Laser Channeling of Bethe-Heitler Pairs

Erik Lötstedt,* Ulrich D. Jentschura, and Christoph H. Keitel
Max-Planck-Institut für Kernphysik, Postfach 103980, 69029 Heidelberg, Germany
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Electron-positron pair creation is analyzed for an arrangement involving three external fields: a high-frequency gamma photon, the Coulomb field of a nucleus, and a strong laser wave. The frequency of the incoming gamma photon is assumed to be larger than the threshold for pair production in the absence of a laser, and the peak electric field of the laser is assumed to be much weaker than Schwinger's critical field. The total number of pairs produced is found to be essentially unchanged by the laser field, while the differential cross section is drastically modified. We show that the laser can channel the angular distribution of electron-positron pairs into a narrow angular region, which also facilitates experimental observation.

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The creation of an electron-positron pair by an external electromagnetic field—the conversion of field energy into matter—remains an intriguing phenomenon, and its exploration continues to enhance our understanding of the foundations of field theory. Usually, pair creation is accomplished by weak, high-frequency fields, gamma photons, with the standard examples being the merging of two high-energy photons into an electron-positron pair or the conversion of a photon into a pair in the vicinity of a nucleus (the Bethe-Heitler process; see Ref. [1]). Strong, static macroscopic fields can also create pairs [2,3]. In the sense of QED perturbation theory, pair creation by a static field is a nonperturbative phenomenon, and its magnitude is controlled by a parameter χ, defined as $\chi = E/E_{\mathrm{crit}} = -eE\hbar/(m^2c^3)$, where E is the peak value of the electric field, $e = -|e|$ and m are the electron charge and mass, respectively, and $E_{\mathrm{crit}} \approx 1.3 \times 10^{16}$ V/cm is the so-called critical field. The basic result, which holds for any strong static electromagnetic field, is that the probability for pair creation is exponentially suppressed unless χ is at least of the order of unity.

If the field is allowed to oscillate, another parameter $\xi = -eE/(mc\omega)$, related to the angular frequency ω of the oscillation, becomes relevant. The value of ξ governs the nature of pair production. Specifically, the regime $\xi \ll 1$ is called the multiphoton regime, while for $\xi \gg 1$ the pairs are created by tunneling through the tilted potential gap of magnitude $2mc^2$, and one may call ξ the Keldysh parameter of vacuum ionization. The transition between the two regimes for an oscillating electric field was treated in [4–6]. Modern lasers achieve $\xi \gg 1$ for infrared frequencies, but even for the strongest lasers available we have $\chi \ll 1$. As is well known, a plane laser wave cannot create pairs by itself, due to energy-momentum conservation. Just like in a pure magnetic field [7], a second particle is necessary to provide the required momentum. In a focused laser pulse [8] or a standing wave [9,10], pair creation is possible, since the field configuration is different from a plane wave. Indeed, pair creation in a strong laser field induced by an additional high-energy gamma photon has been studied both theoretically [11–13] and experimentally [14,15]. In [16–20], the probing particle is replaced by a nuclear field.

In this Letter, we study the creation of an electron-positron pair by three fields: a high-energy gamma photon, the Coulomb field of a nucleus, and an intense, low-frequency laser field, as schematically shown in Fig. 1. Numerical estimates of the cross section for pair production in this field configuration are absent in the literature, to the best of our knowledge. Previous studies of related processes [21,22] obtained approximate analytical formulas for weak ($\xi \ll 1$) fields or ultrarelativistic gamma photon energies [23]. Here we find that an interesting laser-channeling phenomenon requires the opposite limit $\xi > 1$, and we have accessed this region by a full numerical treatment.

For the laser field, we employ a low-frequency ($\hbar\omega \sim 10$ eV), high-power laser beam with typically $\xi \sim 10$, corresponding to an intensity $I_l = 9 \times 10^{21}$ W/cm^2.

