Dissipation Induced Coherence of a Two-Mode Bose-Einstein Condensate

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We discuss the dynamics of a Bose-Einstein condensate in a double-well trap subject to phase noise and particle loss. The phase coherence of a weakly interacting condensate as well as the response to an external driving show a pronounced stochastic resonance effect: Both quantities become maximal for a finite value of the dissipation rate matching the intrinsic time scales of the system. Even stronger effects are observed when dissipation acts in concurrence with strong interparticle interactions, restoring the purity of the condensate almost completely and increasing the phase coherence significantly.

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In our naive understanding thermal noise is generally distracting, hindering measurements and degrading coherences in quantum mechanics. A paradigmatic counterexample to this assertion is the effect of stochastic resonance (SR), where the response of a system to an external driving assumes its maximum in the presence of a finite amount of thermal noise, when the time scales of the noise and the driving match [1]. In this case the noise is strong enough to cause a large dynamical effect when it adds up constructively with the driving, whereas it is still weak enough not to make the dynamics completely random. By now, SR has been shown in a variety of systems, an overview is given in the review articles [2–5].

In addition to numerous examples in classical dynamics, SR has also been found in a variety of quantum systems (see [5] and references therein). Recently, there has been an increased interest in controlling and even exploiting dissipation in interacting many-body quantum systems. For instance, the entanglement in a spin chain assumes an SR-like maximum for a finite amount of thermal noise [6]. Furthermore, it has been shown that dissipative processes can be tailored to prepare arbitrary pure states for quantum computation and strongly correlated states of ultracold atoms [7] or to implement a universal set of quantum gates [8]. Actually, a recent experiment has even proven that strong inelastic collisions may inhibit particle losses and induce strong correlations in a quasi one-dimensional gas of ultracold atoms [9].

In this Letter we demonstrate the constructive effects of dissipation for an interacting many-particle quantum system realized by ultracold atoms in a double-well trap with biased particle dissipation. It is shown that a proper amount of dissipation maximizes the coherence of the two condensate modes in the fashion of the SR effect. In this case the particle loss is strong enough to significantly increase the condensate purity, whereas it is still weak enough not to dominate the complete dynamics. These effects are of considerable strength for realistic parameters, especially in the case of strong interparticle interactions, and thus should be observable in ongoing experiments [10–13].

The unitary dynamics of ultracold atoms in a double-well trap is described by the two-mode Bose-Hubbard Hamiltonian [14–16]

$$\hat{H} = -J(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1) + \epsilon(\hat{n}_2 - \hat{n}_1) + \frac{U}{2}[\hat{n}_1(\hat{n}_1 - 1) + \hat{n}_2(\hat{n}_2 - 1)], \qquad (1)$$

where $\hat{a}_j$ and $\hat{a}_j^\dagger$ are the bosonic annihilation and creation operators in the jth well and $\hat{n}_j = \hat{a}_j^\dagger \hat{a}_j$ are the number operators. In general we consider a biased double-well trap, where the ground state energies of the two wells differ by 2ϵ. We set $\hbar = 1$, thus measuring all energies in frequency units.

The main source of decoherence is phase noise due to elastic collisions with atoms in the thermal cloud [17,18] which effectively heats the system. The heating rate is fixed as $\gamma_p = 5\ \mathrm{s}^{-1}$ in the following, which is a realistic value for the experiments in Heidelberg [10,11]. Methods to attenuate this source of decoherence were discussed only recently [19]. Amplitude noise, i.e., the exchange of particles with the thermal cloud due to inelastic scattering, drives the system to thermal equilibrium. However, this effect is usually much too weak to produce the effects discussed below in present experiments (cf. the discussion in [18]). In contrast, a strong and tunable source of dissipation can be implemented artificially by shining a resonant laser beam onto the trap, that removes atoms with the site-dependent rates γ_{aj} from the two wells $j = 1, 2$. Nontrivial effects of dissipation such as the stochastic resonance discussed below require strongly biased loss rates, i.e., $\gamma_{a1} \neq \gamma_{a2}$. For a laser beam focused on one of the wells an asymmetry of $f_a = (\gamma_{a2} - \gamma_{a1})/(\gamma_{a2} + \gamma_{a1}) = 0.5$ should be feasible. Thus we consider the dynamics generated by the master equation

