Distinguishing Newly Born Strange Stars from Neutron Stars with g-Mode Oscillations

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The gravity-mode (g-mode) eigenfrequencies of newly born strange quark stars (SQSs) and neutron stars (NSs) are studied. It is found that the eigenfrequencies in SQSs are much lower than those in NSs by almost 1 order of magnitude, since the components of a SQS are all extremely relativistic particles while nucleons in a NS are nonrelativistic. We therefore propose that newly born SQSs can be distinguished from the NSs by detecting the eigenfrequencies of the g-mode pulsations of supernovae cores through gravitational radiation by LIGO-class detectors.

Since their discovery 40 years ago, pulsars have been serving as unprecedented astro-laboratories, especially due to their extreme density and magnetic field. However, the basic nature of pulsarlike stars remains murky, since, besides the conventional neutron stars (NSs), strange quark stars (SQSs) are also potential candidates for the stars [1–7], due to the quark deconfinement phase transition at high density. More recently, the composition of pulsars, especially the existence of SQSs, has become a hotly debated topic (see, for example, Refs. [6–10]), since the observation of the object pulsating source of radio (PSR) $J0751 + 1807$ yielded a mass $(2.1 \pm 0.2)M_\odot$ with 1σ error bars [11], and one of the pulsars Ter 5 I and J has a mass larger than $1.68M_\odot$ to 95% confidence [12]. However, in general view, the equation of state (EOS) of quark matter is much softer, the quark star may not be so massive [8]. The composition of pulsars and the method to distinguish the SQSs from NSs becomes a fundamental topic not only in astrophysics but also in high energy physics and nuclear physics.

Many potentially observable differences have been proposed to distinguish SQSs from NSs. For example, SQSs have much larger dissipation rate of radial vibrations [13] and higher bulk viscosity [14]; the spin rate of SQSs can be much closer to the Kepler limit than that of NSs [15]; SQSs may cool more rapidly than NSs within the first 30 years [16]. In this Letter, we propose that the eigenfrequencies of the gravity-mode (g-mode) pulsations of supernova cores provide another sharp criteria to distinguish SQSs from NSs after their birth in a core-collapse supernovae.

Recent simulations of core-collapse supernovae have indicated that core g-mode pulsations may be excited by turbulence and downstream accretion [17], due to the stalled supernova shock undergoing a standing-accretion-shock instability and becoming highly deformed [18,19]. Following this discovery, Ott *et al.* have found that these g-mode pulsations of supernova cores serve as efficient sources of gravitational waves [20]. It is natural to expect that more information about the interior constituents of supernova cores carried by gravitational waves, such as whether SQSs can be formed during the processes of supernovae, could be obtained by ground-based detectors such as LIGO and VIRGO. In this Letter, we present our calculations of the g-mode eigenfrequencies for newly born NSs and SQSs in the supernova cores. Our results show that the frequencies of newly born SQSs are lower than those of NSs by as much as 1 order of magnitude. Such a large difference may provide an appropriate means to distinguish SQSs from NSs that are newly formed in a core-collapse supernovae. The present work is then of general interest in nuclear/quark physics, gravitation-wave physics, and supernova physics.

Our calculations start with the differential equations in the Newtonian formulation which governs the linear oscillations of a nonrotating, unmagnetized, and fluidic star (for details, see, for example, Refs. [21–23]). If the equilibrium configuration of a star is assumed to be spherically symmetric, the pressure p_0, the density ρ_0, and the gravitational potential ϕ_0 depend only on the radial coordinate r. Oscillations of the star can be described by a vector field $\boldsymbol{\xi}(\boldsymbol{r}, t)$ of displacement and the Eulerian (or "local") perturbations of the pressure, density, and the gravitational potential are δp, $\delta\rho$, and $\delta\phi$. We adopt the Cowling approximation, i.e., neglecting $\delta\phi$ [24]. Let the radial component of $\boldsymbol{\xi}(\boldsymbol{r}, t)$ be $\xi_r Y_{lm}(\theta, \varphi)e^{-i\omega t}$, following the equations of motion and continuity, we have the linear oscillation equations as

$$\frac{d}{dr}(r^2\xi_r) = \frac{g}{c_s^2}(r^2\xi_r) + \left[\frac{l(l+1)}{\omega^2} - \frac{r^2}{c_s^2}\right]\left(\frac{\delta p}{\rho_0}\right), \quad (1)$$

$$\frac{d}{dr}\left(\frac{\delta p}{\rho_0}\right) = \frac{\omega^2 - \omega_{BV}^2}{r^2}(r^2\xi_r) + \frac{\omega_{BV}^2}{g}\left(\frac{\delta p}{\rho_0}\right), \quad (2)$$

where the dependence of δp on the spherical harmonics $Y_{lm}(\theta, \varphi)$ and the time factor $e^{-i\omega t}$ is separated analogous to ξ_r; ω is the eigenfrequency of a mode with the index

