Optical Bloch Oscillations and Zener Tunneling with Nonclassical Light

Stefano Longhi
Dipartimento di Fisica and Istituto di Fotonica e Nanotecnologie del CNR, Politecnico di Milano, Piazza L. da Vinci 32, I-20133 Milano, Italy
(Received 3 September 2008; published 6 November 2008)

A quantum theory of optical Bloch oscillations and Zener tunneling (ZT) in arrays of coupled waveguides is theoretically presented, and the particlelike behavior of photons undergoing ZT is highlighted. In singly-periodic arrays excited by a photon-number-state input beam, each photon behaves as a classical particle which independently undergoes a coin-toss ZT event with a probability described by classical Zener theory. In binary arrays, excitation with two tilted beams enables us to observe the Hong-Ou-Mandel interference for two photons undergoing Bloch-Zener oscillations.

DOI: 10.1103/PhysRevLett.101.193902 PACS numbers: 42.82.Et, 42.50.Dv

Bloch oscillations (BO) and Zener tunneling (ZT) are important phenomena associated with the coherent motion of quantum particles in periodic potentials driven by an external force. In quantum systems, BO have been observed for electrons in biased semiconductor superlattices and for Bose-Einstein condensates in accelerating optical lattices. In their essence, BO and ZT are wave phenomena. As such, they can be observed for classical waves as well, such as electromagnetic or acoustic waves. In the past decade, several experiments have shown optical analogues of BO and ZT in specially designed photonic lattices [1–6]. In particular, arrays of coupled waveguides with transverse refractive index gradients enabled a direct visualization of BO and ZT in the spatial domain [6]. These experiments, which used classical light to excite the lattices, exploited the wave nature of light and some analogies between Maxwell and Schrödinger wave equations in periodic media. However, it is known that in certain conditions light may behave more as a stream of photons rather than as an electromagnetic wave. This is the case, for instance, of nonclassical light passing through a beam splitter (see, for instance, [7,8]). Coupled waveguides are known to behave similarly to beam splitters [9], and are thus suited to probe strictly quantum signatures of light. For instance, Hong-Ou-Mandel quantum interference [7] has been recently observed using silicon-based coupled waveguides [10]. In another recent experiment, Bromberg *et al.* [11] suggested nontrivial photon correlations in coupled waveguide arrays illuminated by two-photon states and mimicked them using classical intensity correlation measurements. These experiments suggest that a particlelike behavior of photons undergoing optical BO and ZT, hidden in classic wave theory, might be manifested using nonclassical fields. It is the aim of this Letter to present a quantum theory of optical BO and ZT in waveguide arrays, highlighting certain particlelike signatures of photons undergoing ZT.

Classical model of optical BO and ZT.—Let us consider propagation of a traveling and quasimonochromatic TE-polarized light wave at carrier frequency $\omega = 2\pi c_0/\lambda$ in a weakling guiding one-dimensional waveguide array, with periodic refractive index profile $n(x)$ and with a superimposed transverse refractive index gradient Fx, which is suited to study optical BO and ZT [6,12]. In the paraxial and quasimonochromatic approximations, the slow evolution of the vector potential envelope $\psi(x, z, t)$ along the paraxial z direction can be readily obtained from Maxwell's equations (see, for instance, [13]) and is governed by the scalar wave equation

