Directional Coupling for Quantum Computing and Communication

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We introduce the concept of directional coupling, i.e., the selective transfer of a state between adjacent quantum wires, in the context of quantum computing and communication. Our analysis rests upon a mathematical analogy between a dual-channel directional coupler and a composite spin system.

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Introduction.—Directional coupling, that is the exchange of power between guided modes of adjacent waveguides, has many applications in (opto)electronics [1–3]. For instance, directional couplers (DCs) may perform a number of useful functions in thin-film devices such as power division, switching, frequency selection, and (de) multiplexing. Typically, a dual-channel DC is a passive device with two input and two output ports. The ports are the ends of two waveguides, the so-called source and drain channels, which are brought in close proximity over a certain region. Varying a control parameter, one may achieve any division of a signal entering the source channel, between the outputs of the two channels.

From the theoretical point of view, directional coupling can be treated in the framework of coupled-mode theory, where one deals with equations of motion for the complex amplitudes pertaining to the two forward-propagating guided modes [1,2]. Usually, back reflection is absent due to technical reasons (e.g., in electronic devices an externally applied voltage may determine the propagation direction for electrons), while in certain configurations pertaining to optical waveguides, it has been shown that excitation of backward-propagating modes can be suppressed by applying adiabatic mode-coupling techniques [4,5], which require an elaborate sequence of pulses.

In this Letter, we propose a dual-channel DC for quantum computing and short-distance communication purposes. In analogy to conventional DCs, a dual-channel quantum DC (QDC) can be defined as a device which allows the selective transfer of quantum signals (i.e., quantum states of information carriers [6]) between two adjacent quantum channels. A QDC is expected to be the key element for various useful quantum information processing tasks, such as quantum switching, (de)multiplexing, etc.

The information carriers involved in some of the most promising proposals for large-scale quantum computing are not compatible with photons [7]. Hence, the engineering of perfect quantum channels for specific information carriers has recently attracted considerable interest [8]. In contrast to their (opto)electronic counterparts, quantum channels are discrete as they typically pertain to arrays of coupled quantum objects (sites). An excitation created somewhere in the array will unavoidably propagate in both directions and, after some time, various sites of the array may be occupied with different probabilities. Despite such delocalization effects, it has been shown that an excitation can be transferred in a perfect and deterministic way between the two ends of the channel by engineering the couplings between adjacent sites [9,10].

Given two perfect quantum channels, our task here is to define interchannel interactions, for which the entire system operates as a dual-channel QDC. In other words, we will discuss the conditions under which a ''flying'' qubit [6], prepared initially in the first site of one of the channels, can be transferred to the last site of either of the two channels, in a controlled and deterministic manner. We are interested in symmetric configurations with minimal external control, i.e., without elaborate sequences of time-dependent pulses and measurements.

Perfect quantum channel.—A Hamiltonian for perfect state transfer (PST) along a chain of N coupled nearly identical sites is of the form ($\hbar = 1$)

$$\hat{\mathcal{H}}_{\mathrm{PST}} = \sum_{j=1}^{N} \varepsilon \hat{c}_j^\dagger \hat{c}_j + \sum_{j=1}^{N-1} \Omega_{j,j+1} (\hat{c}_j^\dagger \hat{c}_{j+1} + \hat{c}_{j+1}^\dagger \hat{c}_j), \tag{1a}$$

where, $\hat{c}_j^\dagger$ is the creation operator for an excitation on the jth site of the channel with energy ε [11], and $\Omega_{j,j+1}$ is the coupling between adjacent sites. Consider a situation when a single excitation is prepared initially in the first site of the chain. The Hamiltonian (1a) preserves the number of excitations, and thus the system is restricted to the one-excitation Hilbert space throughout its evolution. The computational basis can be chosen as $\{|j\rangle\}$, where $|j\rangle \equiv \hat{c}_j^\dagger |\{0\}\rangle$ is the state with one excitation on the jth site, and $|\{0\}\rangle$ denotes the vacuum state of the system. As has been shown in [9,10], the chain acts as a perfect quantum channel; i.e., one can achieve PST from the 1st to the Nth site by judiciously engineering the coupling strengths according to

$$\Omega_{j,j+1} = \Omega \sqrt{(N-j)j}. \tag{1b}$$

Moreover, setting $J = (N-1)/2$ and $m = j - (N+1)/2$, one may define a one-to-one correspondence between the angular-momentum (AM) basis $\{|J, m\rangle\}$ and the computational basis $\{|j\rangle\}$. In view of this correspon-

dence, the evolution of the excitation under the influence of the PST Hamiltonian (1) is analogous to the evolution of the spin-J system, which is rotated around the x axis [12]. In the following we discuss how such a quantum channel can be used as a building block for a QDC. Our analysis rests upon the aforementioned mathematical analogy, which turns out to be a rather useful theoretical tool.

