Perfect Reflection of Light by an Oscillating Dipole

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We show theoretically that a directional dipole wave can be perfectly reflected by a single pointlike oscillating dipole. Furthermore, we find that, in the case of a strongly focused plane wave, up to 85% of the incident light can be reflected by the dipole. Our results hold for the full spectrum of the electromagnetic interactions and have immediate implications for achieving strong coupling between a single propagating photon and a single quantum emitter.

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Common treatments of light-matter interaction consider an electromagnetic beam homogeneous over an area $\mathcal{A}$ that is perpendicular to the propagation direction and use the concept of an atomic cross section σ to arrive at the probability $\sigma/\mathcal{A}$ for exciting an atom. In conventional spectroscopy experiments, this ratio is very small because either σ is reduced by various broadening effects or $\mathcal{A}$ is large for technical reasons. However, recent experiments have shown that it is possible to overcome these difficulties for the optical excitation of single molecules, quantum dots, or atoms [1–5]. The intriguing question that arises is whether the experimentally observed coupling efficiencies are close to or far from theoretical limits. In particular, is it possible to excite an atom with probability equal to one by a single photon? Is it possible for an atom to imprint a large phase shift on a photon that passes by? Indeed, new publications address some of these issues [6–8].

On the theoretical side, the interaction of freely propagating photons with the dipolar transition of a two-level system (TLS) has been investigated for a quasi-one-dimensional case with emphasis on the quantum statistics of the incident light [9]. In three dimensions, methods of expansion of the focused beam in terms of vectorial mode functions and decomposition of the focused beam in dipolar and nondipolar vectorial modes have been employed [10,11]. These latter studies concluded that only the dipolar component of the excitation light can couple to a dipole and that the transmitted power is only weakly attenuated. Interestingly and somewhat in parallel, the literature on the interaction of a TLS with light confined to a waveguide claims that very strong attenuation is possible [12–14]. In this Letter, we examine the interaction of different light fields with a dipolar emitter in the framework of Debye diffraction and vectorial multipole expansion. We show that an incident directional dipolar wave experiences strong coupling to the emitter and is fully reflected. We first consider a classical oscillator and then extend the analysis to a TLS.

The classical interaction of light with an oscillating pointlike dipole located at the origin O is described by the Abraham-Lorentz equation [15]. After calculating the differential scattering cross section, one arrives at the total scattered power [15,16]

$$P_{\mathrm{sca}} = \frac{1}{2} c\epsilon_0 \int_{4\pi} r^2 |\mathbf{E}_{\mathrm{sca}}(\mathbf{r})|^2 d\Omega = 2cW^{\mathrm{el}}_{\mathrm{inc}}(O)\sigma, \quad (1)$$

where $\mathbf{E}_{\mathrm{sca}}$ is the field scattered by the dipole and the distance r lies in the far field $kr \gg 1$. $W^{\mathrm{el}}_{\mathrm{inc}}(O) = \epsilon_0 |\mathbf{E}_{\mathrm{inc}}(O)|^2/4$ is the time-averaged electric energy density at O. The parameter

$$\sigma = \sigma_0 \frac{\Gamma^2}{4\Delta^2 + \Gamma^2} \quad (2)$$

denotes the total scattering cross section of the oscillator, where Γ is the damping rate dictated by radiation reaction and $\Delta = \omega_L - \omega_0$ is the detuning between the incident light and oscillator frequencies ω_L and ω_0, respectively. The quantity $\sigma_0 = 3\lambda^2/(2\pi)$ denotes the cross section at resonance ($\Delta = 0$). We now consider the scattering ratio [16]

$$\mathcal{K}_0 = \frac{P_{\mathrm{sca}}}{P_{\mathrm{inc}}} = \frac{2cW^{\mathrm{el}}_{\mathrm{inc}}(O)\sigma_0}{\int \mathbf{S}(\mathbf{r}) \cdot \mathbf{n} d^2 r} = \frac{\sigma_0}{\mathcal{A}} \quad (3)$$

at resonance. Here P_{inc} is the incident power, $\mathbf{S}$ is the time-averaged Poynting vector of the incident field, and $\mathbf{n}$ is a unit vector normal to the integration surface. The integration can be taken over a plane at the incident aperture, over the Gaussian reference sphere (GRS), or over the focal plane (FP).