$$i[\psi_z + (1/v_g)\psi_t] = -(1/2\beta)\psi_{xx} + V(x)\psi, \qquad (1)$$

where $\beta = (\omega/c_0)n_s$ is the propagation constant, n_s is the substrate refractive index, $v_g = (d\beta/d\omega)^{-1}$ is the group velocity of light, and $V(x) = \beta[n_s - n(x) - Fx]/n_s$. In writing Eq. (1), I neglected nonlinearities and group-velocity dispersion, and assumed $\mathbf{A}(x, z, t) = [\hbar/(2\epsilon_0 n_s c_0 \omega_0)]^{1/2}[\psi \exp(-i\omega t + i\beta z) + \text{c.c.}]\mathbf{u}_y$ for the vector potential. With such a normalization, the cycle-averaged total energy of the electromagnetic field [13] is given by $U = (\hbar\omega/v_g)\int dxdz\psi^\dagger\psi = \hbar\omega\int dxdt\psi^\dagger\psi$. For monochromatic beams, the envelope $\psi(x, z)$ can be decomposed as a superposition of orthonormal Bloch states $\varphi_n(x, \kappa)$ of the array as $\psi(x, z) = \sum_n \int_{-\pi/a}^{\pi/a} d\kappa c_n(z, \kappa)\varphi_n(x, \kappa)$, where n is the band index, $-\pi/a < \kappa < \pi/a$, κ is the Bloch wave number (quasimomentum), and a is the lattice period. $\varphi_n(x, \kappa) \times \exp[-2\pi i E_n(\kappa)z/\lambda]$ satisfies Eq. (1) with $F = 0$, where $E_n(\kappa)$ is the dimensionless dispersion curve for the nth band of the array [14]. The evolution equations for the spectral coefficients $c_n(z, \kappa)$ govern the onset of BO and ZT, and have been previously considered in [12,14]. In particular, the fractional light power trapped in the nth band of the array (band occupancy) is given by $Z_n(z) = \int d\kappa|c_n(z, \kappa)|^2/\sum_n \int d\kappa|c_n(z, \kappa)|^2$. The value of the spectral coefficient $c_n(z, \kappa)$ at the entrance plane $z = 0$ is determined by the angular spectrum $\tilde{g}(\kappa) = (2\pi)^{-1/2} \times \int dx g(x)\exp(-i\kappa x)$ of the incident beam $g(x) = \psi(x, 0)$ according to [14] $c_n(0, \kappa) = \sum_{l=-\infty}^{\infty} B_n(\kappa + 2\pi l/a)\tilde{g}(\kappa + 2\pi l/a)$, where $B_n(\kappa)$ is the plane-wave excitation function for the band n defined as in Ref. [14]. For broad beam excitation with an incidence angle $\theta = \kappa\lambda/(2\pi)$ smaller than the Bragg angle $\theta_B = \lambda/(2a)$, the array is mostly excited in its first band [14], and for negligible interband coupling the single-band wave packet would undergo undamped BO with a spatial periodicity $z_B = \lambda F/a$ [6,14]. As the gradient F increases, ZT cannot be neglected and BO are damped. In singly periodic arrays, ZT typically manifests as a cascading of transitions to higher-order bands [6]. Figure 1 shows an example of BO in a 40-mm-long array excited at $\lambda = 1.44$ μm by a broad Gaussian beam at normal incidence. Owing to ZT, the BO motion is damped [Fig. 1(d)], with a characteristic staircase profile of band occupancy Z_1 which drops at each ZT burst [Fig. 1(e)]. Radiation tunnelled into higher-order bands is rapidly refracted away from the main wave packet [Fig. 1(d)]. In doubly periodic (binary) arrays [Fig. 2(a)], light remains trapped in the first two bands of the array, i.e., ZT to higher-order bands is negligible, and a sequence of periodic or aperiodic beam splitting and recombination, superimposed to BO and referred to as Bloch-Zener (BZ) oscillations [12,15], are observed. Examples of BZ oscillations in a binary array, for broad beam excitation at two different incidence angles, are shown in Fig. 2.

Quantum description.—To describe propagation of quantized fields along the array, I follow a procedure similar to that adopted in the quantum theory of fiber solitons [16], which consists in writing Eq. (1) in Hamiltonian form assuming z as an independent variable. Introducing the new field $\Pi = i\hbar\psi^\dagger$ and the Hamiltonian $H = \int dx dt \mathcal{H}$ with density $\mathcal{H} = -(i/2\beta)\Pi_x\psi_x - (1/v_g)\times \Pi\psi_t - iV(x)\Pi\psi$, the Hamilton equations $\psi_z = (\delta H/\delta\Pi)$, $\Pi_z = -(\delta H/\delta\psi)$ yield Eq. (1) and its complex conjugate, so that Π is canonically conjugated to ψ.

