α Channeling in a Rotating Plasma

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(Received 17 August 2008; published 13 November 2008)

The wave-particle α-channeling effect is generalized to include rotating plasma. Specifically, radio frequency waves can resonate with α particles in a mirror machine with $E \times B$ rotation to diffuse the α particles along constrained paths in phase space. Of major interest is that the α-particle energy, in addition to amplifying the rf waves, can directly enhance the rotation energy which in turn provides additional plasma confinement in centrifugal fusion reactors. An ancillary benefit is the rapid removal of alpha particles, which increases the fusion reactivity.

DOI: 10.1103/PhysRevLett.101.205003 PACS numbers: 52.20.−j, 52.55.Jd, 52.55.Pi

In magnetic mirror fusion devices, centrifugal forces can enhance the magnetic confinement [1–3]. A radial electric field induces rapid $\mathbf{E} \times \mathbf{B}$ plasma rotation, leading to the centrifugal force that directly confines ions axially. Electrons are then confined axially through the ambipolar potential. The radial field not only enhances the plasma confinement, but also produces the necessary plasma heating, as injected cold neutral fuel atoms are seen as moving at the rotation velocity in the rotating frame. Lately there has been a renewed interest in this effect [4–7], strengthened by recent findings of reduced turbulence due to sheared rotation [8,9].

What we show here is that in a deuterium-tritium centrifugal fusion reactor, the energy of α particles, the byproducts of the fusion reaction, might be advantageously induced to directly produce this rotation. The predicted effect relies on exploiting the population inversion of the birth distribution of α particles. This is a generalization of the α-*channeling effect*, where injected wave energy can be amplified at the expense of the α-particle energy, with the alpha particles concomitantly removed as cold particles [10]. In tokamaks, if the wave energy is damped on ions, the fusion reactivity might be doubled [11]. Similar advantageous uses of α channeling can be expected in mirror machines [12]. With several waves, a significant amount of the α-particle energy can be advantageously channeled in both tokamaks [13] and mirrors [14]. However, in previous considerations of α channeling, the plasma was not rotating strongly.

In strongly rotating plasma, significant new effects can occur because there are two further reservoirs of particle energy, namely, rotational and potential energy. For example, through a suitable choice of wave parameters, particles can now absorb wave energy yet cool in kinetic energy, with the excess energy being stored in potential energy. Alternatively, particle potential energy might be lost to wave energy with kinetic energy constant. These possibilities could not be achieved through particle manipulation in stationary systems, where the only coupling is between the kinetic energy and the wave energy. What is important for centrifugal mirror fusion is that radio frequency waves can drive a radial α-particle current, with the dissipated power extracted from the α-particle birth energy, thereby maintaining the radial potential which produces the necessary plasma rotation.

To derive the new effects, define the angular rotation frequency $\mathbf{\Omega}_E = \Omega_E \hat{z}$, so that the $\mathbf{E} \times \mathbf{B}$ drift velocity can be written as $\mathbf{\Omega}_E \times \mathbf{r} = \mathbf{E} \times \mathbf{B}/B^2$. For simplicity, consider constant Ω_E (solid-body rotation). Although some aspects may vary with the rotation profile, the concept should be applicable to arbitrary profiles $\Omega(r)$. The electric and magnetic field in the rotating frame are [15]

$$\tilde{\mathbf{E}} = \mathbf{E} + \frac{m}{q}\Omega^2\mathbf{r} + (\mathbf{\Omega} \times \mathbf{r}) \times \mathbf{B}, \tag{1}$$

$$\tilde{\mathbf{B}} = \mathbf{B} + 2\frac{m}{q}\mathbf{\Omega}. \tag{2}$$

The second term in Eq. (1) produces the centrifugal force, and the second term in Eq. (2) is due to the Coriolis effect. For $\mathbf{\Omega} = \mathbf{\Omega}_E$, the first and third terms in Eq. (1) will cancel. However, there will still be drifts due to the centrifugal force. We define as $\Omega_E^\star$ the unique frame of reference in which $\hat{\theta} \cdot \tilde{\mathbf{E}} \times \tilde{\mathbf{B}} = 0$ for the species of interest. Note that the magnetic moment is seen as invariant only in the frame rotating with frequency $\Omega_E^\star$. Our notational convention is to denote terms in this frame with a tilde.