$$\dot{\hat{\rho}} = -i[\hat{H}, \hat{\rho}] - \frac{\gamma_p}{2} \sum_{j=1,2} (\hat{n}_j^2 \hat{\rho} + \hat{\rho}\hat{n}_j^2 - 2\hat{n}_j \hat{\rho}\hat{n}_j) - \frac{1}{2} \sum_{j=1,2} \gamma_{aj} (\hat{a}_j^\dagger \hat{a}_j \hat{\rho} + \hat{\rho}\hat{a}_j^\dagger \hat{a}_j - 2\hat{a}_j \hat{\rho}\hat{a}_j^\dagger). \qquad (2)$$

The macroscopic dynamics of the atomic cloud is well described by a mean-field approximation, considering only the expectation values $s_j = 2\mathrm{tr}(\hat{L}_j \hat{\rho})$ of the angular momentum operators $\hat{L}_x = (\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1)/2$, $\hat{L}_y = i(\hat{a}_1^\dagger \hat{a}_2 - \hat{a}_2^\dagger \hat{a}_1)/2$, $\hat{L}_z = (\hat{a}_2^\dagger \hat{a}_2 - \hat{a}_1^\dagger \hat{a}_1)/2$ and the particle number $n = \mathrm{tr}[(\hat{n}_1 + \hat{n}_2)\hat{\rho}]$. The time evolution of the Bloch vector $\mathbf{s}$ and the particle number is then given by [20]

$$\begin{aligned} \dot{s}_x &= -2\epsilon s_y - U s_y s_z - T_2^{-1} s_x, \\ \dot{s}_y &= 2J s_z + 2\epsilon s_x + U s_x s_z - T_2^{-1} s_y, \\ \dot{s}_z &= -2J s_y - T_1^{-1} s_z - T_1^{-1} f_a n, \\ \dot{n} &= -T_1^{-1} n - T_1^{-1} f_a s_z. \end{aligned} \qquad (3)$$

As usual expectation values of products have been factorized in the U-dependent interaction terms to obtain a closed set of evolution equations [14–16], whereas the dissipation terms are exact. Furthermore, we have defined the transversal and longitudinal damping times by

$$T_1^{-1} = (\gamma_{a1} + \gamma_{a2})/2 \quad \text{and} \quad T_2^{-1} = \gamma_p + T_1^{-1}. \qquad (4)$$

These equations of motion resemble the celebrated Bloch equations in nuclear magnetic resonance [21,22] with some subtle but nevertheless important differences. The longitudinal relaxation is now associated with particle loss and, more important, the dynamics is substantially altered by the interaction term [10,14,15].

In the following, we will show that a finite amount of dissipation induces a maximum of the coherence which can be understood as an SR effect. We have to distinguish between two different kinds of coherence, which will both be considered. First of all we consider the phase coherence between the two wells, which is measured by the average *contrast* in interference experiments as described in [10,11] and given by

$$\alpha = \frac{2|\langle \hat{a}_1^\dagger \hat{a}_2 \rangle|}{\langle \hat{n}_1 + \hat{n}_2 \rangle} = \frac{\sqrt{s_x^2 + s_y^2}}{n}. \qquad (5)$$

Second, we will analyze how close the many-particle quantum state is to a pure Bose-Einstein condensate (BEC), which is a coherent state for the $SU(2)$ operator algebra [23]. This property is quantified by the purity $p = 2\mathrm{tr}(\hat{\rho}_{\mathrm{red}}^2) - 1 = |\mathbf{s}|^2/n^2$ of the reduced single-particle density matrix $\hat{\rho}_{\mathrm{red}}$ cf. [16].