$\alpha \equiv \{n, l, m\}$ being abbreviated; g is the local gravitational acceleration, and the Brunt-Väisälä frequency ω_{BV} is defined as

$$\omega_{BV}^2 = g^2 \left(\frac{1}{c_e^2} - \frac{1}{c_s^2} \right), \qquad (3)$$

with c_s, c_e being, respectively, the adiabatic, the equilibrium sound speeds defined as

$$c_s^2 \equiv \left(\frac{\partial p}{\partial \rho} \right)_{adia}, \qquad c_e^2 \equiv \left(\frac{dp}{d\rho} \right)_{equi} = \frac{dp_0/dr}{d\rho_0/dr}. \qquad (4)$$

Equations (1) and (2) determine the eigenfrequency ω for the modes, when complemented by proper boundary conditions,

$$r^2 \xi_r = \frac{l}{\omega^2} \frac{\delta p}{\rho_0} r \sim r^{l+1} (r \to 0), \qquad (5)$$

at the stellar center, and

$$\Delta p = \delta p - \rho_0 g \xi_r = 0 (r = R) \qquad (6)$$

for a vanishing Lagrangian perturbation of pressure at the stellar surface $r = R$.

The equilibrium configurations of newly born NSs and SQSs are determined by solving the Newtonian hydrostatic equations, which are consistent with Eqs. (1) and (2). The sound speeds c_s and c_e, thus the Brunt-Väisälä frequency ω_{BV}, are calculated as functions of the radius r. Equations (1) and (2) are numerically integrated from a small value of r approaching to zero to the stellar surface $r = R$. The outer boundary condition of Eq. (6) is enforced by a shooting routine. The eigenfrequency ω is computed iteratively after a trial value. The order n of a mode counts the number of nodes in the radial displacement ξ_r.

To proceed, a NS is assumed to consist of nucleons, electrons, thermal photons, and neutrinos trapped in the dense matter, in the first tens of seconds after the core bounce in a core-collapse supernova [25]. A fluid element inside the star is characterized by the baryon density ρ_B, the entropy per baryon S, and the lepton fraction $Y_L = Y_e + Y_{\nu_e}$ (or Y_e related with Y_L through beta equilibrium conditions, here $Y_i = \rho_i/\rho_B$), all depending on the radial coordinate r. In this work, all the components except the nucleons are assumed to be noninteracting, while the interactions between nucleons are described by the relativistic mean field (RMF) theory at finite temperature (see, for example, Refs. [26,27]). We take the formalism of the Lagrangian density for RMF as the same as that used in Ref. [28]. Five parameters for the theory are fixed by fitting the properties of the symmetric nuclear matter at saturation density: saturation nucleon number density $\rho_0 = 0.16$ fm^{-3}, binding energy $E/A = -16$ MeV, nucleon effective mass $m^* = 0.75 m_N$ (the bare mass $m_N = 938$ MeV), incompressibility $K = 240$ MeV, and the symmetry energy $E_s = 30.5$ MeV.

Within the framework of RMF theory, we can obtain the energy density and the pressure of the hot NS matter. In the absence of neutrino diffusion, the adiabatic sound speed reads

$$c_s^2 = \left(\frac{\partial p}{\partial \rho} \right)_{S, Y_L}. \qquad (7)$$

It is remarkable that, in the interior of a newly born NS, the temperature is of order 10 MeV and the time scale of establishing the beta equilibrium can be smaller than $\sim 10^{-8}$ s [28], which is much smaller than the period of the g-mode pulsations about 10^{-3}–10^{-2} s. Therefore, in our calculation, we assume that Y_L for each fluid element is constant and the system is in beta equilibrium during the process of pulsations, which is consistent with the Ledoux convective criterion in supernova simulations [29]. This situation is different from that in cold NS where beta equilibrium cannot be obtained during the pulsations and the electron fraction Y_e rather than Y_L remains unchanged [22]. The equilibrium sound speed c_e can also be fixed for an equilibrium configuration, where ρ_B, S, and Y_L (Y_e) are simply functions of the radius r.

For a newly born SQS, it is usually believed to be composed of three flavors of quarks (u, d and s), electrons, three flavor neutrinos, thermal photons, and gluons. In our present calculation, along the line of Ref. [30], we implement the MIT bag model [31] for its EOS. We choose $m_u = m_d = 0$, $m_s = 150$ MeV, and a bag constant $B^{1/4} = 154.5$ MeV, yielding a binding energy per baryon of 928 MeV for the SQS matter in equilibrium ($p = 0$), which is comparable to 931 MeV per nucleon for the ^{56}Fe nucleus [30]. Like in the NS case, all the thermodynamical properties and the two sound speeds can be determined as the equilibrium configuration of a SQS is known.