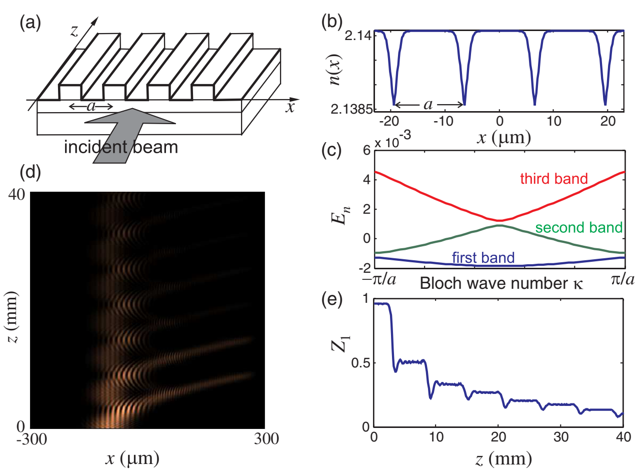

FIG. 1 (color online). (a) Schematic of a singly periodic waveguide array. (b) Refractive index profile. (c) Band diagram. (d) Damped optical BO (pseudocolor map of $|\psi(x,z)|^2$) for a broad Gaussian input beam at normal incidence (index gradient $F = 18.46$ m^{-1}, $\lambda = 1.44$ μm). (e) Fractional light power $Z_1(z)$ trapped in the first band of the array.

Quantization is accomplished by replacing the classical fields ψ and Π with the operators $\hat{\psi}(x,t)$ and $\hat{\Pi} = i\hbar\hat{\psi}^\dagger(x,t)$ satisfying the commutation relations $[\hat{\psi}(x,t), \hat{\psi}^\dagger(x',t')] = \delta(x-x')\delta(t-t')$ and $[\hat{\psi}(x,t), \hat{\psi}(x',t')] = [\hat{\psi}^\dagger(x,t), \hat{\psi}^\dagger(x',t')] = 0$. By introducing the spectral decomposition $\hat{\psi}(x,t) = (2\pi)^{-1/2}\int d\Omega\hat{\phi}(x,\Omega)\exp(-i\Omega t)$, the second-quantized Hamiltonian operator reads

$$\hat{H} = \hbar\int dx d\Omega\left\{\frac{1}{2\beta}\hat{\phi}_x^\dagger\hat{\phi}_x + \left[V(x) - \frac{\Omega}{v_g}\right]\hat{\phi}^\dagger\hat{\phi}\right\} \quad (2)$$

whereas the field energy is described by the operator $\hat{U} = \hbar\omega\int dx d\Omega\hat{\phi}^\dagger(x,\Omega)\hat{\phi}(x,\Omega)$. The commutation relations $[\hat{\phi}(x,\Omega), \hat{\phi}^\dagger(x',\Omega')] = \delta(x-x')\delta(\Omega-\Omega')$ and $[\hat{\phi}(x,\Omega), \hat{\phi}(x',\Omega')] = [\hat{\phi}^\dagger(x,\Omega), \hat{\phi}^\dagger(x',\Omega')] = 0$ hold. In the Schrödinger picture, the quantum field is described by a vector state $|\mathcal{Q}(z)\rangle$ which evolves according to $i\hbar(d/dz|\mathcal{Q}\rangle) = \hat{H}|\mathcal{Q}\rangle$. The state $|\mathcal{Q}\rangle$ can be expanded in Fock space as $|\mathcal{Q}\rangle = \sum_n a_n|f^{(n)}(\mathbf{x},\mathbf{\Omega},z)\rangle$, where the n-photon number state $|f^{(n)}\rangle$ is defined by [16]