Let us first discuss the weakly interacting case, where the mean-field equations of motion (3) provide an excellent description of the dynamics, which is exact for $U = 0$. Obviously, only the trivial solution $\mathbf{s} = 0$ and $n = 0$ is a steady state in the strict sense. However, the system rapidly relaxes to a quasisteady state of the form $\mathbf{s}(t) \sim \mathbf{s}_0 e^{-\kappa t}$ and $n(t) \sim n_0 e^{-\kappa t}$, where the internal dynamics is completely frozen out and all components of the Bloch vector and the particle number decay at the same rate κ. Figure 1 shows the contrast α for this quasisteady state as a function of the tunneling rate J and the dissipation rate $1/T_1$ for $U = 0$. For a fixed value of one of the parameters, say J, one observes a typical SR-like maximum of the contrast for a finite value of the dissipation rate $1/T_1$. In particular, the contrast is maximal if the time scales of the tunneling and the dissipation are matched according to $4J^2 \approx f_a T_1^{-1}(f_a T_1^{-1} + \gamma_p)$ [24]. This scenario is robust and not altered by weak interparticle interactions. Changes in the system parameters such as ϵ preserve the general shape of $\alpha(1/T_1, J)$ and the existence of a pronounced SR-like maximum. At the most, the function $\alpha(1/T_1, J)$ is stretched, shifting the position of the SR-like maximum.

The occurrence of a maximum of the contrast is explained by Fig. 2(b), where the results of a Monte Carlo wave function (MCWF) simulation [25] of the many-body dynamics are shown for three different values of J and $U = 0.1\ \mathrm{s}^{-1}$. We have plotted a histogram of the probabilities to observe the relative population imbalance s_z/n and the relative phase ϕ in a single experimental run for three different values of the tunneling rate J after the system has relaxed to the quasisteady state. With increasing J, the atoms are distributed more equally between the two wells so that the single shot contrast increases. Within the mean-field description this is reflected by an increase of $\sqrt{s_x^2 + s_y^2}/|\mathbf{s}|$ at the expense of $|s_z|$. However, this effect also makes the system more vulnerable to phase noise so that the relative phase of the two modes becomes more and more random and $|\mathbf{s}|/n$ decreases. The average contrast (5) then assumes a maximum for intermediate values of J as shown in Fig. 2(a). In this example, the trap is assumed to be weakly biased, shifting the position of the SR-like maximum to a value of J which is more easily accessible in ongoing experiments [10,11].

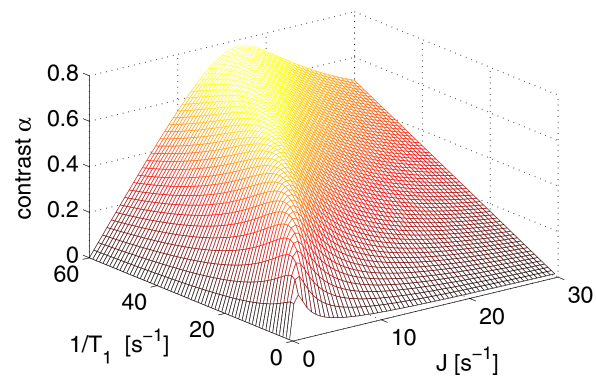

FIG. 1 (color online). Contrast α in the quasisteady state in dependence on the tunneling rate J and the dissipation rate $1/T_1$ for $U = 0$ and $\epsilon = 0$.

