

A Spin-1 Top Quark Superpartner

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We construct a supersymmetric model where the left-handed top and bottom quarks are mainly the gauginos of a vector supermultiplet and hence their superpartners are spin 1. The right-handed top quark is unified with the Higgs field; the top Yukawa coupling arises from the gaugino coupling.

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In the standard model (SM), the Higgs field mass squared receives quadratically divergent corrections from interactions with other SM fields. These contributions should be cut off at scales not much higher than the electroweak symmetry breaking scale so that the electroweak scale is stable. The largest contribution comes from a top quark loop through the top Yukawa coupling. Requiring that there is no more than 10% fine-tuning in the Higgs mass parameter, the top loop should be cut off below ~ 2 TeV. This implies new physics at or below the TeV scale, which will be explored at the Large Hadron Collider (LHC).

The most popular and promising candidate for new physics at the TeV scale is supersymmetry (SUSY). In the minimal supersymmetric standard model (MSSM), there is a superpartner for each SM particle with the opposite spin-statistics, and the quadratic radiative corrections are cancelled between bosons and fermions. If SUSY is responsible for stabilizing the electroweak scale, the SUSY-breaking masses of the superpartners are expected to be around the TeV scale. In particular, the superpartners of the top quark, which are scalar particles in the MSSM, are required to have TeV or smaller masses in order to avoid fine-tuning. The top's superpartner, the top squark, is necessarily colored and can be copiously produced at the LHC.

In recent years, several other possibilities of cancelling the quadratic divergences to the Higgs mass squared were discovered. In little Higgs theories [1], the one-loop quadratic divergences from the SM particles are cancelled by new particles at the TeV scale with the *same* spins as the corresponding SM particles. The top partner which cuts off the top loop is a spin-1/2 fermion in this case. Again it also carries color so it will be copiously produced at the LHC if it exists. It is also possible to cancel the top loop by new particles which are not charged under color, but another $SU(3)$ gauge group, as in the twin Higgs model [2] and folded supersymmetry [3]. There can be very exotic phenomenology associated with these models.

In this Letter we will explore the possibility that the top partner responsible for cancelling the top loop is a spin-1 particle. Such a possibility should not be unexpected, as SUSY relates particles with spins that differ by 1/2. Usually in supersymmetric models both the left- and right-handed top quarks belong to chiral supermultiplets

and their superpartners are spin-0 particles. One can consider the alternative possibility of assigning the top to a vector supermultiplet so that its superpartner is a spin-1 vector boson. In this Letter we construct a relatively simple model where the superpartner of the left-handed top quark is a spin-1 vector particle. (The superpartner of the right-handed top quark is still a scalar.) There are many interesting aspects of this model and we will briefly discuss its phenomenology.

One immediate challenge to assign the top to a vector supermultiplet is that the vector supermultiplet transforms as an adjoint representation under some gauge group while the top quarks are fundamentals under the SM gauge group. However, if we consider an enlarged gauge group such as $SU(5)$, it contains gauge bosons transforming as $(\mathbf{3}, \mathbf{2})$ under the $SU(3) \times SU(2)$ subgroup, and we can try to identify them as the superpartner of the left-handed top quark. By identifying the left-handed top quark as a gaugino, the top Yukawa coupling should come from a gaugino interaction, which implies that the right-handed top and the Higgs field should be unified into a chiral supermultiplet transforming under the $SU(5)$ gauge group.

Our model is based on the gauge group $SU(3) \times SU(2) \times U(1)_H \times U(1)_V \times SU(5)$. It is broken at the TeV scale down to the diagonal SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ by the nonzero vacuum expectation values (VEVs) of the fields $\Phi_3, \bar{\Phi}_3, \Phi_2, \bar{\Phi}_2$. We include three generations of matter fields (quarks and leptons) which transform under $SU(3) \times SU(2) \times U(1)_H$. The two Higgs doublets belong to $\mathbf{5}$ and $\bar{\mathbf{5}}$ of the $SU(5)$ gauge group. They also carry $U(1)$ charges in order to arrange the correct hypercharges after symmetry breaking. The field content is given in Table I.

