapped spin have been observed very recently applying a low magnetic field in the Voigt configuration [1].

Important experimental achievements have also been reported recently with a *weak* magnetic field ($\leq$1 T) applied *along* the crystal growth axis. In this Faraday configuration, the spin of a single trapped electron can be prepared along the field, with a fidelity close to one [2]. Time-averaged state readout of the electron spin has been reported [15]. However, the optical control of a single spin appears rather difficult experimentally: It requires two laser beams propagating perpendicularly [16].

In this Letter, we also consider a Faraday configuration but we address the interaction of a charged quantum dot with a *single* laser beam *not* collinear to a *weak* magnetic field. We show that the virtual excitation of quantum dot electron light-hole pairs with π symmetry yields a precession of the trapped electron spin around an effective axis which is fully controlled by the laser polarization: In this tilted geometry, the relative effect of the weak magnetic field is beautifully increased at will by changing the photon polarization. This makes the resulting precession of the electron spin possible to achieve around any direction. Within this Faraday configuration, a single electron spin can thus be manipulated through a rather simple experimental setup [17].

The efficiency of electron spin rotations strongly depends on the lifetime of the quantum dot light holes, which induces an effective broadening of electronic levels. To the best of our knowledge, this lifetime has not been measured yet and appears difficult to estimate as it is controlled by the dot conformation. Therefore, we use the light-hole lifetime measured in quantum wells, namely, 50 ps [18], and deduce performances for electron spin rotations already comparable to the ones predicted with a Voigt configuration. Moreover, let us note that the structure of the valence band can be inverted by an appropriate tensile strain [19]. Hence light holes become lowest lying hole states, thereby exhibiting a lifetime *a priori* comparable to the one of heavy holes (≥ 20 ns [20]). In such cases, the fidelity for ultrafast quantum control is almost unity.

Physical idea.—A trapped spin embedded in a static magnetic field has a clockwise precession around the field axis. Irradiated by an unabsorbed laser beam it can also rotate around the photon propagation axis due to a possible exchange between the trapped electron and the electron of the virtual exciton coupled to the unabsorbed photons [21]. This leads to a clockwise or counterclockwise precession, depending on the photon circular polarization, the precession stopping completely when the polarization is linear.

We here add the two effects induced by a weak magnetic field applied along the crystal growth axis and a slightly tilted pump beam: The spin of a trapped electron experiences a precession around an effective axis which results from the competition between the precession induced by the pump laser and the precession induced by the magnetic field. When the pump is powerful and the magnetic field weak, the pump controls the precession [1], except for linearly polarized photons since these photons do not act on trapped spin. We can thus evolve from a laser-induced to a field-induced precession by changing the photon polarization from circular to linear. This shows that the magnetic field relative efficiency can be increased at will by just changing the photon polarization.

We then note that due to large semiconductor refractive indices, the angle between the magnetic field and the laser beam, α in Fig. 1(a), cannot be very large ($\alpha \leq \pi/10$). Hence, when the photon circular polarization and the magnetic field both induce the same clockwise precession, the possible change in the effective axis when going from circular to linear has to remain small to ensure continuity [see Fig. 1(b)]. By contrast, if the photon circular polarization induces a counterclockwise precession, the effective axis can rotate by as much as π [see Fig. 1(c)]. We can then fully control the spatial orientation of the effective precession axis through the pump photon ellipticity, even for a slightly tilted laser beam.

Note that, by taking a laser beam not collinear to the magnetic field, the photon momentum has a nonzero projection along the magnetic field. This allows one to engineer arbitrary rotations of the trapped electron spin through optical excitations with π symmetry, i.e., excitations of virtual electron light-hole pairs [see Fig. 1(d)]. The tilted geometry is a crucial component of our proposal.