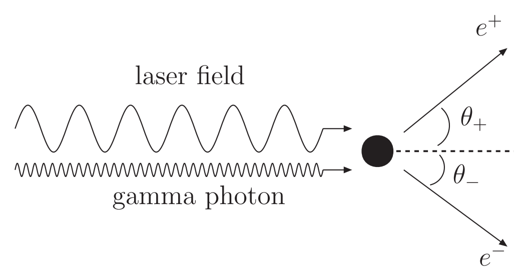

FIG. 1. Schematic picture of the considered pair creation process. A low-frequency, high-intensity laser beam of linear polarization and a high-frequency gamma photon, propagating in the same direction, impinge on a stationary nucleus, depicted as a filled black circle, to produce an electron and a positron. The angle $\theta_\pm$ denotes the ejection angle in the plane spanned by the propagation direction and the polarization direction of the laser. Note that the wavelengths of the two waves are not drawn to scale; in reality we consider the case where the laser wavelength is many orders of magnitude larger than the wavelength of the gamma photon.

Since we still have $\chi \ll 1$, and we consider a nucleus at rest, the laser field will affect the total probability of the pair creation only marginally. This can be justified by the following heuristic argument: The electron-positron pair is expected to be created over a distance of the order of the Compton wavelength $\lambda_C = \hbar/(mc)$. Over this distance, the peak electric field of the laser accomplishes an amount of work $W = \xi\hbar\omega$, which is much smaller than the threshold $2mc^2$ to create a pair, since we assume $\hbar\omega \ll mc^2$. We thus expect that the total number of pairs, or the total cross section for pair production, is not changed even for a strong laser. However, the differential cross section, that is the dependence on the directions and energies of the produced pairs, is expected to differ drastically from the laser-free case, due to the interaction with the laser field after the actual creation. In particular, we find that the laser field strongly focuses the pairs to form a narrow beam.

From here on, we use natural units with $\hbar = c = \epsilon_0 = 1$. Furthermore, we denote four-vector dot products with a dot, so that $a_\mu b^\mu = a \cdot b = a_0 b_0 - \boldsymbol{a} \cdot \boldsymbol{b}$ for two four-vectors a^μ and b^μ. Contraction with the Dirac gamma matrices γ^μ is written with a hat, $\gamma^\mu b_\mu = \hat{b}$. QED in strong laser fields can be treated in analogy to the Furry picture [24]. Since the laser is strong, $\xi \gtrsim 1$, the lepton-laser interaction needs to be treated nonperturbatively to all orders, while we include the interaction with the Coulomb field and the gamma photon in first-order perturbation theory. We consider linear polarization of the laser field, described by the vector potential $A^\mu(\phi) = a^\mu \cos\phi$, where $a^\mu = (0, \boldsymbol{a})$ is the polarization vector, and $\phi = k \cdot x$ is the Lorentz invariant laser phase, expressed through the wave vector $k^\mu = (\omega, \boldsymbol{k})$. The amplitude of the vector potential is related to the parameter ξ as $\xi = -e|\boldsymbol{a}|/m$.

The basis states for the electron and positron are the Volkov states $\psi_p^\mp(x)$ [24], which are exact solutions of the Dirac equation coupled to an external laser field:

$$[i\hat{\partial} - m - e\hat{A}(\phi)]\psi_{p_\mp}^\mp(x) = 0. \tag{1}$$

Here $p_\mp$ is the asymptotic momentum of the electron or positron outside the laser, and we define $q_\mp^\mu = (Q_\mp, \boldsymbol{q}_\mp) = p_\mp^\mu + k^\mu e^2|\boldsymbol{a}|^2/(4k \cdot p_\mp)$, the effective momentum of the electron or positron inside the laser. From the Volkov states, one constructs the Dirac-Volkov propagator $G(x, x')$, Green's function of Eq. (1) [24,25]. We also need the potential $A_C^\mu(x) = -Ze\delta^{\mu 0}/(4\pi|\boldsymbol{x}|)$ of the nucleus with atomic charge number Z and the vector potential $A_\gamma^\mu(x) = \epsilon_\gamma^\mu e^{-ik_\gamma \cdot x}/\sqrt{2\omega_\gamma}$ of the high-energy photon with wave vector $k_\gamma^\mu = (\omega_\gamma, \boldsymbol{k}_\gamma)$ and polarization vector $\epsilon_\gamma^\mu = (0, \boldsymbol{\epsilon}_\gamma)$.

