

Gravitational Waves from Fragmentation of a Primordial Scalar Condensate into Q Balls

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A generic consequence of supersymmetry is the formation of a scalar condensate along the flat directions of the potential at the end of cosmological inflation. This condensate is usually unstable, and it can fragment into nontopological solitons, Q balls. The gravitational waves produced by the fragmentation can be detected by the Laser Interferometer Space Antenna, Advanced Laser Interferometer Gravitational-Wave Observatory, and Big Bang Observer, which can open an important window to the early Universe and the physics at some very high energy scales.

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Supersymmetry is widely regarded as a likely candidate for physics beyond the Standard Model. While many variants of supersymmetry have been considered, all of them have scalar potentials with some flat directions lifted only by the supersymmetry-breaking terms. At the end of cosmological inflation, the formation of a scalar condensate along the flat directions can have a number of important consequences [1]. In particular, it can be responsible for generation of the matter-antimatter asymmetry via the Affleck-Dine (AD) mechanism [2], and, in some models, dark matter can be produced in the same process [3,4]. Some flat directions could be responsible for the primordial inflation [5,6].

The formation of an AD condensate is a generic phenomenon, relying only on the assumptions of inflation and supersymmetry. In general, this condensate is unstable: an initially homogeneous condensate can break up into lumps of the scalar field, called Q balls [7], under some very generic conditions [3]. All phenomenologically acceptable supersymmetric generalizations of the standard model admit Q balls [8], which can be stable, or can decay into fermions [8,9]. The formation of Q balls is accompanied by a coherent motion of the scalar condensate, which creates the source of gravity waves. We will show that fragmentation of the scalar condensate into Q balls can produce gravitational waves detectable by LISA [10], LIGO III [11], and BBO [12].

The physics of AD condensate fragmentation has been studied both analytically [3,13–16] and numerically [17–23]. At the end of inflation (assuming that the inflation occurs in a hidden sector at high scales) the condensate has a uniform density, with small perturbations of the order of 10^{-5} [1]. Under some rather generic conditions, the instabilities develop and lead, eventually, to the formation of Q balls, which can either decay or remain as stable relics [3,13]. Although the final product of such evolution, Q balls in the ground state, are spherically symmetric, the coherent motions associated with the condensate fragmentation and rearrangement are not spherically symmetric.

Moreover, the newly formed Q balls first appear in their excited states and oscillate until they settle in the spherically symmetric ground states [18,19]. The lack of spherical symmetry in the process of fragmentation is essential for generating the gravity waves.

Following the general picture developed in Refs. [3,15], the scalar condensate undergoing fragmentation can be approximated, in the linear regime, as $\phi(x, t) = \phi(t) \equiv R(t)e^{i\Omega(t)}$, plus a perturbation $\delta R, \delta\Omega \propto e^{S(t) - i\vec{k}\cdot\vec{x}}$. One finds that the homogeneous solution is unstable due to some exponentially growing modes, $\text{Re}\alpha > 0$, where $\alpha = dS/dt$ [3,15,16]. The mass density of the condensate undergoing fragmentation can be written as

$$\rho(x, t) = \rho_0 + \rho_1(x, t), \quad (1)$$

where

$$\rho_1(x, t) = \epsilon\rho_0 \int d^3k e^{\alpha_k t} \cos(\omega t - \vec{k}\cdot\vec{x}). \quad (2)$$

The instability develops when there is a band of growing modes with positive and large enough α_k [3]. The linear approximation breaks down when $\epsilon \exp(\alpha_k t) \sim 1$, but we will use this representation, up to its limit of applicability, to get the estimates of the gravity waves produced.

The quadrupole moment that generates gravity waves is given by [24]

$$D_{ij} = \int d^3x x_i x_j T^{00}(x, t), \quad (3)$$

where the energy-momentum tensor $T^{00}(x, t) \approx \rho(x, t)$. The space integration is over some arbitrary volume.

The power emitted in gravity waves in one frequency mode is given by

$$P(\omega) = \frac{2}{5}G\omega^6 (D_{ij}^*(\omega)D_{ij}(\omega) - \frac{1}{3}|D_{ij}(\omega)|^2), \quad (4)$$

and the total energy emitted in gravitational waves, in all frequencies, is given by

$$E \sim \left(2\pi \int p(\omega) d\omega\right) \Delta t, \quad (5)$$

where Δt is the duration of the fragmentation.