Theoretical support.—We follow the procedure developed in [21], except that, due to the magnetic field, the electron states now have slightly different energies $\varepsilon_s^{(e)} = \varepsilon^{(e)} - su$, with $s = (\pm 1/2)$ and u positive for spin quantized along the magnetic field. Let us anticipate that a

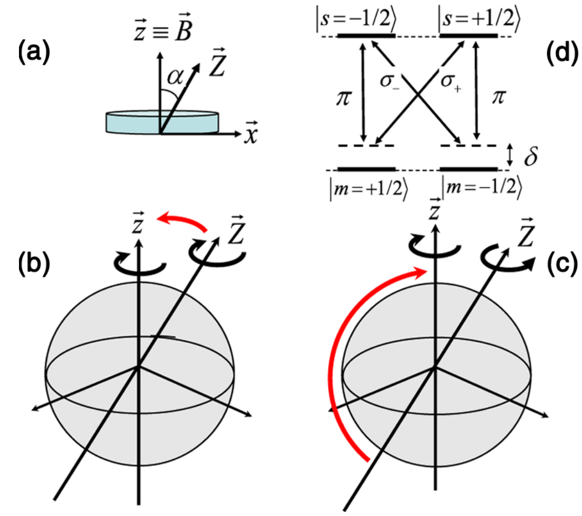

FIG. 1 (color online). (a) The magnetic field **B** is along the sample growth axis **z**. The photon propagation axis **Z** is at α in the (**x**, **z**) plane. (b) When the angle between **z** and **Z** is small, the rotation of the precession axis stays small for pump photons inducing a clockwise precession, i.e., for polarization possibly going from circular to linear. (c) By contrast, it can be as large as π if the photon induced precession is counterclockwise. (d) Excitations with π symmetry are between electrons and light holes, for photon detuning δ.

nonzero value of u is crucial in our proposal: Indeed, u appears through the dimensionless parameter

$$\tilde{u} = \frac{u}{\Delta_0 \sin 2\xi}, \qquad (1)$$

where Δ_0 is an energylike quantity characteristic of the unabsorbed pump beam and ξ is the photon ellipticity. The effect of a small but finite u can thus be infinitely increased by acting on ξ.

The coupled dot-photon Hamiltonian reads $H = H_e + H_p + W$. For small dots, the electronic part reduces to

$$H_e = \sum \varepsilon_s^{(e)} b_s^\dagger b_s + \sum \varepsilon_m^{(h)} c_m^\dagger c_m, \qquad (2)$$

where $b_s^\dagger$ creates a trapped electron $s = (\pm 1/2)$, while $c_m^\dagger$ creates a trapped hole $m = (\pm 3/2, \pm 1/2)$. Let us note that corresponding energies depend on s or m due to the applied magnetic field. Even if we can excite the dot light holes ($m = \pm 1/2$) selectively when the confinement is strong, it is appropriated to keep the two types of holes in the calculation, in order to show that light holes are the key ones in our proposal.

The photon part of the Hamiltonian reads $H_p = \omega_p a^\dagger a$, where $a^\dagger$ creates a photon, while the electron-photon coupling can be written as $W = aU^\dagger + a^\dagger U$ with $U^\dagger = \Omega \sum_s b_s^\dagger d_s^\dagger$ where Ω is the vacuum Rabi energy of the dot. To prove in a simple way that arbitrary rotations cannot be engineered through heavy-hole excitations, let us consider a single Ω for both heavy and light hole transitions. Moreover, $d_s^\dagger$ denotes the combination of hole states coupled to the s electron. For a photon polarization vector $\vec{\varepsilon} = \varepsilon_x \mathbf{x} + \varepsilon_y \mathbf{y} + \varepsilon_z \mathbf{z}$ with $\mathbf{z}$ along the magnetic field, $d_s^\dagger$ reads [22]

$$d^\dagger_{\pm 1/2} = \varepsilon_\mp c^\dagger_{\mp 3/2} \pm \sqrt{\frac{2}{3}} \varepsilon_0 c^\dagger_{\mp 1/2} - \frac{1}{\sqrt{3}} \varepsilon_\pm c^\dagger_{\pm 1/2}, \quad (3)$$

where we have set $\varepsilon_0 = \varepsilon_z$ and $\varepsilon_\pm = (\varepsilon_x \mp i\varepsilon_y)/\sqrt{2}$ with $(\mathbf{x}, \mathbf{y})$ chosen such that $\mathbf{y} \cdot \mathbf{Z} = 0$ for convenience, $\mathbf{Z}$ being along the photon momentum [see Fig. 1(a)]. By writing the polarization vector $\vec{\varepsilon} = \mathbf{X}\cos\xi + i\mathbf{Y}\sin\xi$ with $-\pi/2 < \xi \leq \pi/2$, for photons with elliptical polarization ξ having $(\mathbf{X}, \mathbf{Y})$ as main axes, one can show that