The derivation of Eqs. (1) and (3) is based on the fact that the oscillator interacts only with the electric field at the location of the oscillator, irrespective of whether the field is homogeneous as for a plane wave or inhomogeneous as in the focal region of a strongly focused beam [17]. Thus, σ can be treated as a universal quantity for a pointlike oscillator regardless of the modal properties of the excitation light. The quantity $\mathcal{A}$ introduced in Eq. (1) represents an effective focal area and depends implicitly on λ through the diffraction phenomenon. It is closely related to the normalized energy density $W^{\mathrm{el}}_{\mathrm{inc}}/P_{\mathrm{inc}}$, which has been studied for various focal systems [18–20]. The peculiarities of the incident field enter $\mathcal{K}_0$ via $\mathcal{A}$. Consequently, as we will show below, the problem of minimal transmittance is shifted to that of a minimal $\mathcal{A}$, and a strong photon-oscillator interaction is reachable for $\mathcal{K}_0 \gtrsim 1$ [12].

We first consider an incident x-polarized plane wave of amplitude E_0. The integration in Eq. (3) over the incident aperture is straightforward and yields $P_{\rm inc} = \frac{1}{2} c \epsilon_0 E_0^2 \pi a^2$ [16], where a is the radius of the entrance aperture. We also have $W_{\rm inc}^{\rm el}(O) = \epsilon_0 [\pi f E_0 |I_0(O)|/2\lambda]^2$, where f is the focal length of the focusing system and $I_0(O)$ is a diffraction integral [16,17]. The resulting value of $\mathcal{A}$ then yields

$$\mathcal{K}_0 = \frac{128}{75} \frac{1}{\sin^2\alpha} \left(1 - \frac{1}{8}(5 + 3\cos\alpha)\cos^{3/2}\alpha\right)^2, \quad (4)$$

where α specifies the incident solid angle Ω_α [see Fig. 1(a)]. For $\alpha = \pi/2$, $\mathcal{K}_0$ reaches the maximum value of $128/75 \simeq 1.7$. Assuming a backward and forward half-space and accounting for the fact that half of the power is scattered in each direction, it follows that up to 85% of the incident light is reflected into the backward half-space. For this configuration, the reflectance and transmittance are thus limited to $\mathcal{R} \lesssim 0.85$ and $\mathcal{T} = 1 - \mathcal{R} \gtrsim 0.15$, respectively.

An alternative way of performing the integration in Eq. (3) is to consider the FP. Because the intensity has cylindrical symmetry about the optical axis, the electric and the magnetic energy densities are equal at the focal spot [17] so that $2cW_{\rm inc}^{\rm el}(O) = S_z(O)$. The calculation of $\mathcal{A}$ then becomes

$$\mathcal{A} = \frac{\int_{\rm FP} S_z d^2 r}{S_z(O)} = \frac{\int_{\rm FP}(|I_0|^2 - |I_2|^2) d^2 r}{|I_0(O)|^2}, \quad (5)$$

where I_2 is again a diffraction integral [17]. The integration in the numerator turns out to be straightforward when an orthogonality relationship for Bessel functions is considered [16]. We note that the fields in a strongly focused beam show vortices in the FP [21] so that S_z takes on positive and negative values as shown in Fig. 2(a) [17]. Thus, in general, $S_z(\mathbf{r})$ cannot be substituted by $2cW_{\rm inc}^{\rm el}(\mathbf{r})$, which is a positive quantity. We remark in passing that Ref. [10] predicts a much lower value than 1.7 for a quantity equivalent to our parameter $\mathcal{K}_0$. We believe one of the origins of this discrepancy is that Ref. [10] takes the integrand in the definition of $\mathcal{A}$ to be $2cW_{\rm inc}^{\rm el}(\mathbf{r})$.

In order to derive an upper limit of $\mathcal{K}_0$ for the general class of transverse axially symmetric systems, we consider the field produced by the combination of an electric and a magnetic dipole which has been suggested for optimal focusing [18,22]. To emulate such a field, one considers the emission field patterns at the GRS of virtual electric and magnetic dipoles orthogonal to each other and placed at O and then reverses the field propagation. Using Eq. (3) for the calculation of $\mathcal{A}$, we obtain [16]

$$\mathcal{K}_0 = \tfrac{1}{4}(7 - 3\cos\alpha - 3\cos^2\alpha - \cos^3\alpha). \quad (6)$$

At $\alpha = \pi/2$, $\mathcal{K}_0 = 7/4$ establishes the maximum value for transverse axially symmetric systems. This is only slightly larger than 128/75 obtained for the plane wave.