$$|f^{(n)}\rangle = \int d\mathbf{x}d\mathbf{\Omega}\frac{f^{(n)}(\mathbf{x},\mathbf{\Omega},z)}{\sqrt{n!}}\hat{\phi}^\dagger(x_1,\Omega_1)\cdots\hat{\phi}^\dagger(x_n,\Omega_n)|0\rangle \quad (3)$$

with the normalization conditions $\sum_n|a_n|^2 = 1$ and $\int d\mathbf{x}d\mathbf{\Omega}|f^{(n)}(\mathbf{x},\mathbf{\Omega},z)|^2 = 1$. Note that, as $\hat{U}|f^{(n)}\rangle = n\hbar\omega|f^{(n)}\rangle$, the Fock state $|f^{(n)}\rangle$ is obtained from the vac-

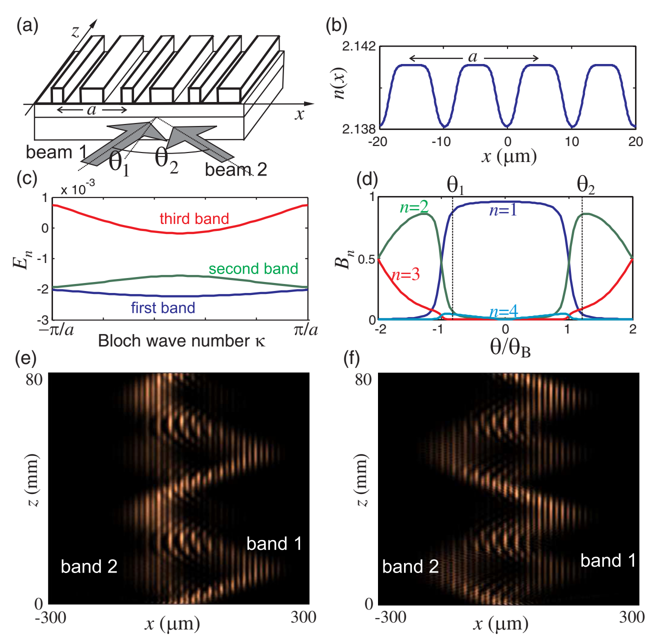

FIG. 2 (color online). (a) Schematic of a binary array made of a sequence of alternating wide and narrow waveguides. (b) Refractive index profile. (c) Band diagram. (d) Plane-wave excitation coefficients B_n for low-order bands versus incidence angle $\theta = \lambda\kappa/(2\pi)$, normalized to the Bragg angle $\theta_B = \lambda/(2a)$. (e) and (f) BZ oscillations when the array is excited by a broad Gaussian beam at either $\theta_1 = -0.8\theta_B$ [in (e)] and $\theta_2 = 1.2\theta_B$ [in (f)]. $F = 3.6$ m^{-1}, $\lambda = 1.44$ μm.

uum state $|0\rangle$ by creating n photons with space-frequency weighting function $f^{(n)}$. The evolution equation for $f^{(n)}$, obtained from the Schrödinger equation and using the commutation relations of field operators, reads

$$i\frac{\partial f^{(n)}}{\partial z}=\sum_{l=1}^{n}\left[-\frac{1}{2\beta}\frac{\partial^2}{\partial x_l^2}+V(x_l)-\frac{\Omega_l}{v_g}\right]f^{(n)}. \quad (4)$$