Quantum directional coupler.—In analogy to its (opto) electronic counterparts, a dual-channel QDC involves two nearly identical channels, the source (s) and the drain (d). Each channel consists of $N > 2$ nearly identical sites denoted by (σ, j), with $\sigma \in \{s, d\}$ and $1 \leq j \leq N$. Accordingly, the computational basis of the system can be chosen as $\{|\sigma; j\rangle\}$, with $|\sigma; j\rangle$ denoting an excitation on the jth site of channel σ. The first two sites $\{(s, 1); (d, 1)\}$ play the role of the two input ports, whereas the output ports are represented by the last sites $\{(s, N); (d, N)\}$.

The source and the drain channels are described by a PST Hamiltonian of the form (1). Our task is to define interactions between them so that an excitation initially occupying one of the input ports can be transferred to either of the two output ports in a controlled and deterministic way. More precisely, consider the excitation initially occupying the first site of the source channel [13]; i.e., the device is initially prepared in the state

$$|\Psi_C(0)\rangle = |s; 1\rangle. \tag{2}$$

At well-defined time instants, a dual-channel QDC should be capable of performing perfectly the following transformations

$$|s; 1\rangle \rightarrow |s; N\rangle, \tag{3a}$$

$$|s; 1\rangle \rightarrow |d; N\rangle \tag{3b}$$

apart, perhaps, from an unimportant global phase. Moreover, one should be able to switch between (3a) and (3b) by adjusting a set of parameters controlling the interchannel interactions.

We discuss two different configurations of sites that may operate as dual-channel QDCs. Both of them pertain to a grid of $M \times N$ nearly identical sites with preengineered couplings, although their operation relies on fundamentally different principles. In second quantization, the dynamics of a single excitation in such a two-dimensional structure is described by a Hamiltonian of the form

$$\hat{\mathcal{H}}_{M\times N} = \frac{1}{2} \sum_{i,i'=1}^{M} \sum_{j,j'=1}^{N} G_{j,j'}^{i,i'} (\hat{a}_{i,j}^{\dagger} \hat{a}_{i',j'} + \hat{a}_{i',j'}^{\dagger} \hat{a}_{i,j}), \tag{4}$$

where $\hat{a}_{i,j}^{\dagger}$ creates an excitation on the jth site of the ith row with energy $\varepsilon = G_{j,j}^{i,i}$, while the coupling strength between two different sites (i, j) and (i', j') is denoted by $G_{j,j'}^{i,i'}$, with $G_{j,j'}^{i,i'} = G_{j',j}^{i',i}$. In this formalism, the two outermost chains represent the source and the drain channels (i.e., $s \equiv 1$ and $d \equiv M \geq 2$), while any intermediate sites (i, j) with $i \neq \{1, M\}$ pertain to the coupler. Depending on the particular quantum-computing realization under consideration, each site of the grid may correspond, for instance, to a quantum dot or a superconducting qubit.

In general, the two channels of a QDC may be coupled directly or indirectly through their interaction with another system (coupler) placed between them. To describe the operation of the device in a unified theoretical framework, we may introduce two AM operators $\hat{\mathbf{J}}_h$ and $\hat{\mathbf{J}}_v$ acting on different subspaces, with

$$J_h = (N - 1)/2, \qquad m_h = j - (N + 1)/2; \tag{5a}$$

$$J_v = (M - 1)/2, \qquad m_v = i - (M + 1)/2. \tag{5b}$$

An orthonormal basis for the state space of the spin-J_α system (with $\alpha \in \{h, v\}$) can be chosen as $\{|J_\alpha, m_\alpha\rangle\}$, where $|J_\alpha, m_\alpha\rangle$ are degenerate eigenvectors of the operator $\hat{J}_\alpha^2$. As we will see later on, this degeneracy specifies the class of QDCs, whose operation can be simulated by the dynamics of the composite spin system with basis states $\{|J_v, m_v; J_h, m_h\rangle\}$, where $|J_v, m_v; J_h, m_h\rangle \equiv |J_v, m_v\rangle \otimes |J_h, m_h\rangle$.