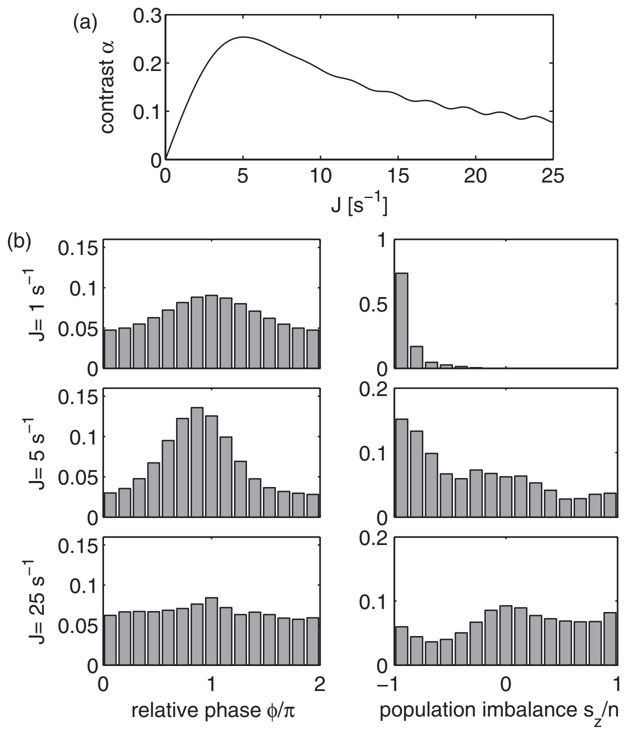

FIG. 2. (a) Average contrast α after $t = 1.5$ s starting from a pure BEC (i.e. a product state) with $s_z = n/2$ and $n(0) = 100$ particles in dependence on the tunneling rate J for $T_1 = 0.5$ s, $\epsilon = 10\ \mathrm{s}^{-1}$, $U = 0.1\ \mathrm{s}^{-1}$. (b) Histogram of the probabilities to measure the relative phase ϕ and the relative population imbalance s_z/n in a single experimental run after $t = 1.5$ s obtained from a MCWF simulation of the many-body dynamics.

So far we have demonstrated a SR of the contrast for a BEC in a static double-well trap with biased particle losses. We will now show that the system's response to a weak external driving also assumes a maximum for a finite dissipation rate—an effect which is conceptually closer to the common interpretation of stochastic resonance. We consider a weak driving of the tunneling rate $J(t) = J_0 + J_1 \cos(\omega t)$ at the resonance frequency $\omega = \sqrt{J_0^2 + \epsilon^2}$, where the amplitude is not more than $J_1/J_0 = 10\%$. This can be readily implemented in optical setups by varying the intensity of the counterpropagating lasers forming the optical lattice. Figure 3(a) shows the resulting dynamics for $T_1 = 0.5$ s and $J_0 = 1.5\ \mathrm{s}^{-1}$. After a short transient period, the relative population imbalance $s_z(t)/n(t)$ oscillates approximately sinusoidally. The system response measured by the amplitude of these forced oscillations shows the familiar SR-like maximum as illustrated in Fig. 3(b). It should be detectable without major problems in ongoing experiments, in which the population imbalance s_z can be measured with a resolution of a few atoms [10,11]. A more detailed study of such a driven case of SR will be discussed in a forthcoming article [24].

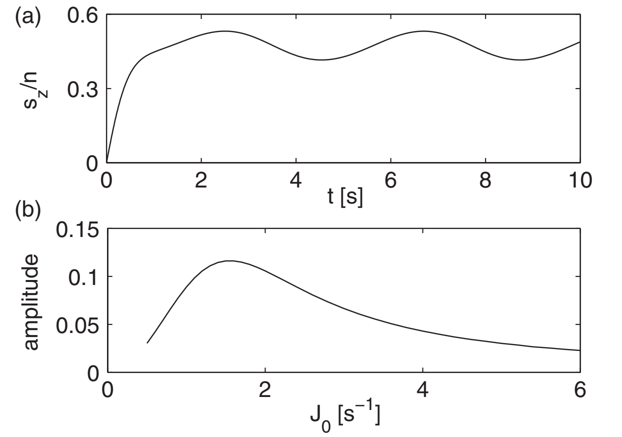

FIG. 3. (a) Oscillation of the relative population imbalance s_z/n of a weakly driven two-mode BEC for $J_0 = 1.5\ \mathrm{s}^{-1}$, $T_1 = 0.5$ s and $\epsilon = 0$. (b) Amplitude of the oscillations in dependence on the tunneling rate J_0.

Even more remarkable values of the coherences are observed in the case of strong interactions, which is experimentally most relevant and theoretically most profound. The interplay between interactions and dissipation significantly increases the coherences in comparison to situations where one of the two is weak or missing. An example for the dynamics of a strongly-interacting BEC is shown in Fig. 4 for an initially pure BEC with $s_z = n/2$, calculated both with the MCWF method and within the mean-field approximation (3). At first, the purity p and the contrast α drop rapidly due to the phase noise and the interactions cf. [16]. For intermediate times, however, the system relaxes to a nonlinear quasisteady state, which is a