Owing to the bosonic nature of photons, solely symmetric functions $f^{(n)}$ should be considered. Here I focus on propagation of monochromatic fields, so that in previous equations I may disregard integration over different spectral components Ω and use a single (renormalized) bosonic creation operator $\hat{\phi}^\dagger(x)$ (e.g., at frequency $\Omega=0$) satisfying the commutation relations $[\hat{\phi}(x),\hat{\phi}^\dagger(x')]=\delta(x-x')$ and $[\hat{\phi}(x),\hat{\phi}(x')]=[\hat{\phi}^\dagger(x),\hat{\phi}^\dagger(x')]=0$. The simplest n-photon number state [Eq. (3)], denoted by $|g\rangle_n$, is obtained by assuming $f^{(n)}=g(x_1,z)g(x_2,z)\dots g(x_n,z)$, where the function $g(x,z)$ satisfies the wave equation (1) with the normalization $\int dx|g(x,z)|^2=1$. In this case one has $|g\rangle_n=(1/\sqrt{n!})(\int dxg(x,z)\hat{\phi}^\dagger(x))^n|0\rangle$, a state which describes the excitation of the optical system with an n-photon number state input beam with a spatial profile $g(x,0)$ at the entrance plane $z=0$. For a given set of normalized and orthogonal solutions $g_1(x,z), g_2(x,z),\dots$ to Eq. (1), using Eqs. (3) and (4) one can also construct the n-photon number state $|g_1,g_2,\dots\rangle_{n_1,n_2,\dots}=|g_1\rangle_{n_1}\otimes|g_2\rangle_{n_2}\otimes\dots$, which describes excitation of the optical system with a set of independent beams carrying $n_1,n_2,\dots$ photons ($n=n_1+n_2+\dots$). The classical picture of BO and ZT is retrieved from the quantum model when the input beam is in a coherent state (classical light). In fact, the vector state $|g;\alpha\rangle_{\rm coh}\equiv\sum_{n=0}^{\infty}a_n|g\rangle_n$, obtained by a superposition of photon number states $|g\rangle_n$ with a Poisson distribution $a_n=\exp(-|\alpha|^2/2)\alpha^n/\sqrt{n!}$ with c number α, is an eigenstate of the field annihilation operator $\hat{\phi}(x)$, $\hat{\phi}(x)|g;\alpha\rangle_{\rm coh}=\alpha g(x,z)|g;\alpha\rangle_{\rm coh}$, where $g(x,z)$ evolves according to Eq. (1). Therefore the expectation value ${}_{\rm coh}\langle g;\alpha|\hat{\phi}(x)|g;\alpha\rangle_{\rm coh}=\alpha g(x,z)$ yields the classical solution of the wave equation (1) for an input beam profile $\alpha g(x,0)$. More generally, for a nonclassical state $|\mathcal{Q}\rangle$ obtained by an arbitrary superposition of photon number states $|g\rangle_n$ with amplitudes a_n, one can readily show that the expectation value of $\hat{\phi}^\dagger(x)\hat{\phi}(x)$ yields the classic wave optics intensity distribution, i.e., $\langle\mathcal{Q}|\hat{\phi}^\dagger(x)\hat{\phi}(x)|\mathcal{Q}\rangle=\langle n\rangle|g(x,z)|^2$, where $\langle n\rangle=\sum_n n|a_n|^2$ is the mean photon number of the input beam. To highlight strictly quantum aspects of BO and ZT using nonclassical light, the photon statistics in the various bands of the array has to be considered.