In our calculation, we employ the equilibrium configuration of newly born NSs resulting from the 2D hydrodynamic simulations of core-collapse supernovae by the Arizona Group [32]. We calculate then the properties of the g-mode oscillations of newly born NSs at the time $t = 100$, 200, and 300 ms after the core bounce, along the line taken in Ref. [32]. The time range is limited to within 300 ms for simplicity since this is guaranteed by the fact that during the first second after the core bounce, the eigenfrequencies vary only a little; for example, the frequencies of g-modes vary within 727 Hz–819 Hz when the time ranges from 0.3 to 1 s in a newly born NS model [33].

Equations (1) and (2) are integrated from the center to a radius of about 20 km where a convective instable region appears. We choose the mass inside the radius of 20 km to be 0.8, 0.95, and $1.05 M_\odot$ at $t = 100$, 200, and 300 ms after the core bounce, respectively, according to the supernovae simulations [32]. The increase of the mass with time is due to accretions of mantle materials of the progenitor star onto the newly born neutron star.

Table I lists the calculated g-mode oscillation eigenfrequencies of an above-modeled newly born NS with order $n = 1, 2, 3$. Here the harmonic index is fixed at $l = 2$, which represents the quadrupole motion of mass and is

TABLE I.   Calculated eigenfrequencies (in unit of Hz) of the g-mode oscillations of the newly born NSs and SQSs at several time t (in unit of millisecond) after the core bounce.

Radial order of g-mode	Neutron Star			Strange Quark Star		
	$t = 100$	$t = 200$	$t = 300$	$t = 100$	$t = 200$	$t = 300$
$n = 1$	717.6	774.6	780.3	82.3	78.0	63.1
$n = 2$	443.5	467.3	464.2	52.6	45.5	40.0
$n = 3$	323.8	339.0	337.5	35.3	30.8	27.8

directly related to the emission of gravitational waves (see, for example, Refs. [34–36]). Our results of 717.6, 774.6, and 780.3 Hz for $n = 1$ at $t = 100$, 200, and 300 ms after the core bounce, respectively, are consistent with the Newtonian supernovae simulation results of about 675 Hz in Ref. [17] and about 950 Hz in Ref. [20]. They are also close to the general relativistic results ranging from ∼620 to ∼820 Hz in the first second after the core bounce [33]. The quantitative differences result from different equations of state and NS models used, general relativistic effects and so on.

For comparison, we proceed to calculate the g-mode oscillation eigenfrequencies of newly born SQSs. Despite the lack of commonly accepted simulations on supernovae with strange quark cores in literatures, following the results of the trial simulations [37,38], general theoretical investigations [39,40], and analysis of some supernovae observations [41,42], we assume that a strange quark core can be formed during the collapse of presupernova.

To carry out the calculation for SQSs, we assume that the radial variation behaviors of S and Y_L for newly born SQSs are the same as for NSs. But a newly born SQS differs from a NS in its smaller size, having a radius of about 10 km where the pressure vanishes. With the same masses as a NS at three time instants after the core bounce, we calculate the $l = 2$ eigenfrequencies of a newly born SQS at first. The obtained results are also listed in Table I. It is apparent that the g-mode oscillation frequencies of 82.3, 78.0, and 63.1 Hz of a SQS are definitely lower than those of a NS by an order of magnitude. Table I also shows the frequencies of modes with radial order higher than one, although such higher-order g-mode oscillations are less likely to be significant sources of gravitational waves, due to the much longer gravitational wave damping time and thus much weaker coupling to gravitational waves [33]. We have also performed calculations in the case of that the entropy per baryon S is increased to $1.5S$ at each fluid element inside the SQS, by considering the release of thermal energy during the conversion from nuclear matter to strange quark matter [43]. The eigenfrequencies of the $n = 1$ g-mode oscillations become 100.7, 105.9, and 96.1 Hz at the same three time instants, respectively, which are still significantly lower than those of a newly born NS. The dependence of the g-mode frequencies on the mass of a SQS has also been investigated. Our finding is that the eigenfrequencies of a newly born SQS increase a little for a larger mass. For example, a SQS core with mass $1.4M_\odot$

holds the $n = 1$ order g-mode frequencies 100.2, 91.4, and 73.0 Hz at the three instants, respectively. And the frequencies change to 108.8, 100.9, and 84.5 Hz accordingly as the mass of the SQS is increased to $1.68M_\odot$. They are also much lower than those of newly born NSs.