Note that by flux conservation, $r^2/r_0^2 \propto \tilde{B}_0/\tilde{B}$. Thus for magnetic mirror ratio $R_m = \tilde{B}_m/\tilde{B}_0$, there is an effective confinement potential $\Phi_c = \frac{1}{2}m\Omega_E^{\star 2}r_0^2(1 - R_m^{-1})$, which varies with the midplane particle radius r_0. The loss-cone diagram is depicted in Fig. 1. The maximum confinement potential is $W_{E0w}(1 - R_m^{-1})$, where $W_{E0w} = \frac{1}{2}m\Omega_E^{\star 2}r_w^2$, and r_w is the midplane radius of the last field line not intersecting a wall.

Now consider a wave with frequency ω, parallel wave number $k_\parallel$, and azimuthal mode number $n_\theta = k_\theta r$. Because of the rotation, the wave frequency in the rotating frame will be $\tilde{\omega} = \omega - n_\theta\Omega_E^\star$. The wave-particle resonance condition is then $\tilde{\omega} - k_\parallel v_\parallel = n\tilde{\Omega}_c$, where the reso-

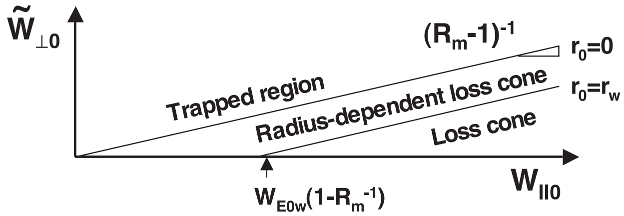

FIG. 1. The loss cone in (rotating) midplane energy coordinates for a rotating plasma, including centrifugal confinement.

nance is at the nth harmonic of the rotating-frame cyclotron frequency $\tilde{\Omega}_c = q\tilde{B}/m$ (q is the ion charge and m is its mass). The parallel velocity $v_\|$ is independent of the rotating frame, and corresponds to energy $W_{\|\mathrm{res}} = mv_\|^2/2$. Unlike in the stationary case, the related midplane parallel energy, $W_{\|0\mathrm{res}}$, will not be constant across the radius of the device. For a rf region at mirror ratio $R_{\mathrm{rf}} = \tilde{B}_{\mathrm{rf}}/\tilde{B}_0$, the resonant parallel energy in rotating midplane coordinates is

$$W_{\|0\mathrm{res}} = W_{\|\mathrm{res}} + W_{E0}(1 - R_{rf}^{-1}), \tag{3}$$

where $W_{E0} = m\Omega_E^{\star 2} r_0^2/2$ depends on the midplane radius r_0. An example resonant region is shown in Fig. 2.

Since the mirror system is axisymmetric, the diffusion paths will be the same as those for tokamaks [10],

$$dP_\theta/d\tilde{W} = n_\theta/\tilde{\omega}, \tag{4}$$

$$d\tilde{\mu}/d\tilde{W} = qn/m\tilde{\omega}, \tag{5}$$

where $\tilde{\mu} = m\tilde{v}_\perp^2/2\tilde{B}$ is the ion magnetic moment in the rotating frame, $\tilde{W} = \tilde{\mu}\,\tilde{B} + mv_\|^2/2$ is the kinetic energy in the rotating frame, and P_θ is the azimuthal canonical angular momentum (which is frame-independent).