Quantum signatures of ZT.—As a first example of particlelike behavior of photons in ZT, let us consider a singly periodic array illuminated by a broad Gaussian beam with (normalized) spatial profile $g(x,0)$ and with a photon statistics a_n. For nearly normal incidence, the lowest band of the array is excited. The state vector of the system is given by $|\mathcal{Q}\rangle=\sum_n a_n|g\rangle_n$, where $g(x,z)$ evolves according to Eq. (1). Let us indicate by $P^{(\boldsymbol{l})}(\mathbf{n})$ the joint photon distribution to find $n_1,n_2,\dots$ photons in the l_1th, l_2th,... bands of the array, respectively, where $\mathbf{n}=(n_1,n_2,\dots)$ and $\boldsymbol{l}=(l_1,l_2,\dots)$. To calculate $P^{(\boldsymbol{l})}(\mathbf{n})$ at a given propagation distance z, I expand the field $g(x,z)$ as a superposition of normalized wave packets $g_1(x,z), g_2(x,z),\dots$ belonging to the different bands of the array (see, e.g., [14]): $g(x,z)=\sqrt{Z_1(z)}g_1(x,z)+\sqrt{Z_2(z)}g_2(x,z)+\dots$, where $Z_l(z)\le 1$ is the fractional occupancy of band l calculated by the classical ZT model. The photon distribution $P^{(\boldsymbol{l})}(\mathbf{n})$ is then readily obtained after a multiple binomial expansion of the operator $(\sqrt{Z_1}\hat{b}_1^\dagger+\sqrt{Z_2}\hat{b}_2^\dagger+\dots)^n$ entering in the photon number state $|g\rangle_n$, where $\hat{b}_k^\dagger\equiv\int dxg_k(x,z)\hat{\phi}^\dagger(x)$. For instance, the (marginal) photon distribution $P^{(l)}(n)$ to find n photons in the lth band of the array, calculated with this procedure, yields

$$P^{(l)}(n)=\sum_{k=n}^{\infty}|a_k|^2\binom{k}{n}Z_l^n(z)[1-Z_l(z)]^{k-n}. \quad (5)$$

The mean photon number in the lth band is readily calculated as $\langle n_l\rangle=\sum_n nP^{(l)}(n)=\langle n\rangle Z_l(z)$, where $\langle n\rangle=(\sum_n n|a_n|^2)$ is the mean photon number of input beam. In particular, the mean photon number $\langle n_1\rangle$ participating in the BO motion decays with the same law $\sim Z_1(z)$ of classic ZT theory shown in Fig. 1(e). The particlelike behavior of photons undergoing ZT is revealed when the photon statistics for a coherent and for a photon number state input beams are compared. In the former case, $|a_n|^2$ is a Poissonian distribution with mean $\langle n\rangle$ and variance $\langle\Delta n^2\rangle=\langle n\rangle$. From Eq. (5) it follows that the photon distribution $P^{(1)}(n)$ remains Poissonian with reduced mean $\langle n_1\rangle=Z_1(z)\langle n\rangle$ and variance $\langle\Delta n_1^2\rangle=\langle n_1\rangle$. Conversely, for a photon number state input beam, i.e., for $a_n=\delta_{n,n_0}$, a binomial distribution $P^{(1)}(n)=[n_0!/n!(n_0-n)!]Z_1^n(z)[1-Z_1(z)]^{n_0-n}$ $[P^{(1)}(n)=0$ for $n>n_0]$ is obtained, with photon mean $\langle n_1\rangle=Z_1(z)n_0$ and variance $\langle\Delta n_1^2\rangle=Z_1(z)[1-Z_1(z)]n_0$. Such a distribution, analogous to that created by a beam splitter excited by a photon number state in one port, and the vacuum state in the other one, provides a clear signature of the particlelike behavior of photons [8]: each photon in the initially excited band behaves, during the BO motion, like a "classical particle" which independently undergoes a Bernoulli trial (coin toss) at each ZT burst with a cumulative tunnelling probability into other bands given by $1-Z_1(z)$. The binomial distribution of photons remaining in the initial band then follows from simple combinatory arguments [8].