The amplitude $S_{p_+p_-}$ for laser-dressed creation of one electron with asymptotic momentum p_- and one positron with asymptotic momentum p_+ can be calculated by adding the contributions from the two Feynman diagrams in Fig. 2. We consider a collinear arrangement of the gamma photon and the laser beam, which, in particular, implies $k \cdot k_\gamma = 0$, and provides for a considerable simplification of the matrix element. We have

$$\begin{aligned} S_{p_+p_-} &= ie^2 \int d^4x d^4x' \bar{\psi}_{p_-}^-(x)[\hat{A}_\gamma(x)G(x,x')\hat{A}_C(x') + \hat{A}_C(x)G(x,x')\hat{A}_\gamma(x')]\psi_{p_+}^+(x') \\ &= 2\pi i \sum_{n=-\infty}^{\infty} \frac{Ze^3 m}{\sqrt{2\omega_\gamma E_+ E_-}} \frac{\delta(Q_+ + Q_- - n\omega - \omega_\gamma)}{(\boldsymbol{q}_- + \boldsymbol{q}_+ - n\boldsymbol{k} - \boldsymbol{k}_\gamma)^2} \sum_{s=-1}^{1} \bar{u}_{p_-}\bigg(M_s^- \frac{\hat{k}_\gamma - \hat{p}_- - m}{2(p_- \cdot k_\gamma + s p_- \cdot k)} F_{n-s} \\ &\quad + F_{n-s} \frac{\hat{p}_+ - \hat{k}_\gamma - m}{2(p_+ \cdot k_\gamma + s p_+ \cdot k)} M_s^+\bigg) u_{p_+}^+, \end{aligned} \tag{2}$$

where $M_{-1,1}^\pm = \pm ea \cdot \epsilon_\gamma \hat{k}/(k \cdot p_\pm)$, $M_0^\pm = \hat{\epsilon}_\gamma$,

$$\begin{aligned} F_n &= A_0(n, \alpha_+ - \alpha_-, -\beta_+ - \beta_-)\gamma^0 \\ &\quad + A_1(n, \alpha_+ - \alpha_-, -\beta_+ - \beta_-)\left(\frac{\gamma^0 e\hat{a}\hat{k}}{2k \cdot p_+} + \frac{e\hat{a}\hat{k}\gamma^0}{2k \cdot p_-}\right) \\ &\quad + A_2(n, \alpha_+ - \alpha_-, -\beta_+ - \beta_-)\frac{e^2 a^2 \omega \hat{k}}{2k \cdot p_- k \cdot p_+}, \end{aligned} \tag{3}$$

$\alpha_\pm = ea \cdot p_\pm/(k \cdot p_\pm)$, $\beta_\pm = e^2 a^2/(8k \cdot p_\pm)$, and $u_{p_\mp}^\mp$ are constant spinors satisfying $(\hat{p}_\mp \mp m)u_{p_\mp}^\mp = 0$. The generalized Bessel function $A_K(n, \alpha, \beta)$ is defined as [11,12]

$$A_K(n, \alpha, \beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos^K\varphi e^{-in\varphi + i\alpha\sin\varphi - i\beta\sin(2\varphi)} d\varphi. \tag{4}$$

The expression (2) was first obtained in Ref. [22]. Energy conservation is enforced by the delta function $\delta(Q_+ + Q_- - n\omega - \omega_\gamma)$, which clarifies the meaning of the integer n: It signifies the net number of photons absorbed during the process. To obtain the total amplitude, one should sum over all photon orders n. Note, however, that energy conservation is expressed through the effective energies $Q_\mp$, and that photon number n is bounded from below by the laser-dressed pair-production threshold condition $n\omega \geq 2m_* - \omega_\gamma$, with $m_* = (q_\mp^2)^{1/2}$.