We next abandon the restriction of axial symmetry and search for an upper limit of $\mathcal{K}_0$ in general. Guided by a mode matching argument [6,10], we consider a directional dipolar incident wave. In this case the incident field stems from the emission pattern at the GRS of a virtual dipole parallel to the x axis and placed at the origin [23]. Following Eq. (3), we obtain [16]

$$\mathcal{K}_0 = \tfrac{1}{2}(4 - 3\cos\alpha - \cos^3\alpha). \quad (7)$$

We remark that $\mathcal{A}$ deduced from Eqs. (3)–(7) is equivalent to the corresponding expression for the normalized energy

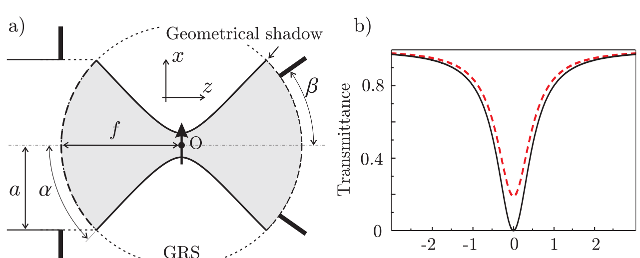

FIG. 1 (color online). (a) Incident light propagating along the z axis is focused with a spherical phase front onto a dipole placed in vacuum. GRS: Gaussian reference sphere; a: entrance-aperture radius; α: entrance half angle; β: collection half angle; f: focal length. (b) The dashed curve plots the transmittance for the focused plane wave and displays an attenuation of $\mathcal{T} \simeq 80\%$ at resonance for $\alpha = \beta = \pi/3$. The solid curve shows that a directional dipolar wave can be completely attenuated for $\alpha = \beta = \pi/2$.

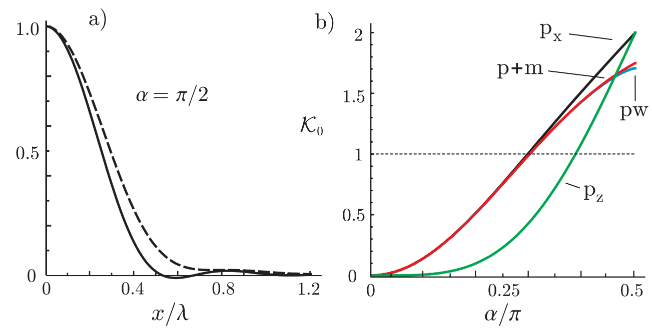

FIG. 2 (color online). (a) The z component of the Poynting vector S_z (solid curve) and the electric energy density $W_{\rm inc}^{\rm el}$ (dashed curve), both along the positive x axis in the focal plane and normalized to their respective values at $x = 0$. (b) Scattering ratio $\mathcal{K}_0$ as a function of α for several focused waves. p_z denotes the dipole wave with the electric dipole along the z axis, pw signifies the focused plane wave, $p + m$ shows the combined electric and magnetic dipole fields, and p_x notes the wave of an electric dipole oriented along the x axis. $\mathcal{K}_0 = 2$ is reached at $\alpha = \pi/2$ for p_x and p_z. The horizontal dashed line separates the regimes $\mathcal{K}_0 \gtrless 1$.

density in Refs. [19,20]. At $\alpha = \pi/2$, $\mathcal{A}$ reaches its minimum value of $\mathcal{A} = \sigma_0/2$ and $\mathcal{K}_0$ its ultimate maximum value of 2, respectively. This value is consistent with the limit $W_{\rm inc}(O)/P_{\rm inc} \leq k^2/(3\pi c)$ given by Bassett for the sum $W_{\rm inc}$ of the time-averaged electric and magnetic energy densities at the focal spot [24]. As a last case study, we consider the interaction of an oscillating dipole oriented along the z axis with a radially polarized dipolar incident field obtained from the radiation of a virtual dipole oriented along the z axis and located at O [18]. Here, too, we find that $\mathcal{K}_0$ reaches the maximum value of 2 at $\alpha = \pi/2$ [16].