The triplet partners of the Higgs doublets in \bar{H} and H have the same quantum numbers as the right-handed top and its conjugate. We can write them as

$$\bar{H} = (\bar{T}, H_1), \quad H = (\bar{T}^c, H_2). \quad (1)$$

The superpotential is given by

$$\begin{aligned} W = & y_1 Q_3 \Phi_3 \bar{\Phi}_2 + \mu_3 \Phi_3 \bar{\Phi}_3 + \mu_2 \Phi_2 \bar{\Phi}_2 + y_2 \bar{u}_3 H \bar{\Phi}_3 \\ & + \mu_H H \bar{H} + Y_{Uij} Q_i \bar{u}_j \bar{\Phi}_2 H + Y_{Dij} Q_i \bar{d}_j \Phi_2 \bar{H} \\ & + Y_{Eij} L_i \bar{e}_j \Phi_2 \bar{H}. \end{aligned} \quad (2)$$

TABLE I. The field content of the model. The last column is the hypercharge of the diagonal hypercharge $U(1)_Y$, where $Y = H + V + aT_{24}$, with $a = 1/\sqrt{15}$.

	$SU(3)$	$SU(2)$	$U(1)_H$	$U(1)_V$	$SU(5)$	$H + V + aT_{24}$
Q_i	\square	\square	$\frac{1}{6}$	0	$\mathbf{1}$	$\frac{1}{6}$
\bar{u}_i	$\bar{\square}$	$\mathbf{1}$	$-\frac{2}{3}$	0	$\mathbf{1}$	$-\frac{2}{3}$
\bar{d}_i	$\bar{\square}$	$\mathbf{1}$	$\frac{1}{3}$	0	$\mathbf{1}$	$\frac{1}{3}$
L_i	$\mathbf{1}$	\square	$-\frac{1}{2}$	0	$\mathbf{1}$	$-\frac{1}{2}$
\bar{e}_i	$\mathbf{1}$	$\mathbf{1}$	1	0	$\mathbf{1}$	1
H	$\mathbf{1}$	$\mathbf{1}$	$\frac{1}{2}$	$\frac{1}{10}$	\square	$(\frac{2}{3}, \frac{1}{2})$
\bar{H}	$\mathbf{1}$	$\mathbf{1}$	$-\frac{1}{2}$	$-\frac{1}{10}$	$\bar{\square}$	$(-\frac{2}{3}, -\frac{1}{2})$
Φ_3	$\bar{\square}$	$\mathbf{1}$	$-\frac{1}{6}$	$\frac{1}{10}$	\square	$(0, -\frac{1}{6})$
Φ_2	$\mathbf{1}$	$\bar{\square}$	0	$\frac{1}{10}$	\square	$(\frac{1}{6}, 0)$
$\bar{\Phi}_3$	\square	$\mathbf{1}$	$\frac{1}{6}$	$-\frac{1}{10}$	$\bar{\square}$	$(0, \frac{1}{6})$
$\bar{\Phi}_2$	$\mathbf{1}$	\square	0	$-\frac{1}{10}$	$\bar{\square}$	$(-\frac{1}{6}, 0)$

There are also the usual soft SUSY-breaking terms, including the gaugino masses, scalar masses, A terms and B terms. We do not specify the origin of these soft SUSY-breaking terms as it is not essential for the discussion here. We assume that due to SUSY breaking the potential for Φ_j , $\bar{\Phi}_j$ is unstable at the origin so that they get the following nonzero VEVs, breaking $SU(3) \times SU(2) \times U(1)_H$ and $SU(5) \times U(1)_V$ down to the diagonal SM gauge group,

$$\begin{aligned} \langle \Phi_3 \rangle &= \begin{pmatrix} f_3 & 0 & 0 & 0 & 0 \\ 0 & f_3 & 0 & 0 & 0 \\ 0 & 0 & f_3 & 0 & 0 \end{pmatrix}, & \langle \bar{\Phi}_3 \rangle^T &= \begin{pmatrix} \bar{f}_3 & 0 & 0 & 0 & 0 \\ 0 & \bar{f}_3 & 0 & 0 & 0 \\ 0 & 0 & \bar{f}_3 & 0 & 0 \end{pmatrix}, \\ \langle \Phi_2 \rangle &= \begin{pmatrix} 0 & 0 & 0 & f_2 & 0 \\ 0 & 0 & 0 & 0 & f_2 \end{pmatrix}, & \langle \bar{\Phi}_2 \rangle^T &= \begin{pmatrix} 0 & 0 & 0 & \bar{f}_2 & 0 \\ 0 & 0 & 0 & 0 & \bar{f}_2 \end{pmatrix}. \end{aligned} \quad (3)$$

