

Formulation of Time-Resolved Counting Statistics Based on a Positive-Operator-Valued Measure

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We propose a derivation of the full counting statistics of electronic current based on a positive-operator-valued measure. Our approach justifies the Levitov-Lesovik formula in the long-time limit, but can be generalized to the detection of finite-frequency noise correlations. The combined action of the projection postulate and the quantum formula for current noise at high frequencies imply an additional white noise. Estimates for this additional noise are in accordance with known experiments. We propose an experimental test of our conjecture by a simultaneous measurement of high- and low-frequency noise.

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The core of quantum measurement theory is the projection postulate [1]. It provides a consistent description of a sequence of measurements. Quantities represented by non-commuting operators cannot be measured simultaneously. The corresponding projection operators have to be time ordered. For continuous variables, the projection postulate should be replaced by a positive-operator-valued measure (POVM) [2]. The idea of the POVM is that one does not measure the exact value for a given operator but a finite accuracy is taken into account due to some interaction with the detector and its internal dynamics. However, due to Naimark's theorem [3], every POVM can be realized by a set of orthogonal projections in an extended Hilbert space. The resulting POVM will depend on the detection scheme.

The statistical behavior of current flow in a quantum electronic contact—e.g., a narrow constriction in a two-dimensional electron gas—can be found by measurement of the correlation functions. The long-time cumulants of the transferred charge can be derived from the Levitov-Lesovik formula [4], which led to the foundation of the electronic version of full counting statistics (FCS) [5–7]. It has been confirmed experimentally for noise [8] and third cumulant [9–11]. On the other hand, the current-current correlation function (noise spectral density) is given by the quantum noise [12,13], which coincides with the FCS result at low frequencies. The high-frequency quantum noise can be obtained by a generalization of FCS to finite frequencies with additional predictions for higher cumulants at arbitrary frequency [14,15]. Also, the semiclassical predictions of the third cumulant are consistent with purely quantum results in some limits [16]. The behavior of quantum noise has been confirmed experimentally also for high frequencies [17–19]. From the fundamental point of view, the low-frequency results can be justified by a proper use of the projection postulate, but there is no unique derivation for high frequencies [20]. A similar problem occurs for a chain of spin-resolved detectors, for which the results depend on the detector properties [21]. While it is reasonable to expect an influence of the detector

on the outcome [7,21,22], it should be possible to separate it from the bare signals of the sample.

In this Letter, we address the question, if a general definition of FCS for finite frequencies is possible—maintaining the probabilistic interpretation. We will demonstrate that the standard definition of FCS, when generalized to finite frequency, can lead to negative probabilities. To cure this deficiency, we show that taking into account a minimal model of a detector, a POVM of FCS can be introduced, which leads to positive definite probabilities.

The definition of the generating functional for a probability distribution ϱ of a given time trace of the current through a quantum point contact, $I(t)$, is

$$e^{S[\chi]} = \langle e^{\int i\chi(t)I(t)dt/e} \rangle_e = \int DI \varrho[I] e^{\int i\chi(t)I(t)dt/e}. \quad (1)$$

On the other hand, one can first define FCS generating function [4,6,7,14]

$$e^{S[\chi, \phi]} = \text{Tr} \hat{\rho} \tilde{\mathcal{T}} e^{\int (idt/2e)[\chi(t) + 2\phi(t)]\hat{I}(t)} \times \mathcal{T} e^{\int (idt/2e)[\chi(t) - 2\phi(t)]\hat{I}(t)}. \quad (2)$$

Here, $\hat{\rho}$ denotes initial state density matrix, $\hat{I}(t)$ is the Heisenberg current operator, ϕ refers to classical phase bias, and \mathcal{T} ($\tilde{\mathcal{T}}$) denotes (anti-) time ordering. A detailed definition of $\hat{I}(t)$ will be given later.

Taking $S[\chi] = S[\chi, 0]$, we obtain ϱ by inverse Fourier transform of (1). However, this gives positive probabilities only in the zero-frequency limit. For time-dependent quantities, we can construct the following counterexample for a single-mode point contact at zero temperature in the tunneling limit (transmission $T \ll 1$). Let us define

$$X = \int_0^{t_0} dt dt' I(t) I(t') [e^{-(t-t')^2/s^2} - 2e^{-(t-t')^2/9s^2}]. \quad (3)$$

Then, following [14], we find $\langle (\delta X)^2 \rangle_e = -T t_0 e^4 / 3s \pi^{3/2}$ for $\delta X = X - \langle X \rangle_e$ and $t_0 \gg s$. This obviously contradicts the interpretation of ϱ as probability.