Figure 2(b) displays $\mathcal{K}_0$ as a function of α for various illuminations considered above. In all cases, $\mathcal{K}_0 \gtrsim 1$ is met for realistic numerical apertures. We are, thus, facing the paradoxical seeming situation that the power emitted by the oscillator may be larger than the incident power. However, this finding does not violate the law of power conservation because there is destructive interference in the forward direction. We analyze this interference by determining now the incident and scattered fields at the GRS for $z > 0$. A particularly insightful approach is to expand an arbitrary excitation field in terms of vectorial multipoles [25–27]. All multipoles become zero at the origin except the electric dipole mode, which for a transverse system reads [25]

$$\mathbf{N}_{e11} = \begin{cases} \frac{2}{3}\hat{\mathbf{e}}_x, & \mathbf{r} = O, \\ (\cos\vartheta\cos\varphi\hat{\mathbf{e}}_\vartheta - \sin\varphi\hat{\mathbf{e}}_\varphi)\frac{e^{i(kr-\pi/2)}}{kr}, & kr \gg 1. \end{cases} \tag{8}$$

We note that here the field for $kr \gg 1$ is given only for the outgoing wave. The electric dipole-wave component $\mathbf{\Psi}$ of the excitation field can be written as

$$\mathbf{\Psi}(\mathbf{r}) = \frac{E_{\rm inc}(O)}{|\mathbf{N}_{e11}(O)|}\mathbf{N}_{e11}(\mathbf{r}), \tag{9}$$

where $E_{\rm inc}(O)$ is taken from the Debye diffraction approach [16]. The field scattered by the oscillator also forms a dipole wave [15]

$$\mathbf{E}_{\rm sca}(\mathbf{r}) = -\frac{3E_{\rm inc}(O)\Gamma}{2(2\Delta + i\Gamma)}\frac{e^{ikr}}{kr}[\hat{\mathbf{e}}_x - (\hat{\mathbf{e}}_x\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}], \tag{10}$$

where $\hat{\mathbf{r}}$ is the unit vector along $\mathbf{r}$ and the polarization of $\mathbf{E}_{\rm inc}(O)$ is along the x axis. At resonance, one finds $\mathbf{E}_{\rm sca} = -\mathbf{\Psi}$ for $kr \gg 1$, $z > 0$. Therefore, the dipole-wave component of the excitation field is completely reflected just as in the reflection of a collimated beam from a perfect metal. The π phase shift of $\mathbf{E}_{\rm sca}$ with respect to $\mathbf{E}_{\rm inc}$ results from the sum of two effects. First, the comparison of Eqs. (8) and (10) reveals a relative Gouy phase shift of $-\pi/2$ [28]. Second, the denominator of the Lorentzian term in Eq. (10) yields a phase shift of $\pi/2$ on resonance.

This approach allows for an easy calculation of the transmittance $\mathcal{T}$ as a function of the angles α and β. For a focused incident plane wave, we find [16]

$$\begin{aligned} \mathcal{T}(\alpha,\beta) &= 1 + 3I_0(\alpha)\frac{X(\beta)I_0(\alpha) - I_0(\min\{\alpha,\beta\})}{2\sin^2(\min\{\alpha,\beta\})}, \\ X(\beta) &= \frac{1}{8}(4 - 3\cos\beta - \cos^3\beta), \\ I_0(\xi) &= \frac{16}{15}\left[1 - \frac{1}{8}(5 + 3\cos\xi)\cos^{3/2}\xi\right]. \end{aligned} \tag{11}$$

The numerical data in Fig. 3 display a rapid decrease of $\mathcal{T}$ with increasing α, while the dependence on β is less pronounced. Of particular experimental relevance is the geometrical shadow boundary where $\beta = \alpha$; i.e., all of the incident light is collected. Along this line, the transmittance experiences a minimum of $\mathcal{T} \simeq 0.1$ at $\alpha \simeq 0.43\pi$. $\mathcal{T}$ as a function of the detuning is presented in Fig. 1(b) for $\alpha = \beta = \pi/3$. We point out that more complicated expressions are expected if the dipole is displaced from the focal spot. Particularly, the phase fronts of the scattered and excitation fields no longer match at the GRS.