The alignment of the VEVs of Φ_3 and $\bar{\Phi}_2$ is ensured by the $y_1 Q_3 \Phi_3 \bar{\Phi}_2$ term in the superpotential. The VEVs f_3 , \bar{f}_3 , f_2 , and \bar{f}_2 are not equal in general as they depend on the soft SUSY-breaking terms and μ_2, μ_3 . The unequal VEVs can generate a $U(1)_H$ D term, $(\hat{g}_{1H}/2)(\bar{f}_3^2 - f_3^2)$, and a $U(1)_V$ D term, $(\hat{g}_{1V}/10)[3(f_3^2 - \bar{f}_3^2) + 2(f_2^2 - \bar{f}_2^2)]$ which in turn gives additional contributions to the scalar masses of fields charged under the $U(1)$'s. We assume that these are small enough not to affect the spectrum dramatically.

With the above VEVs, the Φ fields split into the following representations under $SU(3)_C \times SU(2)_L \times U(1)_Y$:

$$\begin{aligned} \Phi_3 &\rightarrow (1, 1, 0) + (\mathbf{8}, 1, 0) + (\bar{\mathbf{3}}, \mathbf{2}, -1/6) \\ \bar{\Phi}_3 &\rightarrow (1, 1, 0) + (\mathbf{8}, 1, 0) + (\mathbf{3}, \mathbf{2}, 1/6) \\ \Phi_2 &\rightarrow (\mathbf{3}, \mathbf{2}, 1/6) + (1, 1, 0) + (1, \mathbf{3}, 0) \\ \bar{\Phi}_2 &\rightarrow (\bar{\mathbf{3}}, \mathbf{2}, -1/6) + (1, 1, 0) + (1, \mathbf{3}, 0). \end{aligned} \quad (4)$$

The gauge couplings for the SM $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group are given by

$$\frac{1}{g_{2,3}^2} = \frac{1}{\hat{g}_{2,3}^2} + \frac{1}{\hat{g}_5^2}, \quad \frac{1}{g_1^2} = \frac{1}{\hat{g}_{1H}^2} + \frac{1}{\hat{g}_{1V}^2} + \frac{1}{15\hat{g}_5^2}, \quad (5)$$

where \hat{g}_i and \hat{g}_5 are the gauge couplings of the original $SU(3)$, $SU(2)$, $U(1)_H$, $U(1)_V$ and $SU(5)$ gauge groups, respectively. There are two broken $U(1)$ gauge bosons whose masses are fixed by

$$\begin{aligned} \mathcal{L} \supset \frac{1}{2} \left\{ 6(f_3^2 + \bar{f}_3^2) \left(\frac{\hat{g}_{1H}}{6} B_{1H} - \frac{\hat{g}_{1V}}{10} B_{1V} - \frac{\hat{g}_5}{\sqrt{15}} B_{24} \right)^2 \right. \\ \left. + 4(f_2^2 + \bar{f}_2^2) \left(\frac{\hat{g}_{1V}}{10} B_{1V} - \frac{\sqrt{15}}{10} \hat{g}_5 B_{24} \right)^2 \right\}. \end{aligned} \quad (6)$$

The masses of the other heavy gauge bosons after symmetry breaking are given by,

$$m_{G'}^2 = (\hat{g}_3^2 + \hat{g}_5^2)(f_3^2 + \bar{f}_3^2), \quad (7)$$

$$m_{W'}^2 = (\hat{g}_2^2 + \hat{g}_5^2)(f_2^2 + \bar{f}_2^2), \quad (8)$$

$$m_{\tilde{Q}}^2 = \frac{1}{2} \hat{g}_5^2 (f_3^2 + \bar{f}_3^2 + f_2^2 + \bar{f}_2^2). \quad (9)$$

The \tilde{Q} gauge boson will be the dominant superpartner of the left-handed top and bottom quark doublet Q as we will see later. We denote it with an arrow above the particle name to emphasize that it is a spin-1 superpartner as opposed to the usual tilde used for superpartners. In $SU(5)$ grand unified models these gauge bosons are often denoted by X and Y . For other particles beyond the MSSM, we use the notation that the particles with odd R parity in a supermultiplet have a tilde above the particle name, and the particles with even R parity do not.