To overcome this fundamental problem, we now construct a positive definite probability of time-dependent FCS based on a POVM. Instead of the projection operator, we define the more general Kraus operator [23]

$$\hat{K}[I] = \int D\varphi \mathcal{T} e^{\int dt [i\varphi(t)[\hat{I}(t) - I(t)] / e^{-\varphi^2(t)/\tau}}. \quad (4)$$

Causality is preserved since the detector affects the measurement only in later times. The time scale τ describes internal fluctuations of the detector and depends on its temperature in general. For $\tau \rightarrow \infty$, the measurement is accurate, but the detector noise strongly affects the system by full projection. A shorter τ reduces the influence of detector but induces a larger measurement error. The integration measure contains also a normalization factor to be determined later. The positive definite probability of a given $I(t)$ is defined as

$$\rho[I] = \text{Tr} \hat{\rho} \hat{K}^\dagger[I] \hat{K}[I], \quad (5)$$

for the given initial density matrix $\hat{\rho}$. We note that our choice of the Kraus operator represents generically the influence of a detector, parametrized by a single parameter τ . What concrete models of detectors lead to our definition of the Kraus operator is an interesting question, which we will not address here.

We now substitute in Eq. (5) $\varphi \rightarrow \phi \pm \chi/2$ in \hat{K} and \hat{K}^\dagger , respectively. The generating functional $\mathcal{S}[\chi] = \ln \langle \exp[i \int dt \chi(t) I(t)] \rangle_\rho$ necessary for the calculation cumulants takes the form

$$\mathcal{S}[\chi] = \ln \int D\phi e^{\mathcal{S}[\chi, \phi] - \int dt [2\phi^2(t) + \chi^2(t)/2]/\tau}, \quad (6)$$

where $\mathcal{S}[\chi, \phi]$ is defined by (2). The measure $D\phi$ is scaled to keep $\mathcal{S}[\chi \equiv 0] = 0$. The measuring device affects the generating function by the additional exponent in (6). In [21], Di Lorenzo and Nazarov used the expression $\tau^2 \dot{\phi}^2 + \phi^2/(\Delta\phi)^2$ instead of ϕ^2 , with $\Delta\phi$ as an additional parameter, and considered low-frequency measurements. In contrast, we rather assume a continuous weak measurement of the system to obtain finite-frequency correlations.

To further model our measuring device, we note that in general a current measurement has also a *spatial* sensitivity, i.e., the point of the measurement is not exact. In experiments, it can be usually related to the finite capacitance of the sample. Therefore, we assume a generic form of the current operator in a quasi-one-dimensional lead as

$$\hat{I}(t) = \int \frac{dx}{\sqrt{2\pi}\Delta x} \hat{I}(x, t) e^{-(x-x_0)^2/2\Delta x^2}. \quad (7)$$

The setup is shown in Fig. 1(a). The real dispersion may be non-Gaussian. However, we stress that our model is general enough to capture the essential physics, but still allows some analytical progress.

We will assume noninteracting electrons and energy- and spin-independent transmission through the M mode junction. We count *all* modes, although most of them are

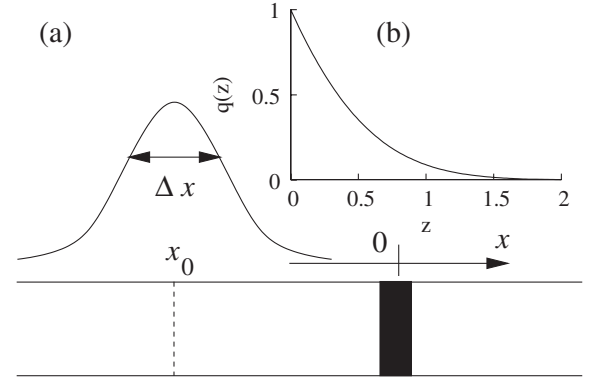


FIG. 1. (a) Sensitivity of current measurement. The Gaussian distribution refers to the dispersion of current measurement. (b) The function $q(z)$.

just reflected and denote the Fermi velocity, the transmission, and the reflection coefficients for mode n by v_n , T_n , and $R_n = 1 - T_n$, respectively. For convenience, we introduce $t_n = |x_0|/v_n$ and $\tau_n = \Delta x/v_n$. The times τ_n are related to RC times of the circuit, which limits the observable frequencies to $\omega \lesssim \tau_n^{-1}$. Furthermore, we assume that $t_n^{-1} \ll \tau_n^{-1}$, which means that the detector sensitivity function is entirely located on one side of the junction.