The role of the spin-J_h system is to describe the dynamics of the excitation in either of the two nearly identical channels (source or drain). For fixed channel parameters $\{N, \varepsilon, \Omega\}$, one may define a one-to-one correspondence between the basis states $\{|j\rangle\}$ and $\{|J_h, m_h\rangle\}$ by means of Eq. (5a); i.e., we have

$$|J_h, m_h\rangle \equiv |j\rangle. \tag{6a}$$

On the other hand, the spin-J_v system has been introduced for the description of the interchannel dynamics, with the only convention being

$$|J_v, -J_v\rangle \equiv |s\rangle, \qquad |J_v, J_v\rangle \equiv |d\rangle. \tag{6b}$$

In view of Eqs. (5b) and (6b), a dual-channel QDC with directly coupled channels is described by a spin-1/2 particle (i.e., for $M = 2$), whereas the presence of a coupler between the two channels is represented by a spin-J_v particle with $J_v > 1/2$ (i.e., for $M > 2$). In the latter case, the spin states $|J_v, m_v\rangle$ with $m_v \neq \pm J_v$ correspond to the coupler.

In AM representation, the dynamics of a single excitation in a dual-channel QDC can be described by a Hamiltonian of the form

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_h + \hat{\mathcal{H}}_v, \tag{7}$$

where $\hat{\mathcal{H}}_\alpha \equiv \hat{\mathcal{H}}_\alpha^{(0)} + \hat{\mathcal{V}}_\alpha$ refers to the spin-J_α system only. The basis states $\{|J_\alpha, m_\alpha\rangle\}$ are degenerate eigenstates of the corresponding unperturbed Hamiltonian $\hat{\mathcal{H}}_\alpha^{(0)} \equiv \varepsilon_\alpha \hat{J}_\alpha^2$, while $\hat{\mathcal{V}}_\alpha$ is the coupling between various states $\{|J_\alpha, m_\alpha\rangle\}$. Because of the degeneracy, conventions (6) imply that a dual-channel QDC can be described in the present theoretical framework if the coupler is on resonance with both channels and, for a given channel, all the states $|\sigma; j\rangle$ have the same energy.

The initial condition (2) reads in AM representation

$$|\Psi_{\mathrm{AM}}(0)\rangle = |J_v, -J_v; J_h, -J_h\rangle, \tag{8}$$

whereas transformations (3) are in one-to-one correspondence with the following transformations:

$$|J_v, -J_v; J_h, -J_h\rangle \rightarrow |J_v, -J_v; J_h, J_h\rangle, \tag{9a}$$

$$|J_v, -J_v; J_h, -J_h\rangle \rightarrow |J_v, J_v; J_h, J_h\rangle. \tag{9b}$$

Transformation (9a) pertains to the evolution of the spin-J_h system only, and is thus expected to be implementable by Hamiltonian (7) for $\hat{\mathcal{V}}_v = 0$ (no interchannel coupling). Recall now that the dynamics of the spin-J_h system have to describe accurately the evolution of the excitation in either of the two (nearly identical) channels. Given that both channels are described by a PST Hamiltonian of the form (1), with fixed parameters $\{N, \varepsilon, \Omega\}$, we can easily specify the form of the Hamiltonian $\hat{\mathcal{H}}_h$ by expressing the PST Hamiltonian (1) in the basis $\{|J_h, m_h\rangle\}$. Using correspondences (5a) we find $\hat{\mathcal{H}}_h = \varepsilon_h \hat{J}_h^2 + 2\Omega \hat{J}_{h,x}$, where $\varepsilon_h = \varepsilon/[J_h(J_h+1)]$, and $\hat{J}_{h,x}$ is the x component of the vector $\hat{\mathbf{J}}_h$. Under the influence of $\hat{\mathcal{H}}_h$, the initial state of the isolated spin-J_h system undergoes a rotation around the x axis, and the transformation (9a) takes place at time $\tau = \pi/(2\Omega)$. Having specified the first part of Hamiltonian (7), we have to determine the interchannel interaction $\hat{\mathcal{V}}_v$, for which transformation (9b) takes place at a well-defined time instant. We discuss two different solutions.