In this work, we have shown that a classical pointlike oscillating dipole can undergo strong coupling with a confined incident beam, reaching 100% efficiency when the illumination consists of a directional dipolar field. In fact, in the limit of weak excitation, many essential features of light-matter interaction are shared by the quantum electrodynamic and classical formalisms alike [29]. A central underlying reason for this phenomenon is that both treatments use the same spatial description of the electromagnetic field. To this end, our classical results can be readily extended to the interaction of light with a TLS. The scattering cross section of a quantum mechanical TLS is known to be [30]

$$\sigma_{\rm TLS} = \sigma_0\frac{\Gamma^2}{4\Delta^2 + \Gamma^2 + 2\mathcal{V}^2}, \tag{12}$$

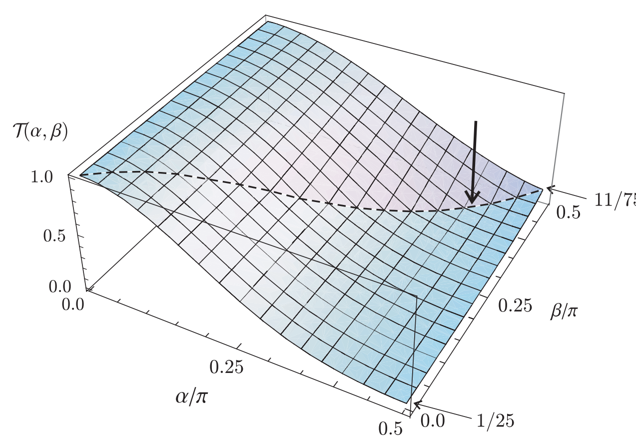

FIG. 3 (color online). Transmittance $\mathcal{T}$ of a focused plane wave as a function of the angles α and β as defined in Fig. 1(a). The dashed curve indicates the edge at the geometrical shadow boundary $\alpha = \beta$, and the vertical arrow indicates the location $\alpha = \beta \simeq 0.43\pi$ of the minimum value approximately equal to 10%. $\mathcal{T}$ values for the cases $\alpha = \pi/2$, $\beta \to 0$, and $\alpha = \beta = \pi/2$ are also noted.

where σ_0 is the same quantity as in Eq. (2), Γ stands for the spontaneous emission rate, $\mathcal{V} = -\mathbf{d}_{12} \cdot \mathbf{E}_{\rm inc}(O)/\hbar$ is the Rabi frequency, and $\mathbf{d}_{12}$ denotes the vectorial transition dipole moment. In a semiclassical treatment, the coherently scattered field by the TLS is [30]

$$\mathbf{E}_{\rm sca}^{\rm coh} = \frac{-3\Gamma(\Delta - i\Gamma/2)E_{\rm inc}(O)}{4\Delta^2 + \Gamma^2 + 2\mathcal{V}^2} \frac{e^{ikr}}{kr}[\hat{\mathbf{e}}_x - (\hat{\mathbf{e}}_x \cdot \hat{\mathbf{r}})\hat{\mathbf{r}}]. \tag{13}$$

At weak excitation $|\mathcal{V}| \ll \Gamma$, Eqs. (12) and (13) become equivalent to Eqs. (2) and (10). Therefore, the results obtained for the classical oscillator also hold for a TLS. We thus conclude that a directional dipole wave can be perfectly reflected from a TLS under weak excitation. However, in the saturation regime $|\mathcal{V}| \gtrsim \Gamma$, $\sigma_{\rm TLS}$ decreases with increasing excitation.

Considering a quantized field, we are led to conclude that a few or even single photon pulses can be fully reflected by a single TLS if the coherence time of the photon is sufficiently long compared to the excited state lifetime [12]. The modal formalism developed in this Letter can be extended in the context of QED to analyze such phenomena and will be the subject of a future study. Furthermore, it would be interesting to investigate the photon autocorrelation function since photon bunching or antibunching is generally expected when there is destructive or constructive interference, respectively [9,10].

In conclusion, we have shown that a single pointlike oscillating dipole can fully reflect an incident light field. For the experimentally important case of a focused plane wave, we have found that the transmission can be attenuated by up to 85%. Our results readily hold for the whole electromagnetic spectrum, and we expect interesting applications in the detection and spectroscopy of subwavelength objects in the infrared to radio-wave domains. In the optical range, we anticipate that a strong coupling between a single photon and a single quantum system can be realized in a directional focal system without the need for high finesse cavities. Such an arrangement would open new doors for quantum information processing using photons as information carriers.

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