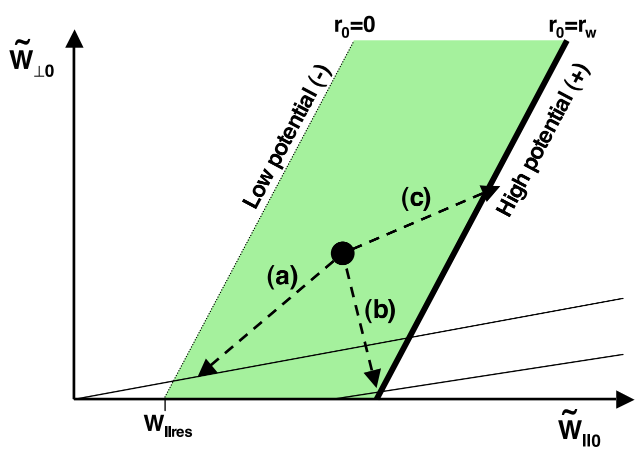

FIG. 2 (color online). The shaded region indicates the wave resonance, dependent on radius. The dashed lines depict diffusion paths that would eject particles using only perpendicular diffusion. Path (a) reduces both the kinetic and potential energy of the particle, path (b) increases potential energy but decreases kinetic energy, and path (c) increases both kinetic and potential energy. The energy balance is assumed by the wave.

The significant difference in the rotating frame is that the interaction of the particle with a wave at axial position z_{rf} changes the particle's perpendicular, parallel, and rotational kinetic energy, as well as its potential energy. The change in perpendicular energy may be written $\tilde{W}_\perp(z_{\mathrm{rf}}) \rightarrow \tilde{W}_\perp(z_{\mathrm{rf}}) + \Delta\tilde{W}_\perp$, the change in parallel energy, $\tilde{W}_\|(z_{\mathrm{rf}}) \rightarrow \tilde{W}_\|(z_{\mathrm{rf}}) + \Delta\tilde{W}_\|$, and the change in rotational energy, $W_E(z_{\mathrm{rf}}) \rightarrow W_E(z_{\mathrm{rf}}) + \Delta W_E$. Thus the wave interaction, breaking the adiabatic invariance of $\tilde{\mu}$, gives stochastic kicks in $\Delta\tilde{W}_\perp$, $\Delta\tilde{W}_\|$, and ΔW_E.

The energy kicks are correlated through the properties of the wave. The relation between $\Delta\tilde{W}_\perp$ and $\Delta\tilde{W}_\|$ is found, by Eq. (5), to be $\Delta\tilde{W}_\| = \Delta\tilde{W}_\perp k_\| v_\|/(n\tilde{\Omega}_c)$. The radial excursion is determined in terms of the perpendicular energy change by Eq. (4), yielding $r\Delta r = \Delta\tilde{W}_\perp n_\theta/(m\tilde{\omega}\tilde{\Omega}_c)$. This then gives the rotational energy change, $\Delta W_E = m\Omega_E^{\star 2} r\Delta r = \Delta\tilde{W}_\perp n_\theta \Omega_E^{\star 2}/(\tilde{\omega}\tilde{\Omega}_c)$, and the potential energy change, $q\Delta\Phi = -qE\Delta r = n_\theta\Omega_E\Omega_c/(\tilde{\omega}\tilde{\Omega}_c)\Delta\tilde{W}_\perp$.

Using the adiabatic invariance of $\tilde{\mu}$, flux conservation ($r^2/r_0^2 \propto \tilde{B}_0/\tilde{B}$), and conservation of energy, we require

$$\Delta\tilde{W}_\perp + \Delta\tilde{W}_\| - \Delta W_E = R_{\mathrm{rf}}^{-1}\Delta\tilde{W}_\perp + \Delta\tilde{W}_{\|0} - R_{\mathrm{rf}}\Delta W_E. \tag{6}$$