The laser-modified differential cross section $d\sigma$ is given by the standard formula $d\sigma = (1/T)|S_{p_+p_-}|^2 d^3p_- d^3p_+/(2\pi)^6$, where the large observation time T is cancelled by the relation $\delta^2(x) = T\delta(x)/(2\pi)$.

We have evaluated the differential cross section for different values of the parameter ξ. In all cases, we have averaged over the polarization of the initial gamma photon and summed over the spins of the final electron and positron. Because of symmetry reasons, the differential cross

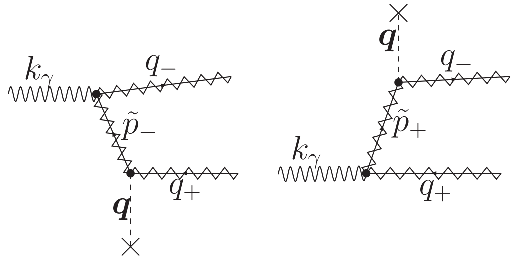

FIG. 2. The Feynman diagrams for laser-assisted pair creation. External electron and positron lines, as well as propagators, are denoted by a wiggly line superimposed on a straight line, to stress that the laser-lepton interaction is treated nonperturbatively. The electron is created with an effective four-momentum q_-, and the four-momentum of the positron is q_+. The intermediate electron propagator momentum is denoted by $\tilde{p}_\pm$. The absorbed nonlaser mode photon has four-momentum k_γ, and the virtual Coulomb field photon, depicted with a dashed line, has three-momentum $\boldsymbol{q}$. Time flows from left to right.

section is symmetric under the exchange of electron and positron, and we show the positron spectrum. This symmetry is known from [1,16], and applies since the Coulomb field interaction is taken into account up to first order. The laser frequency is chosen as $\omega = 10$ eV. However, we expect that the qualitative behavior of the cross sections is independent of ω, as long as $\chi \ll 1$.

In Fig. 3, we show the cross section $d\sigma/d\Omega_+$, resulting from fourfold Monte Carlo integration, which remains differential only in the solid angle $d\Omega_+ = d\theta_+ d\varphi_+ \sin\varphi_+$ of the created positron outside the laser field. Here φ_+ is the corresponding polar angle, with the polar axis being perpendicular to $\boldsymbol{a}$ and $\boldsymbol{k}$, so that $\varphi_+ = \pi/2$ corresponds to the plane spanned by $\boldsymbol{a}$ and $\boldsymbol{k}$ (see Fig. 1). The gamma photon energy is $\omega_\gamma = 1.25$ MeV $> 2m$, so that pair-production is possible without the laser. However, as is clearly seen in Fig. 3, the angular distribution in the field-free case [1] is broad. Quite to the contrary, the laser-dressed curves show sharp peaks, with the peak height increasing with increasing laser intensity, and the peak position given roughly by $\theta_{\rm peak} = 1/\xi$. This value can be understood intuitively by classical arguments: We assume that the positron is created from the photon energy $\omega_\gamma = 1.25$ MeV by the Bethe-Heitler process, with resulting momentum p^μ_{0+}, and subsequently evolves in the laser field according to the classical equations of motion [26]. An estimate for the angle $\theta_+ \ll 1$ then follows as

$$\theta_+(\phi_0) \approx \frac{p_+^\perp}{p_+^\parallel} \approx \frac{2k \cdot p_{0+}}{\omega|e\boldsymbol{A}(\phi_0)|}. \tag{5}$$