As a second example, I consider two-photon excitation of a binary array by two nearly plane-wave beams g_1 and g_2 at incidence angles θ_1 and θ_2 [Fig. 2(a)]. Each wave is assumed to be prepared in a single photon number state. The angle θ_1 of the first beam is chosen to efficiently excite the first-band Bloch mode $\varphi_1(x,\kappa)$ with transverse wave number $\kappa=2\pi\theta_1/\lambda$, whereas the angle θ_2 of the second

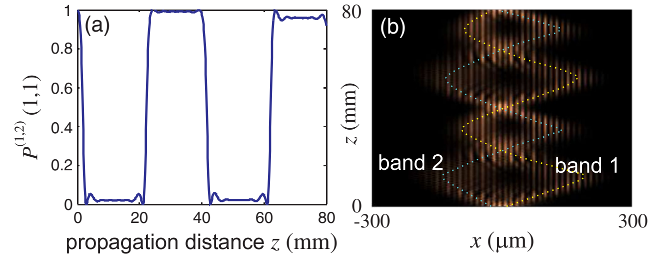

FIG. 3 (color online). (a) Numerically computed joint photon probability in the binary array of Fig. 2 excited by two-photon beams in the geometry of Fig. 2(a). (b) Pseudocolor map of the expectation value $\langle \hat{\phi}^\dagger(x)\hat{\phi}(x)\rangle$.

beam is correspondingly tuned to excite the second-band Bloch mode $\varphi_2(x, \kappa)$ with the same wave number κ. This is achieved by assuming $\theta_2 = \theta_1 + 2\theta_B$. For example, for the array of Fig. 2 one can assume $\theta_1 = -0.8\theta_B$ and $\theta_2 = 1.2\theta_B$, which ensure efficient single-band excitation [see Fig. 2(d)]. As ZT to higher-order bands is negligible, the two photons remain trapped in the two lowest-order bands. According to the acceleration theorem, the quasi-momentum drifts as $\kappa' = \kappa - 2\pi Fz/\lambda$ [14] and propagation in the array yields $(g_1(x, z), g_2(x, z))^T = \exp(i\alpha)\mathbf{S}(\varphi_1(x, \kappa'), \varphi_2(x, \kappa'))^T$, where $\mathbf{S}(z)$ is the ZT matrix and $\alpha(z)$ an unimportant phase term. The ZT matrix is unitary, with $S_{n,l}(0) = \delta_{n,l}$, $S_{22} = S_{11}^*$, and $S_{21} = -S_{12}^*$. After the introduction of the bosonic creation operators $\hat{a}_1^\dagger = \int dx \varphi_1(x, \kappa')\hat{\phi}^\dagger(x)$ and $\hat{a}_2^\dagger = \int dx \varphi_2(x, \kappa')\hat{\phi}^\dagger(x)$ of modes $\varphi_1(x, \kappa')$ and $\varphi_2(x, \kappa')$, the state vector of the system may be written as

$$|Q\rangle = [S_{11}(z)\hat{a}_1^\dagger + S_{12}(z)\hat{a}_2^\dagger][S_{21}(z)\hat{a}_1^\dagger + S_{22}(z)\hat{a}_2^\dagger]|0\rangle. \quad (6)$$

The joint probability $P^{(1,2)}(1, 1)$ to find one photon in each of the two bands is $P^{(1,2)}(1, 1) = |S_{11}S_{22} + S_{12}S_{21}|^2$. The two terms $S_{11}S_{22}$ and $S_{12}S_{21}$ describe two possible paths which correspond, the former to the absence of ZT (each photon remains in its original band), the latter to two ZT events (each photon undergoes ZT into the other band). $P^{(1,2)}(1, 1)$ vanishes when the two paths interfere destructively. This leads to a photon bunching effect and entanglement of photons in the two bands, fully analogous to the Hong-Ou-Mandel interference in a 50% beam splitter [7,8]. An example of photon bunching in a binary array is shown in Fig. 3(a). Here the index gradient F has been tuned to achieve ~50% ZT probability at each ZT burst. The joint probability $P^{(1,2)}(1, 1)$ thus switches from ~1 to ~0 at successive bursts. While each photon alone would undergo a sequence of BZ oscillations shown in Figs. 2(e) and 2(f), the expectation value $\langle \hat{\phi}^\dagger(x)\hat{\phi}(x)\rangle$ for the two-photon state, shown in Fig. 3(b), is given by the incoherent superposition $|g_1(x, z)|^2 + |g_2(x, z)|^2$, the absence of interference being due to the lack of a definite phase relationship between the two photons [17]. The wave packets belonging to the two Bloch bands follow distinct paths [denoted by dashed lines in Fig. 3(b) for eye guiding] and are spatially separated after each ZT burst. Therefore, photon coincidence measurements of the two spatially separated beams can be performed in near field to check photon bunching. It should be noted that imperfections of the array, such as nonuniformity of waveguide spacing, affect the scattering ZT matrix, and hence the joint probability $P^{(1,2)}(1, 1)$; however, BZ oscillations and quantum interference are not destroyed for slight deviations [18].