To model the electron transport, we apply the standard scattering picture around the Fermi level [12]. The Hamiltonian can be approximated by $\hat{H} = \sum_{\bar{n}} \int dx \hat{\mathcal{H}}_{\bar{n}}(x)$, where

$$\begin{aligned} \hat{\mathcal{H}}_{\bar{n}} = & i\hbar v_n [\hat{\psi}_{L\bar{n}}^\dagger(x) \partial_x \hat{\psi}_{L\bar{n}}(x) - \hat{\psi}_{R\bar{n}}^\dagger(x) \partial_x \hat{\psi}_{R\bar{n}}(x)] \\ & + q_n \delta(x) [\hat{\psi}_{L\bar{n}}^\dagger(x) \hat{\psi}_{R\bar{n}}(x) + \hat{\psi}_{R\bar{n}}^\dagger(x) \hat{\psi}_{L\bar{n}}(x)] \\ & - eV\theta(x) [\hat{\psi}_{L\bar{n}}^\dagger(x) \hat{\psi}_{L\bar{n}}(x) + \hat{\psi}_{R\bar{n}}^\dagger(x) \hat{\psi}_{R\bar{n}}(x)]. \end{aligned} \quad (8)$$

The scattering states obey standard fermionic anticommutation relations $\{\hat{\psi}_{A\bar{n}}^\dagger(x), \hat{\psi}_{B\bar{m}}(y)\} = \delta_{AB} \delta_{\bar{n}\bar{m}} \delta(x-y)$ and $\{\hat{\psi}_{A\bar{n}}(x), \hat{\psi}_{B\bar{m}}(y)\} = 0$. Here, $A = L, R$ denote left and right going state, $\bar{n} = (n, \sigma)$ denotes mode number n and spin orientation σ . The transmission coefficient is given by $T_n = \cosh^{-2}(q_n/\hbar v_n)$. The current operator is defined as $\hat{I}(x) = \sum_{\bar{n}} e v_n \hat{\psi}_{L\bar{n}}^\dagger(x) \hat{\psi}_{L\bar{n}}(x) - L \leftrightarrow R$. The initial density matrix for a thermal state is $\hat{\rho} = e^{-\hat{H}/k_B T} / \text{Tr} e^{-\hat{H}/k_B T}$, and the time evolution is governed by the Heisenberg operator $\hat{I}(x, t) = e^{i\hat{H}t/\hbar} \hat{I}(x) e^{-i\hat{H}t/\hbar}$.

For our model, the mean current is independent of the detector, $\langle I(t) \rangle_\rho = GV$, where the conductance $G = \sum_n T_n G_Q$ and $G_Q = e^2/\pi\hbar$. We define the noise spectral density as a second cumulant $e^2 P(\omega) = \int dt e^{i\omega t} \langle \delta I(0) \delta I(t) \rangle_\rho$, where $\delta I(t) = I(t) - \langle I(t) \rangle_\rho$. It is calculated from the functional derivative

$$\langle \delta I(0) \delta I(t) \rangle_\rho = -e^2 \frac{\delta^2 \mathcal{S}[\chi]}{\delta \chi(t) \delta \chi(0)} \Big|_{\chi=0}, \quad (9)$$

where $\mathcal{S}[\chi]$ is defined by Eq. (6). In our construction, the noise is a classical quantity and, hence, symmetric with respect to ω . We consider frequencies $|\omega\tau_n| \ll 1$ since we do not include capacitive effects and obtain

$$P(\omega) = P_{\text{off}} + P_S(\omega) + P_0(\omega) + P_\tau(\omega) + P_\Delta(\omega). \quad (10)$$

Let us discuss the behavior of all terms of this expression. The first one, $P_{\text{off}} = 1/\tau$ is a white offset noise, independent of temperature and voltage bias. Defining $w(\omega) = \omega \text{cth}(\hbar\omega/2k_B T)$ and $w_\pm(\omega) = w(\omega \pm eV/\hbar)$, the second term

$$P_S(\omega) = \frac{1}{2\pi} \sum_n T_n \{ 2T_n w(\omega) + R_n [w_+(\omega) + w_-(\omega)] \} \quad (11)$$