Here $p_+^\parallel$ is the momentum component in the laser propagation direction, $p_+^\perp$ is the momentum component in the laser polarization direction, and $|\boldsymbol{A}(\phi_0)|$ is the amplitude of the vector potential at the moment of creation. The momentum $\boldsymbol{p}_+$ at the detector thus depends on the laser phase at the moment of creation. Taking into account the initial momentum distribution given by the Bethe-Heitler formula, one finds that the angular distribution after the laser pulse, integrated over all phases of the laser, has a peak close to $\theta_+(\phi_0 = 0)$, so that $\theta_{\rm peak} = \theta_+(0) \approx 1/\xi$, assuming the typical $E_{0+} = \omega_\gamma/2$ and $\boldsymbol{p}_{0+} = (p_{0+}^\parallel, 0, 0)$. Note that the angle $\theta_{\rm peak}$ is independent of ω. The differential cross section $d\sigma/d\Omega_+$ as a function of the polar angle φ_+ is peaked sharply around $\varphi_+ = \pi/2$ for fixed $\theta+ \approx \theta_{\rm peak}$, with a peak width of approximately 0.01 rad for the case $\xi = 10$. The angular spectrum is furthermore symmetric around $\theta_+ = 0$, so that there is another identical peak centered around $\theta_+ \approx -1/\xi$. This means that essentially half the number of positrons produced per laser cycle are contained in the peak shown in Fig. 3 ($0 \leq \theta_+ \leq 0.4$ rad, $-0.03 \leq \varphi_+ - \pi/2 \leq 0.03$). For large $\omega_\gamma \gg m$, when the pairs are created preferentially in the forward direction even without the laser, the focusing effect still persists, with instead $\theta_{\rm peak} \approx 2m/(\omega_\gamma \xi)$, so that the pairs are focused into an even smaller angular region. Summarizing the intuitive picture given above, the Lorentz force of the laser field, with rising intensity, transfers at the moment of creation an increasingly larger amount of momentum to the positron in the propagation direction, compared to the amount transferred in the polarization direction, which consequently leads to the described laser channeling of the pairs into a narrow angular region.

To demonstrate the assertion that the total cross section, or the total number of produced pairs, is unchanged even by a laser field as strong as $\xi = 10$, we show in Fig. 4 the total cross section $\sigma_{\rm tot} = \int d\sigma$, resulting from sixfold numerical integration over the created electron and positron momenta. Because of energy conservation, the laser-

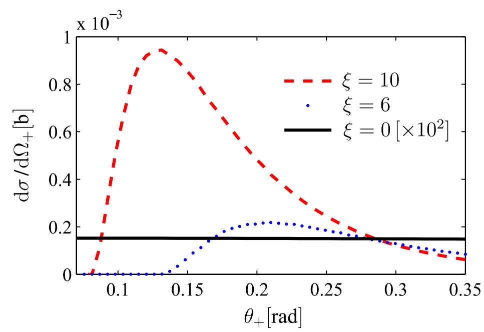

FIG. 3 (color online). The differential cross section $d\sigma/d\Omega_+$ as a function of the angle of ejection θ_+ (see Fig. 1), in the plane spanned by the propagation direction $\boldsymbol{k}/\omega$ and the polarization direction $\boldsymbol{a}/|\boldsymbol{a}|$ (polar angle $\varphi_+ = \pi/2$). The laser frequency is $\omega = 10$ eV, $\omega_\gamma = 1.25$ MeV, and $Z = 1$. Here, the parameter values $\xi = 6$ and $\xi = 10$ correspond to laser intensities $I_l = 3.2 \times 10^{21}$ W/cm^2 and $I_l = 8.9 \times 10^{21}$ W/cm^2, respectively. The numerical error bars of the Monte Carlo integration are not visible on the scale of the graph. For comparison, the solid black line shows the laser-free case, multiplied by a factor of 10^2. The total cross section for $\omega_\gamma = 1.25$ MeV coincides practically with the Bethe-Heitler value of 8×10^{-6} b.