$$\sin 2\xi \cos\alpha = |\varepsilon_+|^2 - |\varepsilon_-|^2, \quad (4)$$

$$\sin 2\xi \sin\alpha = \sqrt{2}(\varepsilon_-^* \varepsilon_0 - \varepsilon_+ \varepsilon_0^*), \quad (5)$$

where α is the angle between $\mathbf{z}$ and $\mathbf{Z}$.

We now look for the eigenstates $|\psi\rangle$ of one electron coupled to N_p photons, $(H - E)|\psi\rangle = 0$, as

$$|\psi\rangle = [|\varphi_+\rangle\langle\varphi_+| + |\varphi_-\rangle\langle\varphi_-| + P_\perp]|\psi\rangle, \quad (6)$$

where $|\varphi_\pm\rangle = |\pm\rangle \otimes |N_p\rangle$ are the eigenstates of the uncoupled electron-photon system, while $P_\perp$ is the projector over the subspace perpendicular to $|\varphi_\pm\rangle$. From $0 = P_\perp(H - E)|\psi\rangle$, we get $P_\perp|\psi\rangle$ in terms of $|\phi_\pm\rangle\langle\phi_\pm|\psi\rangle$. When inserted into $\langle\varphi_\pm|H - E|\psi\rangle = 0$, this vector gives two equations for $\langle\varphi_\pm|\psi\rangle$ which have a nonzero solution for $E = N_p\omega_p + \mathcal{E}$ such that

$$\begin{vmatrix} \varepsilon^{(e)}_{+1/2} + D_{++} - \mathcal{E} & D_{+-} \\ D_{-+} & \varepsilon^{(e)}_{-1/2} + D_{--} - \mathcal{E} \end{vmatrix} = 0. \quad (7)$$

The coupling $D_{\eta'\eta}$ reads at lowest order in W as

$$D_{\eta'\eta} = \langle\phi_{\eta'}|W \frac{1}{\mathcal{E} + N_p\omega_p - H_p - H_e} W|\phi_\eta\rangle = \Delta_0 d_{\eta'\eta}, \quad (8)$$

where $\Delta_0 = N_p\Omega^2/3\delta$, $\Omega\sqrt{N_p}$ being the interaction strength between the quantum dot and the laser field. For detuning $\delta = \omega_p - \varepsilon^{(e)} - \varepsilon^{(h)}$ large compared to both the Rabi energy $\Omega\sqrt{N_p}$ and the energy splitting u induced by the magnetic field, the dimensionless coupling $d_{\eta'\eta}$ reduces to $3\Omega^{-2}\langle\eta'|UU^\dagger|\eta\rangle$, with

$$d_{\pm\pm} = 3p_H|\varepsilon_\pm|^2 + p_L[|\varepsilon_\mp|^2 + 2|\varepsilon_0|^2], \quad (9)$$

$$d_{+-} = d^*_{-+} = \sqrt{2}[\varepsilon_-^*\varepsilon_0 - \varepsilon_+\varepsilon_0^*]p_L. \quad (10)$$

p_H and p_L are introduced to characterize couplings to photons. In bulk materials, heavy and light holes are degenerate such that $p_H = p_L = 1$. In confined geometries, we must set $p_L = 1$ and $p_H = 0$ when the photon detuning makes the light holes predominantly coupled to photons, while $p_L = 0$, $p_H = 1$ when the dominant coupling is to the heavy holes. These various values of (p_H, p_L) allow us to cover all configurations with the same equations (9) and (10) and to underline the key role played by light holes in engineering arbitrary rotations.