The transformation (9b) involves a simultaneous rotation of the initial states of both spins. This leads us to introduce the total angular momentum $\hat{\mathbf{J}} = \hat{\mathbf{J}}_h + \hat{\mathbf{J}}_v$, with $|J_h - J_v| \le J \le J_h + J_v$ and $|m| \le J$, while the corresponding basis states $\{|J, m\rangle\}$ can be expanded on the basis $\{|J_v, m_v; J_h, m_h\rangle\}$ in the usual way [14]. In the basis $\{|J, m\rangle\}$, the initial condition (8) reads $|\Psi_{\mathrm{AM}}(0)\rangle = |J, -J\rangle$, while for the transformation (9b) we have $|J, -J\rangle \rightarrow |J, J\rangle$. In view of the previous discussion, this transformation can be performed by defining the interchannel coupling $\hat{\mathcal{V}}_v$, such that $\hat{\mathcal{H}} \sim 2\Omega \hat{J}_x$. One may choose $\hat{\mathcal{V}}_v = 2K\hat{J}_{v,x}$, with K denoting the interchannel coupling strength. The total Hamiltonian (7) then reads

$$\hat{\mathcal{H}} = \varepsilon_h \hat{J}_h^2 + \varepsilon_v \hat{J}_v^2 + 2\Omega \hat{J}_{h,x} + 2K\hat{J}_{v,x}, \tag{10}$$

and acquires the desired form for $K = \Omega$.

The Hamiltonian (10) describes the operation of a perfect dual-channel QDC in an AM representation. Hence, a quantum network involving a number of coupled sites may operate as a QDC if the Hamiltonian of the entire system in an AM representation acquires the form (10). For instance, one can readily show, using Eqs. (5), that Hamiltonian (4) reduces to Hamiltonian (10) when adjacent sites are coupled with strengths $G^{i,i}_{j,j+1} = \Omega\sqrt{j(N-j)}$ and $G^{i,i+1}_{j,j} = K\sqrt{i(M-i)}$, which incidentally underscores the usefulness of the AM representation. This coupling configuration has also been investigated in [15], albeit in a different context. The present work, however, reveals another aspect of such a structure, namely, its use as a QDC with source and drain channels the two outermost chains, and control parameter K.

The main disadvantage of this configuration, however, is that all the sites of the source channel have to be coupled (directly or indirectly) to the corresponding sites of the drain channel via nearest-neighbor interactions. Depending on the physical realization under consideration, this might be very restrictive as it may imply that the two quantum channels have to be close to each other. The question therefore is, can one achieve directional coupling between two chains by defining interchannel interactions for a certain number of sites only?

Clearly, this question cannot be answered in the framework of Hamiltonian (7), as it describes independent evolutions of the vectors $\hat{\mathbf{J}}_v$ and $\hat{\mathbf{J}}_h$. Instead, one has to consider more general Hamiltonians involving coupled angular momenta. Consider, for instance, an interchannel interaction $\hat{\mathcal{W}}$, represented by the coupling between a spin $J_v = 1/2$ and an angular momentum $J_h = 1$, such that $\hat{\mathcal{W}} = K\hat{J}_{v,y}\hat{J}_{h,y}$. In view of the previous discussion, we may write the total Hamiltonian

$$\hat{\mathcal{H}} = \varepsilon_h \hat{J}_h^2 + \varepsilon_v \hat{J}_v^2 + 2\Omega \hat{J}_{h,x} + K\hat{J}_{v,y}\hat{J}_{h,y}, \tag{11}$$

and transformation (9a) can be achieved for $K = 0$, when only the vector $\hat{\mathbf{J}}_h$ is rotated around the x axis. Turning on the interchannel interaction, i.e., setting $K \neq 0$, both vectors $\hat{\mathbf{J}}_h$ and $\hat{\mathbf{J}}_v$ can be rotated simultaneously around the y axis. In this case, the initial state of the system evolves under the influence of both $\hat{\mathcal{V}}_h$ and $\hat{\mathcal{W}}$. Hence, we have two distinct evolution routes that may interfere either constructively or destructively, and the transformation (9b) can be achieved by choosing judiciously the ratio K/Ω. Indeed, for the initial condition (8), one can show that the transformation (9b) occurs at $t = \tau/\sqrt{2}$ for $K = -4\Omega$.

The Hamiltonian (11) can be implemented in the 2×3 grid depicted in Fig. 1(a), for $g = \sqrt{2}\Omega$ and $\kappa = K/(2\sqrt{2})$. Such a configuration can be used as a coupler for selective transfer of an excitation between two chains involving an arbitrary odd number of sites $N > 3$. As shown in Fig. 1(b), the coupler involves intermediate sites of the two chains, with indices $j_c = (N+1)/2$ and $j_\pm = j_c \pm 1$. The corresponding coupling constants are $G^{i,i}_{j_-,j_c} = G^{i,i}_{j_c,j_+} = g$, $G^{s,d}_{j_-,j_c} = -G^{d,s}_{j_-,j_c} = \kappa$, $G^{s,d}_{j_c,j_+} = -G^{d,s}_{j_c,j_+} = \kappa$. In spin networks, the adjustment of geometric phases is possible by looping around magnetic fields along the relevant sections [16], while for optical networks one may use phase shifters.