is just the symmetrized quantum noise $\int dt \cos(\omega t) \text{Tr} \hat{\rho} \hat{\delta} \hat{I}(0) \hat{\delta} \hat{I}(t) / e^2$ [12]. However, for energy independent transmission, as we assume here, the asymmetric noise contains only the additional term $\sum_n T_n \omega / \pi$, which is independent of temperature and voltage. The next term is $P_0(\omega) = \sum_n 2R_n \sin^2(\omega t_n) w(\omega) / \pi$. This is a contribution to the quantum noise due to the finite flight time to the detector, as it depends on t_n . Note that it is independent of voltage and sensitivity. The problem of flight time has been already discussed in context of third cumulant [10,24], but there is no experimental evidence of its influence on the noise. The detection noise

$$P_\tau(\omega) = \frac{\tau}{4\pi^2} \left| \sum_n \left[\omega (1 + R_n e^{2i\omega t_n}) + \frac{i}{\sqrt{\pi}\tau_n} \right] \right|^2 \quad (12)$$

combines the effects of the measurement sensitivity τ and flight times t_n but is independent of voltage and temperature. Finally,

$$P_\Delta(\omega) = \int \frac{d\alpha}{(2\pi)^2} \sum_n f_n(\omega - \alpha) R_n T_n [w_+(\alpha) + w_-(\alpha)] \quad (13)$$

is an additional mixed noise. Here, the sensitivity amplitude is given by

$$f_n(\alpha) = \int dt \left[\exp\left(-\frac{1 - e^{-t^2/4\tau_n^2}}{8\sqrt{\pi}\tau_n/\tau}\right) - 1 \right] e^{i\alpha t}. \quad (14)$$

It is independent of the flight times, but all other parameters enter in a rather complicated way. Equation (13) is the only term depending on τ_n in the limit $|\omega\tau_n| \ll 1$. However, voltage and temperature are arbitrary. For $\tau \ll \tau_n$, the amplitude reduces to $f_n(\alpha) = \tau [e^{-\alpha^2 \tau_n^2} - \delta(\alpha)] \times \sqrt{\pi}/\tau_n / 4$.

We assume that most modes are closed, $\sum_n T_n \ll M$, which is true in many experimental setups (e.g., tunnel barrier, diffusive wire, quantum point contact). We will consider several interesting limits: short and long wire

(flight time), low and high frequencies, and zero temperature. They correspond to the most common experimental setups. For short flight times, $|\omega t_n| \ll 1$, we have $P_0 = 2\omega^2 w(\omega) \sum_n t_n^2$. For long flight times, $|\omega t_n| \gg 1$, we get $P_0 = M w(\omega) / \pi$ because random flight times imply $\sin^2(\omega t_n) \rightarrow 1/2$. In both cases, $P_\tau = (\sum_n \tau_n^{-1})^2 \tau / 4\pi^3$ is independent of voltage and yields additional offset noise. For low frequency and a slow detector, $\hbar|\omega| \ll (\hbar/\tau, \hbar/\tau_n) \ll eV, k_B T$ the mixed noise is negligible since $P_\Delta \ll P_S(0)$. However, for $k_B T \ll \hbar|\omega| \ll \hbar/\tau_n \ll \hbar/\tau$, we have $P_\Delta = (\tau/8\pi^2) \sum_n R_n T_n q(\tau_n |eV|/\hbar) / \tau_n^2$, where $q(z) = e^{-z^2} - 2z \int_z^\infty e^{-t^2} dt$. The decay of $q(z)$ is shown in Fig. 1(b). It vanishes for $eV \gg \hbar/\tau_n$, which means that the size of the wave packet becomes smaller than the spatial sensitivity of the detector.

From the above results, we conclude that the POVM reproduces the standard quantum result in the case $|\omega| \ll (1/\tau_n, 1/t_n) \ll 1/\tau \ll eV/\hbar, k_B T/\hbar$. It can be shown that corrections to higher zero-frequency cumulants are negligible in this case because P_Δ is small. Hence, the Levitov-Lesovik formula and FCS [4–7] are justified, as expected.

The situation becomes more interesting, if we look at the high-frequency quantum noise. We can make P_τ and P_Δ negligible by choosing a very small τ , which corresponds to a weak detection. For small t_n , also P_0 gives only a small contribution. Moreover, higher cumulants then have also negligible corrections to predictions of generalized FCS as small τ corresponds to $\phi \rightarrow 0$ in Eq. (2). What remains is the large white Gaussian offset noise P_{off} —the price we have to pay for small τ . Conversely, lowering P_{off} will increase P_Δ , which additionally depends on voltage. They become of the same order at $\tau \sim \tau_n$, and P_Δ is growing as $\sqrt{\tau/\tau_n}$ for $\tau \gg \tau_n$. Hence, one cannot get rid of the additional noise by increasing τ since it increases the back-action noise.