To explore the reason for the large difference in the g-mode oscillation eigenfrequencies between newly born NSs and SQSs, we reexpress the Brunt-Väisälä frequency in Eq. (3) as

$$\omega_{BV}^2 = -\frac{g}{\rho}\left[\left(\frac{\partial\rho}{\partial Y_L}\right)_{p,S}\frac{dY_L}{dr} + \left(\frac{\partial\rho}{\partial S}\right)_{p,Y_L}\frac{dS}{dr}\right], \quad (8)$$

where we further have

$$\left(\frac{\partial\rho}{\partial Y_L}\right)_{p,S} = -\frac{1}{c_s^2}\left(\frac{\partial p}{\partial Y_L}\right)_{\rho,S}, \quad (9)$$

$$\left(\frac{\partial\rho}{\partial S}\right)_{p,Y_L} = -\frac{1}{c_s^2}\left(\frac{\partial p}{\partial S}\right)_{\rho,Y_L}. \quad (10)$$

Comparing the calculations for SQSs with those for NSs, we find that the values of the $(\partial p/\partial S)_{\rho,Y_L}$ in Eq. (10) for SQSs are much smaller than those for NSs, which result in the values of the Brunt-Väisälä frequency ω_{BV} for SQSs much smaller than those for NSs as well. Quantitative comparisons of $(\partial p/\partial S)_{\rho,Y_L}$ and ω_{BV} between SQSs and NSs are displayed in Fig. 1 for an illustrative time
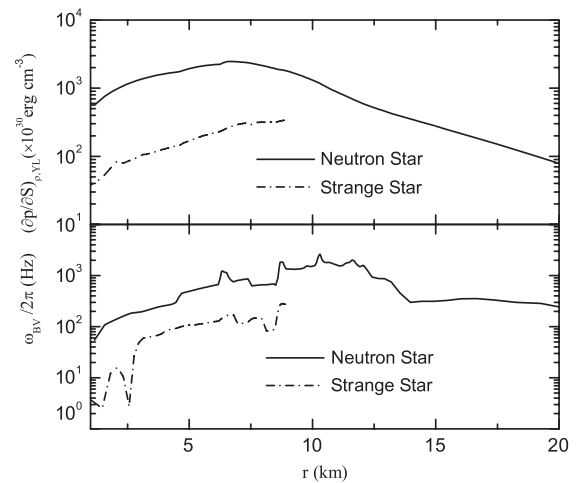

FIG. 1.   Calculated $(\partial p/\partial S)_{\rho,Y_L}$ (top) and Brunt-Väisälä frequency ω_{BV} (bottom) as functions of the radial coordinate at $t = 200$ ms after the core bounce for newly born NSs and SQSs. A sharp surface appears at about $r = 9$ km for the SQS.

$t = 200$ ms. Those calculated results can be understood easily as follows: a SQS is composed of particles which are all extremely relativistic. In the ideal case of neglecting the mass of the strange quark, i.e., $m_s = 0$, the EOS of a SQS has a simple form as $p = \rho/3 - 4B/3$ in the MIT model. Substituting this EOS into Eqs. (9) and (10), we find both $(\partial p/\partial S)_{\rho,Y_L}$ and $(\partial p/\partial Y_L)_{\rho,S}$ are vanishing, and thus the g-mode frequency of the SQS in this extreme situation is zero. This simple analysis is confirmed in our numerical calculations when setting $m_s = 0$. Therefore, the nonvanishing eigenfrequencies of g-mode pulsations for a realistic SQS as given in this work are due to the nonzero value of the strange quark mass. It should be emphasized that this result does not depend on the details of the model for the strange quark matter but only on the relativistic properties of the components of SQSs. Once the matter is relativistic and the EOS can be approximately parameterized as $p = \rho/3 + \text{const.}$, the g-mode frequencies are highly suppressed. While in a NS, beside the relativistic leptons, there are nonrelativistic nucleons (or, simply, with quite large masses), which are responsible for the much higher g-mode eigenfrequencies.

In summary, we have calculated the eigenfrequencies of g-mode pulsations in newly born neutron stars and strange quark stars in this Letter. Special emphasis is given to the g-mode oscillations with harmonic index $l = 2$ and order $n = 1$ due to the potential detectability of their gravitational radiation. Because their components are all relativistic particles, the g-mode oscillation frequencies of newly born strange quark stars are around or below 100 Hz for $l = 2$ and $n = 1$. They are definitely much lower than those of newly born neutron stars (~ 650–950 Hz, combining our results and those in literatures) by almost 1 order of magnitude. Since gravitational waves and the difference of the g-mode oscillation frequencies may be detected by LIGO-class detectors, our work illustrates an important signature to distinguish newly born strange quark stars from the neutron stars.

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