Outside the coupler, only neighboring sites of the same channel are coupled according to Eq. (1b); i.e., we have

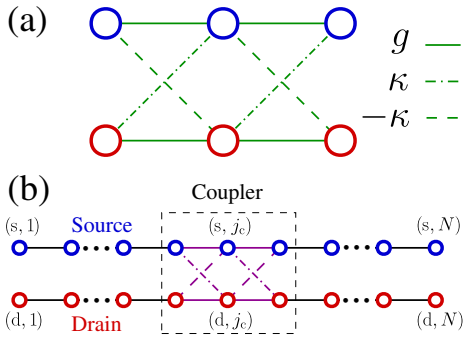

FIG. 1 (color online). (a) A coupler consisting of six identical sites. (b) The coupler integrated in a dual-channel system.

$G^{i,i}_{j,j+1} = \Omega\sqrt{j(N-j)}$ for $i = \{s, d\}$. A qubit state initially prepared at the input port $(s, 1)$ can be transferred selectively to either of the two output ports $\{(s, N), (d, N)\}$ at time $t = \tau$ by adjusting the ratio g/κ. In particular, the transformation (9a) is performed for $g = \Omega\sqrt{(N^2-1)/4}$ and $\kappa = 0$, whereas transformation (9b) takes place for $g = \kappa = \Omega\sqrt{(N^2-1)/8}$. For the sake of illustration, in Fig. 2 we present numerical results pertaining to the transfer of a single excitation between two chains of $N = 7$ sites each. The results have been obtained through the solution of the Schrödinger equation in the computational basis. The excitation, which occupies initially the site $(s, 1)$, splits into two parts at the entrance of the coupler (not shown here). The two parts follow different paths and they split into smaller fractions in the middle of the coupler. The various fractions acquire different phases as they propagate through the sections of the coupler, and they interfere constructively only on the site (d, j_+). In closing, it is worth pointing out that a four-site configuration similar to Fig. 1(a) may operate as a Hadamard gate [16], and one may consider judicious combinations of such gates for directional coupling between two channels as well. Our three-site configuration, however, cannot be expressed in terms of Hadamard gates.

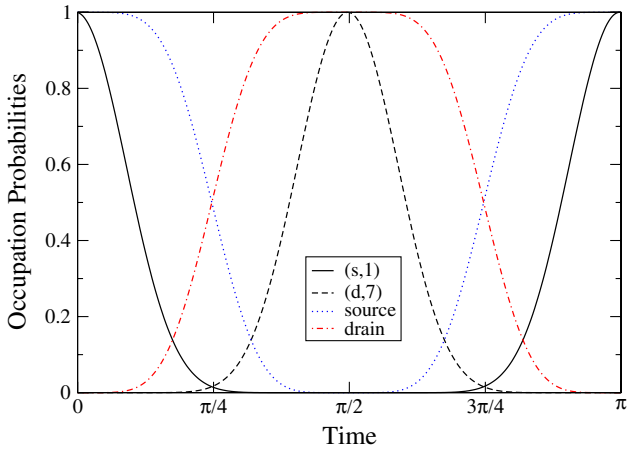

FIG. 2 (color online). Time evolution of an excitation in the QDC of Fig. 1(b), with $N = 7$. The excitation is transferred from the input port $(s, 1)$ to the output port of the drain channel (d, N) at $t = \tau$. The time is in units of Ω^{-1}.

Summary and outlook.—We have introduced the notion of QDC in the context of quantum computation and communication, presenting also a general mathematical analogy to a composite spin system. Employing this analogy, we have been able to specify criteria for perfect and deterministic directional coupling of "flying" qubit states [6] between two quantum channels that rely on existing schemes for state transfer. The present work does not cover all the possible solutions to the problem of directional coupling, which is very general and is not associated with a particular coupling configuration. A number of interesting questions, such as the existence of other configurations for directional coupling between two or more quantum channels, the transfer of arbitrary multiqubit states, the effect of imperfections, as well as the extension of (de)multiplexing processes to the quantum world, deserve further investigation.

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