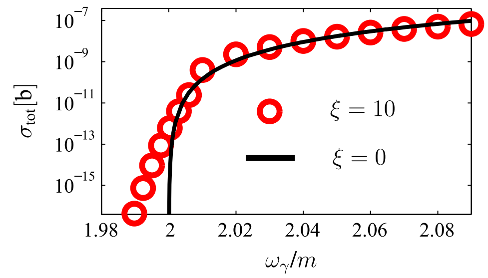

FIG. 4 (color online). The total cross section for electron-positron pair creation close to the threshold, displayed in logarithmic scale as a function of the perturbative photon energy ω_γ. The solid black line shows the field-free case [1], which vanishes below the pair-production threshold $\omega_\gamma/m = 2$. Below threshold, the laser-induced (rather than laser-assisted) pair creation cross section exhibits an exponential decrease. Parameters in the calculation are the same as in Fig. 3: $\omega_\gamma = 10\,\mathrm{eV}$, $\xi = 10$, $Z = 1$.

free cross section has a sharp cutoff at $\omega_\gamma = 2m$, below which it vanishes. When the laser is included, absorption of a sufficient number of laser photons to overcome the laser-modified threshold $2m_*$ results in a finite, but small, total cross section. Note that close to threshold, Coulomb corrections to the total cross section are expected to become important. Since we may speak of a laser-assisted pair creation process for $\omega_\gamma > 2m$, but laser-induced for $\omega_\gamma < 2m$, the considered process enables us to study the transition between laser-assisted and laser-induced by varying the frequency ω_γ.

A few remarks on possible background processes are appropriate. In principle, pairs can also be created by the laser wave together with the Coulomb field only [16]. However, in the regime of subcritical laser fields $\chi \ll 1$ that we are considering here (the parameters used in Fig. 4 give $\chi = \xi\omega/m = 2 \times 10^{-4}$), the rate is exponentially small $\sim \exp(-\chi^{-1})$ [17,19] and can safely be neglected. The photon together with the laser wave only is unable to create pairs in our geometry since $k \cdot k_\gamma = 0$ [12]. As for the experimental verification of our results, we note that Bethe-Heitler pair creation for small photon energies $\omega_\gamma \gtrsim 2m$ has been successfully measured [27], using germanium detectors. Here, it may be preferred to instead use a thin metal foil when measuring the laser-dressed cross section. If an incoming gamma photon flux of 10^7 photons per bunch, in bunches of duration 1 ps, repetition rate 1 Hz, with hard x-ray energy $\omega_\gamma = 1.25$ MeV, obtainable by Compton backscattering of a free electron laser on storage ring electrons [28,29], together with a lead ($Z = 82$) target of 1 mm thickness is assumed, the total number of pairs $N_{\rm tot}$ produced in 1 s can be estimated as $N_{\rm tot} \approx 2 \times 10^3$, in accordance with the Bethe-Heitler cross section. If now the gamma photon beam is synchronized with a strong laser field with $\xi = 10$, corresponding to pulse energy 1.4 J, 1 ps pulse duration, wavelength 1054 nm focused to one wavelength (available at the Vulcan laser facility [30], at repetition rate 1 Hz), all produced pairs will emerge in the angular cone $\theta_+ \approx 6° \pm 2°$, $\varphi_+ = 90° \pm 0.3°$ relative to the laser propagation direction, in the plane spanned by the propagation direction and the polarization direction. Placing a detector in this direction will therefore detect essentially all of the created pairs. We conclude that the proposed scheme of pair creation by a gamma photon together with channeling of the pairs with a strong laser is a realistic way to observe nonlinear laser effects, accessible to current laser facilities, without resorting to ultrahigh-energy photon or proton beams.

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*Erik.Loetstedt@mpi-hd.mpg.de

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