In rotating midplane coordinates, we have

$$\Delta\tilde{W}_{\perp 0} = \Delta\tilde{W}_\perp/R_{\mathrm{rf}}, \tag{7}$$

$$\Delta\tilde{W}_{\|0} = \left[\frac{k_\| v_\|}{n\tilde{\Omega}_c} + (R_{\mathrm{rf}} - 1)\frac{n_\theta\Omega_E^{\star 2}}{\tilde{\omega}\tilde{\Omega}_c} + (1 - R_{\mathrm{rf}}^{-1})\right]\Delta\tilde{W}_\perp, \tag{8}$$

$$\Delta r_0 = \frac{R_{\mathrm{rf}} n_\theta}{m r_0 \tilde{\omega}\tilde{\Omega}_c}\Delta\tilde{W}_\perp. \tag{9}$$

As the particle diffuses in radius it also changes its rotation energy W_E. This will lead to a change in midplane parallel energy for $R_{\mathrm{rf}} > 1$, as can be seen in Eq. (3). This is the source of the second term in brackets in Eq. (8). Note that the particle remains in resonance with the wave on its entire path in the limit $n_\| \rightarrow 0$.

With reference now to Fig. 2, note three ways particles might be extracted from a rotating mirror, with perpendicular diffusion only ($\Delta\tilde{W}_\| = 0$). The particle may be removed through the loss cone by path (a) at a low potential energy and a low kinetic energy. This requires the wave phase velocity in the rotating frame to be positive ($\tilde{v}_p = k_\theta/\tilde{\omega} > 0$). The same wave may be used to remove particles through the last flux surface at high kinetic and potential energy [shown by path (c)]. The energy balance in each case is carried by the interacting wave. Path (b) describes a diffusion path where the particle is removed with less kinetic energy than at its birth but at a higher potential energy. The particle may be removed either through the loss cone or the last flux surface. This is the useful case for maintaining the radial electric field.

To calculate the *branching ratio* f_E, the ratio of energy going into the radial potential to the total energy change, consider that the change in rest-frame kinetic energy is

$$\Delta W = \left(\frac{\omega}{\tilde{\omega}} + \frac{k_\parallel v_\parallel}{n\tilde{\Omega}_c}\right)\Delta\tilde{W}_\perp, \tag{10}$$

giving the branching ratio

$$f_E = \frac{-n_\theta \Omega_E \Omega_c}{\omega\tilde{\Omega}_c + \tilde{\omega} k_\parallel v_\parallel / n - n_\theta \Omega_E \Omega_c}, \tag{11}$$

$$\approx \frac{-n_\theta \chi}{1 + 4\chi + n_\theta \chi^2}, \tag{12}$$

where the approximation in Eq. (12) uses $\Omega_E, k_\parallel v_\parallel \ll \Omega_c$, with the dimensionless variable $\chi = \Omega_E/\Omega_c$, and resonance condition $\tilde{\omega} = \tilde{\Omega}_c + k_\parallel v_\parallel$. If these conditions are sufficiently strong, the fraction of the total energy change provided to the radial electric field is $f_E \approx -n_\theta \chi$. In the case $f_E > 1$, the particle reduces its kinetic energy and simultaneously absorbs wave energy, which can be expected because the direction of the rf wave phase velocity in the rotating frame, $\tilde{v}_p = \tilde{\omega}/k_\theta$, is opposite that in the laboratory frame, $v_p = \omega/k_\theta$. Path (b) in Fig. 2 describes a diffusion path in which the wave will be amplified if $f_E < 1$ (not all kinetic energy is converted to potential) or damped if $f_E > 1$ (wave energy transferred to potential energy).

The wave requirements for the α-channeling effect were calculated for the static mirror case [12,14]. Two conditions were considered to be important. In order for the diffusion path to be favorable, it must connect a dense area of phase space near the birth population to a less dense area of phase space near the loss boundary. In addition, the α-particle heating should be limited. It was shown that waves with purely perpendicular diffusion ($\Delta W_\parallel = 0$, $n = 1$, $k_\parallel v_\parallel \ll \tilde{\Omega}_c$) and $T_i \ll W_{\parallel\mathrm{res}} \ll W_{\alpha 0}$ satisfy the first requirement. In the rotating system, the results are the same; perpendicular diffusion with $W_{E0} < W_{\parallel\mathrm{res}} \ll W_{\alpha 0}$ will provide connection to the velocity-space loss cone, and α particles will leave at low energy. If particles gaining energy move inward, they are less likely to be lost at high energy. For an outward current to maintain the potential, Ω_E must be positive. This is also the preferred polarity for rotating mirror systems [1].