Based on the analytical and numerical calculations of the condensate fragmentation [3,18,25], we take the typical parameters of the fastest-growing mode:

$$k \sim \xi_k 10^2 H_*, \quad \omega_k \sim vk \sim \xi_k 10^2 v H_*, \quad (6)$$

where H_* is the Hubble constant at the time of the condensate fragmentation, and v is the typical group velocity of the wave front in the evolution of the condensate, and we expect that the dimensionless factor $\xi_k \sim 1$, based on the results of Refs. [3,18,25].

Since no cancellations are expected in the absence of spherical symmetry, we replace the $x_i x_j$ by $(f_k \times 10^2 H_*)^{-2}$ in the space integration, take the volume to be $V \sim H_*^{-3}$, and assume that $\epsilon \exp(\alpha_k t) \sim 1$. Then, for the leading mode,

$$D_{ij}(t) \sim H_*^{-3} (10^2 H_*)^{-2} \rho_0 \cos(\omega_k t - kx), \quad (7)$$

and, in frequency space,

$$D_{ij}(\omega) \sim 10^{-4} \xi_k^{-2} \frac{\rho_0}{H_*^5}. \quad (8)$$

For $\omega \sim 10^2 v H_*$, we estimate the power in gravitational waves in a Hubble volume:

$$P \sim 10^4 \xi_k^{-2} G \frac{\rho_0^2 v^6}{H_*^4}. \quad (9)$$

To estimate the velocity of the wave front in the process of fragmentation, we note that, for the mode $\phi(x, t) \approx R(t) \times \exp\{\alpha_k t\} \cos(\omega_k t - kx)$, where $R(t)$ is a slowly changing function of time [26],

$$v \sim \xi_v |\dot{\phi}/\phi_x| \sim \xi_v \alpha_k/k, \quad \xi_v \sim 1, \quad (10)$$

where the uncertainty factor $\xi_v \sim 1$ will be retained to keep track of the uncertainty in the final answer.

The relation between α_k and k is given by a dispersion relation [3], which takes a simple form,

$$(\alpha_k^2 + k^2)(\alpha_k^2 - (\dot{\Omega}^2 - V''(R))) + 4\dot{\Omega}^2 \alpha_k^2 = 0, \quad (11)$$

under the following assumptions: $H \ll k \sim \alpha_k \ll \sqrt{\dot{\Omega}^2 - V''(R)} \sim \dot{\Omega} \sim m_\phi$ valid in the case of the fastest-growing mode in gravity-mediated supersymmetry-breaking models (the latter is essential for the $\dot{\Omega} \sim m_\phi$ condition [1]). This equation has an approximate solution: $\alpha_k \approx k/\sqrt{3}$, and

$$v^6 \sim \xi_v^6 (\alpha_k/k)^6 \sim \xi_v^6 (1/\sqrt{3})^6 \sim 10^{-2} \xi_v^6. \quad (12)$$

The fragmentation takes place on the time scale of the order of $\Delta t \sim \alpha_k^{-1} \sim \xi_k^{-1} 10^{-2} H_*^{-1}$. (Here we neglect the possible contributions from the collisions and oscillations of Q balls, which can take place on a much longer time scale [3,27].) The total energy in gravity waves generated in the Hubble volume is

$$E \sim P \Delta t \sim G \frac{\rho_0^2}{H_*^5} \xi_k^{-3} \xi_v^6. \quad (13)$$

This corresponds to the energy density in gravitational waves at the time of production

$$\rho_{\text{GW}*} \sim 10^{-3} \xi_k^{-3} \xi_v^6 \frac{\rho_0^2}{H_*^2 M_{\text{Pl}}^2}, \quad (14)$$

where $M_{\text{Pl}} = 1/\sqrt{8\pi G}$ is the reduced Planck mass. Hence, the fraction of the energy density in gravitational waves at the time of production is

$$\Omega_{\text{GW}*} \sim 10^{-3} \xi_k^{-3} \xi_v^6 \frac{\rho_0^2}{(H_* M_{\text{Pl}})^4}. \quad (15)$$

If the energy density of the condensate is comparable to the total energy density, or if the condensate energy dominates the energy in the Universe, then $\rho_0 \sim 3H_*^2 M_{\text{Pl}}^2$, and $\Omega_{\text{GW}*} \sim 10^{-3}$.