In conclusion, a quantum theory of BO and ZT in optical lattices has been presented, and particlelike aspects of photons undergoing BO and ZT in singly and doubly periodic lattices have been highlighted.

[1] R. Morandotti, U. Peschel, J. S. Aitchison, H. S. Eisenberg, and Y. Silberberg, Phys. Rev. Lett. **83**, 4756 (1999).
[2] T. Pertsch, P. Dannberg, W. Elflein, A. Bräuer, and F. Lederer, Phys. Rev. Lett. **83**, 4752 (1999).
[3] R. Sapienza, P. Costantino, D. Wiersma, M. Ghulinyan, C. J. Oton, and L. Pavesi, Phys. Rev. Lett. **91**, 263902 (2003).
[4] M. Ghulinyan, C. J. Oton, Z. Gaburro, L. Pavesi, C. Toninelli, and D. S. Wiersma, Phys. Rev. Lett. **94**, 127401 (2005).
[5] H. Trompeter, W. Krolikowski, D. N. Neshev, A. S. Desyatnikov, A. A. Sukhorukov, Yu. S. Kivshar, T. Pertsch, U. Peschel, and F. Lederer, Phys. Rev. Lett. **96**, 053903 (2006).
[6] H. Trompeter, T. Pertsch, F. Lederer, D. Michaelis, U. Streppel, A. Bräuer, and U. Peschel, Phys. Rev. Lett. **96**, 023901 (2006).
[7] C. K. Hong, Z. Y. Ou, and L. Mandel, Phys. Rev. Lett. **59**, 2044 (1987).
[8] R. A. Campos, B. E. A. Saleh, and M. C. Teich, Phys. Rev. A **40**, 1371 (1989).
[9] W. K. Lai, V. Bužek, and P. L. Knight, Phys. Rev. A **43**, 6323 (1991).
[10] A. Politi, M. J. Cryan, J. G. Rarity, S. Yu, and J. L. O'Brien, Science **320**, 646 (2008).
[11] Y. Bromberg, Y. Lahini, R. Morandotti, and Y. Silberberg, arXiv:0807.3938v1.
[12] S. Longhi, Europhys. Lett. **76**, 416 (2006).
[13] T. A. B. Kennedy and E. M. Wright, Phys. Rev. A **38**, 212 (1988).
[14] S. Longhi, M. Lobino, M. Marangoni, R. Ramponi, P. Laporta, E. Cianci, and V. Foglietti, Phys. Rev. B **74**, 155116 (2006).
[15] B. M. Breid, D. Witthaut, and H. J. Korsch, New J. Phys. **8**, 110 (2006).
[16] Y. Lai and H. A. Haus, Phys. Rev. A **40**, 844 (1989);E. M. Wright, Phys. Rev. A **43**, 3836 (1991).
[17] R. Ghosh and L. Mandel, Phys. Rev. Lett. **59**, 1903 (1987).
[18] For instance, numerical simulations show that the joint photon probability in the binary array of Fig. 2 appreciably deviates from the behavior depicted in Fig. 3(a) when adjacent waveguides are randomly spaced by more than 5% around the mean (uniform) distance $a/2 = 10$ μm.