For α channeling in tokamaks, the mode-converted ion-Bernstein wave met the necessary wave characteristics [16,17], possibly in conjunction with other waves [13,18]. Whereas the wave requirements are identified here for the rotating mirror plasma, it remains to identify the precise wave. It would appear though, that similar to tokamaks, a wave in the ion cyclotron range of frequencies would be most suitable. With a suitable wave, efficient conversion of kinetic energy to potential energy can be achieved if $f_E \sim 1$. For $B_0 = 1$ T, this means that $-n_\theta \Omega_E \sim 50 \times 10^6\ \mathrm{s}^{-1}$. An electric field of 50 kV/cm at $r = 1$ m produces $\Omega_E = 5 \times 10^6$/s. To convert all of the kinetic energy change into electric potential energy, we may choose $n_\theta = 10$, or at $r = 1$ m, $k_\theta \approx 0.1$/cm.

We simulated this α-channeling effect in a rotating mirror plasma by following the full equations of motion for 1800 particles interacting with ten rf regions. The wave parameters $W_{\parallel\mathrm{res}0} = 750$ keV, $k_\parallel = -0.03$/cm, $E_{\mathrm{rf}} = 30$ kV/cm were constant across each wave, but $n_\theta = 12$–20 and $R_{\mathrm{rf}} = 1.2$–4.5 varied. The mirror ratio for the simulation was $R_m = 6$, and the device length was 20 m. We estimate that 65% of the energy would be channeled into the radial potential. Figure 3 depicts, for these parameters, the relative effectiveness of energy extraction from alpha particles of different pitch angles, as well as the relative time for extraction. Each point represents the average of 100 particles, with apparent fluctuations since not all injected ions are precisely on resonance with a wave. Particles with $\theta < 0.4$ are not confined (and exit through the loss cone), while particles with $\theta > 1.1$ do not have enough parallel energy to be resonant. The energy extraction needs to be completed before the collisional slowing down of the alpha particle.

The α-channeling effect proposed here uses the energy of α particles to support the rotation of the plasma, the main power requirement in centrifugal fusion reactors; the heating of the fuel is then automatic since through ionization new fuel particles are born with a high kinetic energy in the rotating frame. Define the fusion energy gain, $Q \equiv P_f/P_{\mathrm{circ}}$, as the ratio of fusion power to circulating power to maintain the rotation. Let η be the fraction of alpha-particle power $P_f/5$ that supports the rotation, then the fusion energy gain in the presence of the α-channeling effect can be written as $Q_{\mathrm{AC}} = Q/(1 - \eta Q/5)$. Thus, if all of the α-particle energy could be converted into rotation energy ($\eta = 1$), a reactor formerly operating at $Q = 5$ would become self-sustaining, requiring no external heating or energy input.

There are several considerations to address in choosing the branching ratio f_E in each wave region. In order to

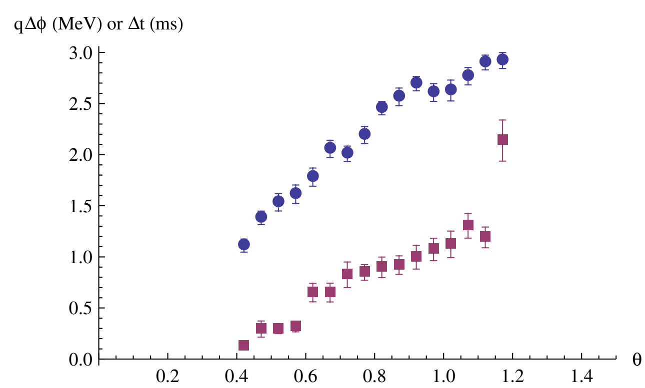

FIG. 3 (color online). Channeled energy (blue circles) and extraction time (purple squares) versus birth pitch angle.