The offset noise for a few mode quantum point contact in most high-frequency experiments [17,18] was usually subtracted. However, the results of the recent experiments [18,19] show relatively high absolute noise temperature $T^* = G_Q P h / 4G k_B$, setting an upper bound to the offset noise. We find $T^* = T_{\text{off}} + T_\Delta$ with $T_{\text{off}} = 24$ K and $T_\Delta = 1.5$ mK for a single-mode quantum point contact with transmission $1/2$, $\tau = 1$ ps, and $\tau_1 = 10$ ps. Although T_{off} is larger, it is constant whereas T_Δ depends on voltage bias and drops to zero according to the function $q(z)$. For zero temperature, this yields a characteristic voltage of $130 \mu\text{V}$ at $z = 1$. Diffusive and tunnel barriers have usually a higher conductance due to the large number of modes, and hence the offset temperature can be much lower. On the other hand, in a Josephson junction or a quantum dot as a detector, the measured quantity is more qualitatively than quantitatively related to the frequency dependent quantum noise [25].

Moreover, as the offset noise is white, it must be visible also at low frequencies. It would be therefore of great interest to test high- and low-frequency noise *simulta-*

neously. The high-frequency detector measures only the difference $P(\text{eV}) - P(0)$ whereas at low frequencies the absolute noise is measured. The fluctuation-dissipation theorem [26] will of course be maintained after subtraction of the offset noise and then taking the limit $\tau \rightarrow 0$.

We generalize our approach to a multiterminal measurement. We define the Kraus operator (4) as

$$\hat{K}[I] = \int D\phi \mathcal{T} e^{\int dt \sum_A \{i\phi_A(t)[\dot{I}_A(t) - I_A(t)]/e - \phi_A^2(t)/\tau_A\}} \quad (15)$$

where A labels the terminals. A simple example is the noise between the left and right sides of a junction, namely $e^2 P_{AB}(\omega) = \int dt e^{i\omega t} \langle \delta I_A(0) \delta I_B(t) \rangle$ where $A, B = L, R$ for left and right terminals (or equivalently $x_0 < 0$ and $x_0 > 0$), respectively. The cross correlation $P_{LR}(\omega)$ is finite only for $|\omega t_n| \ll 1$ since otherwise one averages over different t_n . In this limit, we have

$$\begin{aligned} P_{LL} &= P_{\text{Loff}} + P_S + P_{\tau_L} + P_{\Delta L} + P_{\Delta R}, \\ P_{RR} &= P_{\text{Roff}} + P_S + P_{\tau_R} + P_{\Delta L} + P_{\Delta R}, \\ P_{LR} &= P_{RL} = -P_S + P_x - P_{\Delta L} - P_{\Delta R}, \end{aligned} \quad (16)$$

where $P_x(\omega) = \omega^2(\tau_L + \tau_R)M\sum_n T_n/4\pi^2$. Here, $P_{\text{off}A} = \tau_A^{-1}$, P_{τ_A} , and $P_{\Delta A}$ are given by Eqs. (12) and (13) with τ and τ_n replaced by τ_A and τ_{nA} . P_{Δ} is here replaced by the sum of contributions of both detectors. The cross noise does not contain the offset noise, so it can help to estimate the measurement time scales, in particular $P_{\Delta L} + P_{\Delta R}$ at low frequency.

Finally, we propose the following test of our definition of a quantum probability. Consider two detectors, similar to [10], measure X_L and X_R defined by (3) for $I = I_L$ and I_R , respectively. The classical expectation $X_L = {}^{cl}X_R$ based on charge conservation in the low-frequency limit, leads to $\langle \delta X_L \delta X_R \rangle_{cl} > 0$. The quantum measurement using the probability density $\rho[I]$, results for $\tau \ll s$ in $\langle \delta X_L \delta X_R \rangle_{\rho} = \langle (\delta X)^2 \rangle_{\rho}$. This is, however, negative as we have shown in beginning.

In conclusion, we have constructed a positive probability measure, based on POVM, that justifies the use of Levitov-Lesovik formula and FCS in long-time (low-frequency) limit. Our approach cures certain deficits in the standard definition of FCS, which lead to negative probabilities. Introducing a generic one-parameter model of the influence of the detector, we predict an intrinsic additional white offset noise. Such a noise is in agreement with recent experiments, and a further verification by simultaneous low- and high-frequency noise measurements would be desirable.

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