The energy density in the condensate depends on the model, and, foremost, on the type of supersymmetry-breaking terms that lift the flat direction. This is because the potential along the flat direction depends on supersymmetry breaking (it vanishes in the limit of exact supersymmetry), and there are many ways to break supersymmetry. In gauge-mediated supersymmetry-breaking scenarios the potential can have the form [3]

$$V(\phi) \approx M_S^4 \log\left(1 + \frac{|\phi|^2}{M_S^2}\right). \quad (16)$$

Here M_S is the scale of supersymmetry breaking, which is of the order of $\mathcal{O}(1)$ TeV. In gravity-mediated scenarios, the flat directions are lifted by mass terms that persist all the way to the Planck scale [13]:

$$V(\varphi) \approx m_\phi^2 \left(1 + K \log\left(\frac{|\phi|^2}{M_{\text{Pl}}^2}\right)\right) |\phi|^2, \quad (17)$$

where $K \sim 0.05$ (for squark directions) describes the running of the mass term [13]. Since $m_\phi \sim M_S \sim 1\text{--}10$ TeV in typical models, both potentials are phenomenologically acceptable near the minimum. However, in AD condensate and inside the Q balls that form in its fragmentation, the vacuum expectation value (VEV) can be very large.

The main difference between gauge and gravity-mediated cases for us is the mass per baryon number stored in the AD condensate and in the Q balls that form eventually as a result of the fragmentation. In the gravity-mediated scenarios, the mass density is $\rho_0 \sim m_\phi^2 \phi^2$, the global charge density is $n_Q \sim m_\phi \phi^2$, and the mass per unit global charge is of the order of m_ϕ , independent of the VEV ϕ_0 . In gauge-mediated scenarios, the mass density is $\rho_0 \sim m_\phi^4$, the global charge density is $n_Q \sim m_\phi \phi^2$, and the mass per unit global charge is $\rho_0/n_Q \sim m_\phi^2/\phi$ [3,9].

The flat directions that carry a nonzero global charge $Q = (B - L)$ contribute to the generation of the baryon asymmetry via AD process [2]. The requirement that $\eta_B = n_B/n_\gamma \sim 10^{-10}$ implies that the total mass density of such

a condensate cannot be of the order of the total density of the universe in generic models. This is true in both gauge-mediated and gravity-mediated supersymmetry-breaking models. The gravity waves from the fragmentation of such a condensate are well below the capabilities of the current and planned detectors.

On the other hand, there are flat directions whose baryon number B and lepton number L are equal to each other [1,28,29]. While the flat directions with $B \neq L$ contribute to baryon asymmetry of the Universe, those with $B = L$, have zero $(B - L)$ density. Electroweak sphalerons destroy any primordial $(B + L)$ asymmetry, and so the corresponding $\eta_{B+L} = n_{B+L}/n_\gamma$ is not constrained. It is possible that, at the time of the fragmentation, $\eta_{B+L} \gg \eta_B$. For $B = L$ flat directions, there is no reason why ρ_0 cannot be of the order of the total energy density. The fragmentation of such flat directions can produce a detectable level of gravitational waves.

There are various examples in the literature of the flat directions that can dominate the energy density of the Universe, while they do not contribute to (and are not constrained by) the observed baryon asymmetry of the Universe. This is the case, for example, when the effective mass for the phase direction is large during inflation, which results in the initial condition with a very small $\dot{\Omega}$ for the scalar condensate $\phi(x, t) = \phi(t) \equiv R(t)e^{i\Omega(t)}$. This is also the case when the inflation is driven by a flat direction, udd, LLe or NH_uL [5,6,30]. Another well-known example is a flat direction that acts as a curvaton and dominates the energy density of the Universe at the time of oscillations and decay [31]. In all of these cases, the net global charge of the condensate is negligible, but the fragmentation can still occur and produce Q balls and anti- Q balls [25,32].