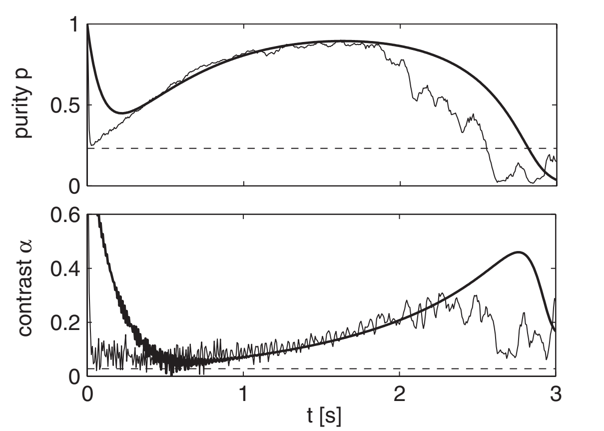

FIG. 4. Time evolution of the purity p and the contrast α for $J = U = 10\ \mathrm{s}^{-1}$, $\epsilon = 0$, $T_1 = 0.5$ s. The initial state is a pure BEC with $s_z = n/2$ and $n(0) = 100$ particles. The results of a MCWF simulation averaged over 100 runs are plotted as a thin solid line while the mean-field results are plotted as a thick line. The dashed line shows the steady state values for $1/T_1 = 1/T_2 = 0$, i.e., without coupling to the environment.

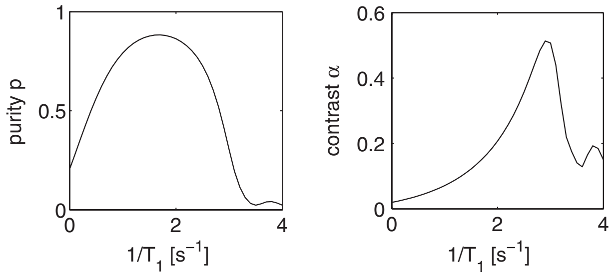

FIG. 5. Purity p and contrast α after $t = 2$ s in dependence on the dissipation rate $1/T_1$ calculated within the mean-field approximation for the same parameters as in Fig. 4.

nearly pure BEC mostly localized in the well with the smaller decay rate. Consequently, the purity p is restored almost completely and the contrast α is relatively large. In close analogy to the celebrated self-trapping effect [10,14,15], this quasisteady state exists only as long as the effective interaction strength $Un(t)$ is larger than a critical value given by [20,24]

$$U^2n^2 \gtrsim 4J^2 - f_a^2 T_1^{-2}. \tag{6}$$

As the particle number n decays, this state ceases to exist so that the system relaxes to a linear quasisteady state with much smaller values of p and α as discussed above.

Moreover, the coherences at intermediate times are also larger than in an interacting, but nondissipative system. The dashed lines in Fig. 4 show the steady state values of the purity p and the contrast α for $1/T_1 = 1/T_2 = 0$, apart from occasional revivals due to the finite particle number. It is observed that the coherences are considerably smaller compared to the strongly-interacting open system. This loss of coherence can be understood by the fact that the interactions lead to an effective decoherence on the single-particle level [16], degrading α and p. This effect is mostly cured by the dissipation.

The behavior illustrated in Fig. 4 is universal, in the sense that the maxima of the purity and the contrast are present for all values of U and $1/T_1$ if only $Un(t = 0)$ is well above the critical value (6) for the existence of the nonlinear quasisteady state. However, the maxima occur later if T_1 or U increase. The purity p and the contrast α after a fixed time $t = 2$ s are plotted in Fig. 5 in dependence on the dissipation rate $1/T_1$, showing pronounced maxima for finite values of $1/T_1$. For smaller dissipation rates, the maximum of the contrast has not been assumed yet while the system has already relaxed to the linear quasisteady state for larger values of $1/T_1$.

To summarize, we have shown that the coherence properties of a weakly and, in particular, also of a strongly interacting Bose-Einstein condensate in a double-well trap can be controlled by engineering the system's parameters and dissipation simultaneously. An important conclusion is that the interplay of interactions and dissipation can drive the system to a state of maximum coherence, while both processes alone usually lead to a loss of coherence. Since the double-well BEC is nowadays routinely realized with nearly perfect control on atom-atom interactions and external potentials [10,11], we hope for an experimental verification and future extensions of the predicted stochastic resonance scheme.

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