extract efficiently the energy of the alpha particles, they must be removed by diffusion (having rate proportional to the standing wave energy) before they are lost collisionally. Thus, the branching ratio should be set first to assure that the waves reach sufficient amplitude for collisionless diffusion. On the other hand, it is not beneficial to amplify the wave beyond what is needed to satisfy this criterion. The power going into the waves in other fusion devices is generally thought to be most effectively channeled into fuel ion heating [10,12]; in contrast, the further heating of ions is not necessary in rotating mirrors, since they are born at high energy. To the extent that extra power is used to support the rotation, beyond the necessary rotation for confinement, the reactor essentially acts as a battery, producing a current source that may be loaded through the end electrodes in the same way as a hydromagnetic capacitor [19]. Most likely, the optimum design would just maintain the rotation and the necessary diffusion time.

Many implementations of open systems attempt the direct conversion of charged fusion product energy into electrical energy [20,21]. A recent suggestion for centrifugal confinement devices [22] captures α-particle energy both through the centrifugal potential (which goes directly into the rotation energy) and through a retarding potential (direct conversion). But the amount of energy extractable to maintain the rotation energy is a small fraction of the alpha-particle birth energy. In contrast, as proposed here, the α-particle energy can be almost entirely converted into potential energy.

The fact that the effect proposed here acts volumetrically—not at a surface—may alleviate the major engineering hurdle facing the use of rotating mirrors as fusion reactors [1,2,5], the end plate electrodes. These electrodes need to support large electric fields and are subject to breakdown, sputtering, and recycling [23].

Because centrifugal fusion reactors are run in the hot ion mode ($T_i > T_e$), the α-channeling effect is particularly fitting. The ions are hot in rotating plasma because they are born at the rotation speed. The rapid removal of α particles, which are slowed down primarily by electrons, then removes an important electron heat source, permitting an even cooler electron temperature. The cooler electron temperature in turn gives rise to a lower ambipolar potential, which means higher ion confinement. Higher ion confinement, in turn, then relaxes the need for additional rotational confinement, so that the mirror can be operated at lower rotation speeds and lower plasma potential. In addition to reducing the electron heating, the quick expulsion of alpha particles by waves reduces the dilution of fuel ions by the alpha particles. Like in a conventional mirror reactor, where the alpha-particle ash can dilute the fuel by as much as 30% [24], the prompt removal of this ash (and the channeling of that energy to fuel ions) can increase greatly the effective fusion reactivity at fixed plasma pressure [12].

In conclusion, we generalized the α-channeling effect to rotating plasma. A new quantity that appears in rotating plasma is the branching ratio, which measures the amount of particle kinetic energy that flows into particle potential energy as compared to the amount which flows into wave energy. By arranging for sufficient channeling of fusion alpha energy directly into electric potential energy, the rotation of the plasma can be maintained against momentum loss. The prompt removal of alpha particles also increases the effective fusion reactivity. Also, the volumetric maintenance of the radial potential should reduce the engineering complexity of the technologically challenging mirror end plates, if not to eliminate the need for these plates entirely. Moreover, the α-channeling effect is particularly well matched to enhance the reactor prospects of centrifugal fusion reactors, since these reactors are imagined to operate best at low electron temperature and high ion temperature. While the channeling of alpha energy in rotating plasma appears to significantly enhance the prospects for controlled nuclear fusion through centrifugal confinement, it remains to identify the specific plasma waves that can accomplish the speculative concepts put forth here.

This work was supported by U.S. DOE Contracts DE-FG02-06ER54851 and DE-AC0276-CH03073.

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