We now note that for eigenstates reading as

$$|\pm\rangle_{\mathbf{Z}^*} = \cos\frac{\theta^*}{2}|\pm\rangle \pm e^{i\varphi^*}\sin\frac{\theta^*}{2}|\mp\rangle, \quad (11)$$

which correspond to $(\pm)$ spins along a $\mathbf{Z}^*$ axis characterized by the Euler angles (θ^*, φ^*) with respect to $(\mathbf{x}, \mathbf{y}, \mathbf{z})$, the electron spin precesses around $\mathbf{Z}^*$. By writing $|\psi\rangle$, eigenstate of Eq. (7), as in Eq. (11) we get $e^{i\varphi^*} = -|\Delta|/\Delta$, while

$$\tan\theta^* = \frac{|\Delta|}{\Delta} \quad \frac{2\sin\alpha}{\tilde{u} + (p_L - 3p_H)\cos\alpha}. \quad (12)$$

Δ is related to the photon polarization through

$$\Delta = \Delta_0 \sin 2\xi \sqrt{4p_L^2 \sin\alpha^2 + [\tilde{u} + (p_L - 3p_H)\cos\alpha]^2}, \quad (13)$$

where $\tilde{u}$ is the dimensionless parameter defined in Eq. (1). It can be seen as the magnetic field efficiency in the presence of an unabsorbed laser beam with characteristic energy Δ_0, dressed by the pump photon polarization which enters $\tilde{u}$ through $1/\sin 2\xi$. This efficiency is thus mainly controlled by the pump beam ellipticity.

The above equations readily show that for photons only coupled to the heavy holes ($p_H = 1$, $p_L = 0$), the eigenstates reduce to $|\pm\rangle$ since $d_{+-} = 0$: The precession axis is then along the magnetic field, whatever the pump beam is. This shows that light holes are crucial to possibly engineer a specific orientation of the effective precession axis $\mathbf{Z}^*$ by acting on the photon polarization. In a quantum dot, this implies the use of photons close in energy to the electron light-hole transition.

Gate proposal.—To prepare a single qubit in an arbitrary state, it suffices for instance to perform a sequence of phase and Hadamard gates. These two operations require the ability to construct two precession axes at $(\pi/4)$. Let us now show how this can be realized applying a weak magnetic field and a slightly tilted excitation laser. First, we consider a gate operation induced by a pump beam with circular polarization σ_+ and negative detuning with respect to the electron light-hole transition ($\delta < 0$). Equation (1) shows that for a magnetic field weak enough to have $|\Delta_0| \gg u$, the key parameter $\tilde{u}$ reduces to $\tilde{u} \simeq 0_-$: The electron spin then has a clockwise precession around an axis $\mathbf{Z}^*_\mathbf{P}$ having as Euler angles $(\theta^*_P, \phi^*_P) \approx (\pi - 2\alpha, 0)$ for α small. To engineer a second rotation around $\mathbf{Z}^*_\mathbf{H}$ at $(\pi/4)$ from $\mathbf{Z}^*_\mathbf{P}$, i.e., $(\theta^*_P - \theta^*_H) = \pi/4$, we keep the same laser beam but adjust the photon ellipticity. This can be easily done since $\tilde{u}$ varies from 0_- to $-\infty$ when the photon ellipticity increases from σ_+ to linear, either X or Y. Equation (12) allows us to deduce the precise ellipticity for the second gate axis, $\mathbf{Z}^*_H$, which reads $\sin 2\xi_H = -(u/\Delta_0)[(2 + \cos\alpha)/(1 + 3\sin^2\alpha)]$.

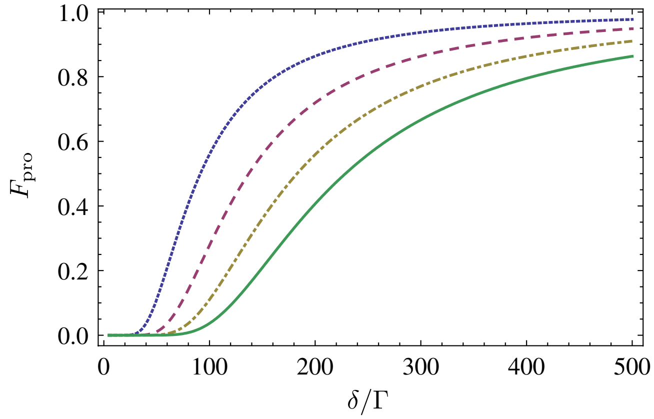

FIG. 2 (color online). Process fidelity F_{pro} for an arbitrary single qubit operation as a function of field detuning δ. The system parameters are $u/\hbar\Gamma = 0.2$, $\alpha = \pi/10$, and four different field strengths are plotted: $\Omega/\Gamma = 50$ (solid curve), 40 (dash–dotted curve), 30 (dashed curve), 20 (dotted curve).