Once the gravitational waves are created, they are decoupled from the rest of the plasma. We can estimate the peak frequency of the gravitational radiation observed today, $f_* = \omega_k/2\pi$:

$$f = f_* \frac{a_*}{a_0} = f_* \left(\frac{a_*}{a_{\text{rh}}} \right) \left(\frac{g_{s,0}}{g_{s,\text{rh}}} \right)^{1/3} \left(\frac{T_0}{T_{\text{rh}}} \right) \\ \approx 0.6 \text{ mHz} \xi_k \xi_v \left(\frac{g_{s,\text{rh}}}{100} \right)^{1/6} \left(\frac{T_{\text{rh}}}{1 \text{ TeV}} \right) \left(\frac{f_*}{10 H_*} \right), \quad (18)$$

where we have assumed that $a_* \approx a_{\text{rh}}$, which also means that during the oscillations of the AD condensate the effect of the Hubble expansion is negligible. The values of relativistic degrees of freedom are $g_{s,\text{rh}} \sim 300$, $g_{s,0} \sim 3.36$. The subscript “rh” denotes the epoch of reheating and thermalization, while the subscript “0” refers to the present time. As we discussed, the typical frequency of the oscillations of the AD condensate is $\omega_k \sim 10^2 H_*$ [3,18,25]. Then for $T_{\text{rh}} \sim 1$ TeV (such a value of the reheat temperature is natural when the flat direction is responsible for reheating the Universe [30,33]), the frequency is of the order of mHz, which is in the right frequency range for LISA [10]. A higher temperature $T_{\text{rh}} \sim 100$ TeV corresponds to the LIGO III frequency range, 10–100 Hz [11].

Signals in both of these ranges will be accessible to BBO [12]. Since the supersymmetry-breaking scale is related to the energy in the condensate, as well as the reheating temperature, LIGO III and BBO could be in the position to probe supersymmetry broken above 100 TeV, beyond the reach of Large Hadron Collider (LHC).

The fraction of the critical energy density ρ_c stored in the gravity waves today is

$$\Omega_{\text{GW}} = \Omega_{\text{GW}^*} \left(\frac{a_*}{a_0} \right)^4 \left(\frac{H_*}{H_0} \right)^2 \\ \approx \frac{1.67 \times 10^{-5}}{h^2} \left(\frac{100}{g_{s,*}} \right)^{1/3} \Omega_{\text{GW}^*} \approx 10^{-8} \xi_k^{-3} \xi_v^6 h^{-2}, \quad (19)$$

where a_0 and H_0 are the present values of the scale factor and the Hubble expansion rate. LISA can detect the gravitational waves down to $\Omega_{\text{GW}} h^2 \sim 10^{-11}$ at mHz frequencies, while LIGO III is sensitive to $\Omega_{\text{GW}} h^2 \sim (10^{-5} - 10^{-11})$ in the $(5 - 10^3)$ Hz frequency band. Therefore, the gravitational waves with $\Omega_{\text{GW}^*} \sim 10^{-3}$ and $\Omega_{\text{GW}} h^2 \sim 10^{-8}$ from the fragmentation of an AD condensate can be detected. The first results of our numerical simulations (work in progress) appear to produce the gravitational-wave signal that is somewhat weaker. We attribute the difference to the value of the uncertainty factor $\xi_k^{-3} \xi_v^6$, which is especially sensitive to the average wave front velocity.

As one can see from Eq. (14), the power generated in frequency ω is proportional to

$$\xi_k^{-3} \sim \left(\frac{\omega}{10^2 H_*} \right)^{-3}.$$

Hence, the spectrum is strongly peaked near the longest wavelength, of the order of the Q ball size [3,18,25]. The relatively narrow spectral width will help distinguish this signal from the gravity waves generated by inflation [34], which are expected to have an approximately scale-invariant spectrum (and a smaller amplitude). Future numerical simulations will help refine the prediction for the signal from Q ball formation, which can help distinguish this source from a phase transition in the early Universe [35]. LISA and LIGO III will be able to discriminate the gravity waves due to fragmentation from those of point sources, such as merging black holes and neutron stars, which have specific “chirp” properties [36]. Furthermore, the signal discussed here will not create a significant background for the cosmic microwave polarization experiments, such as B-Pol [37], which can detect the gravity waves with extremely long wavelength.

Some additional gravitational waves can be generated by collisions and oscillations of Q balls [3,27]. We leave the discussion of the magnitude of this additional contribution to future studies.

To summarize, the fragmentation of a scalar condensate into Q balls, which is a generic consequence of supersymmetry and inflation, can produce a detectable level of gravitational waves, up to $\Omega_{\text{GW}} h^2 \sim 10^{-8}$, near the peak