Estimation of the fidelity.—The efficiency of spin rotations is controlled by the lifetime of light holes, which induces an effective level broadening. To estimate the fidelity of our gate proposal, we include the light-hole decay through an effective Hamiltonian $\tilde{H}$ deduced from H by replacing the light-hole energy $\varepsilon^{(h)}$ by $(\varepsilon^{(h)} - i\Gamma/2)$, with Γ being the light-hole relaxation rate. If we neglect heavy holes for simplicity, as reasonable in small dots for photon energy close to the electron light-hole transition, this effective Hamiltonian $\tilde{H}$ only acts on a 4-level subspace made of the two electron states $b^\dagger_{\pm 1/2}|v\rangle$, and the two "trion" states $|t_\pm\rangle = b^\dagger_{1/2} b^\dagger_{-1/2} c^\dagger_{\pm 1/2}|v\rangle$.

In order to evaluate the quality of the implemented gates, we adopt the measure of process fidelity between an ideal quantum operation $\mathcal{E}'$ and the real quantum operation $\mathcal{E}$: $F_{pro}(\mathcal{E}, \mathcal{E}') = \mathrm{Tr}(\sqrt{\sqrt{\rho_{\mathcal{E}'}}\rho_{\mathcal{E}}\sqrt{\rho_{\mathcal{E}'}}})^2$, where $\rho_{\mathcal{E},\mathcal{E}'}$ are the induced Jamiołkowski state representations of the maps [23]. Here the ideal operation is a single qubit unitary gate U and the implemented one is the nonunitary evolution $V = e^{-i\tilde{H}\tau_{gate}}$ generated by $\tilde{H}$ on the qubit subspace for the gate time τ_{gate}, in which case we can write $F_{pro} = |\mathrm{Tr}[U^\dagger V]|^2/4$. Calculated fidelities for a generic single qubit gate are presented in Fig. 2. Two field polarization settings are used in an Euler decompositions of the gate and are optimized such that $\tau_{gate} < 45/\Gamma$.

As a concrete example, let us characterize the Hadamard rotation of a single electron spin. We take for the light-hole lifetime in quantum dot the quantum well value, 50 ps, leading to $\Gamma/2\pi \approx 30$ GHz. For a magnetic field of 1 T, we have $u \approx 0.2\Gamma$. The optimum photon ellipticity then yields a total gate time of ≈ 300 ps with an excellent fidelity 97%, using $\delta/\Gamma = 200$ and $\tilde{\Omega}/\Gamma = 60$ for $\alpha = \pi/10$.

Regarding the evaluation of process fidelity, let us stress that instead of a light-hole effective lifetime, the coupling between heavy and light holes can be directly included in the quantum dot Hamiltonian. This results in hole eigenstates that contain a light-hole fraction ensuring a channel for the mechanism of our proposal [24]. This fraction also controls performances of spin manipulations. Moreover, we evaluate process fidelities using the light-hole lifetime measured in quantum wells which can seem questionable. However, while considering such short-lived light holes we obtain performances already comparable to the ones predicted with long-lived heavy holes in the Voigt configuration. Furthermore, as previously underlined, light holes can become lowest energy hole states in strained structures [19]. Thereby, light holes are long-lived, and ultrafast quantum control reaches almost unity fidelity.

Conclusion.—We have shown that the quantum control of a single electron spin can be simply obtained by acting on a single pump-laser polarization. This is possible in a Faraday configuration, when the pump beam is slightly tilted with respect to a small magnetic field. This tilted geometry allows one to excite virtual electron light-hole pairs with π symmetry, a requirement to engineer arbitrary rotations. Finally, let us note that while the unabsorbed pump laser induces a precession of the trapped spin, it also experiences a rotation of its plane of polarization. Hence, measurements of Kerr or Faraday rotations of the pump laser appear as natural means to also readout prepared spin states [1,15].

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