The Yukawa couplings for the light SM fermions arise from the last three terms of the superpotential (2). After substituting in the VEVs of the Φ_2 and $\bar{\Phi}_2$ fields, they become the usual Yukawa terms. The fact that they come from nonrenormalizable interactions can explain why they are small. Note that the scalar components of the Φ_2 and $\bar{\Phi}_2$ fields that get VEVs must have the same R -parity assignments as the Higgs fields, i.e. $+$. We will come back to the issue of the R -parity assignments later.

For the top quark, Q_3 and \bar{u}_3 mix with other states of the same quantum numbers under the SM gauge symmetry through the y_1, y_2 , and μ terms. The mass matrix for the fermions in the $(\mathbf{3}, \mathbf{2}, 1/6)$ sector is

$$\begin{array}{ccccc} & \lambda & \Phi_{2t} & \bar{\Phi}_{3t} & Q_3 \\ \bar{\lambda} & M_5 & \hat{g}_5 f_2 & \hat{g}_5 \bar{f}_3 & 0 \\ \Phi_{3t} & \hat{g}_5 f_3 & 0 & \mu_3 & y_1 \bar{f}_2 \\ \bar{\Phi}_{2t} & \hat{g}_5 \bar{f}_2 & \mu_2 & 0 & y_1 f_3 \end{array}, \quad (10)$$

where λ , and $\bar{\lambda}$ are the gauginos corresponding to the broken generators in $SU(5)$ which are in the vector supermultiplets with \tilde{Q} and \tilde{Q}^* ; M_5 is the SUSY-breaking gaugino mass for all $SU(5)$ gauginos; Φ_{2t} , $\bar{\Phi}_{3t}$ are the color triplet fermions from Φ_2 , $\bar{\Phi}_3$, and Φ_{3t} , $\bar{\Phi}_{2t}$ are the color

antitriplet fermions. The massless left-handed top quark is a linear combination of λ , Φ_{2t} , $\bar{\Phi}_{3t}$, and Q_3 . For $M_5 \ll \hat{g}_5 f_2$, $\hat{g}_5 f_3 \ll \mu_3(\hat{g}_5 \bar{f}_2)$, and $\hat{g}_5 \bar{f}_2 \ll \mu_2$, the massless left-handed top quark state is mostly made of the gaugino λ . For example, if we take $\bar{f}_2 = 1.5$ TeV, $f_2 = 1.7$ TeV, $\bar{f}_3 = 0.6$ TeV, $f_3 = 0.4$ TeV, $M_5 = 0.7$ TeV, $\mu_2 = 5$ TeV, $\mu_3 = 2$ TeV, $\hat{g}_5 = 1.2$, and $y_1 = 1.5$, then the massless combination is

$$Q \equiv (t, b)_L \approx 0.93\lambda - 0.31\Phi_{2t} - 0.02\bar{\Phi}_{3t} - 0.18Q_3. \quad (11)$$

For the right-handed top quark, \bar{u}_3 mixes with the colored fermion component of \bar{H} through the y_2 term in the superpotential Eq. (2). The mass matrix is:

$$\begin{array}{cc} \bar{T} & \bar{u}_3 \\ \bar{T}^c & \mu_H \quad y_2 \bar{f}_3. \end{array}$$

One can see that for $y_2 \bar{f}_3 \gg \mu_H$, the massless combination is mostly \bar{T} . For example, if we take $\mu_H = 0.3$ TeV, $y_2 = 1.5$, and $\bar{f}_3 = 0.6$ TeV, then

$$\bar{t}_R = 0.95\bar{T} - 0.32\bar{u}_3. \quad (12)$$

As the left-handed and right-handed top quarks lie mostly in λ and \bar{T} respectively, the top Yukawa coupling predominantly comes from the gaugino interaction,

$$\hat{g}_5 H_1^\dagger \lambda \bar{T}. \quad (13)$$

This explains why the top Yukawa coupling is $\mathcal{O}(1)$. Note that the top quark gets its mass mostly from H_1 , which is the same Higgs field that gives masses to the *down* type quarks and the charged leptons, while the up and charm quarks get masses from H_2 . This is because unlike the holomorphic superpotential, the gaugino interaction involves the conjugate of the Higgs field. The top quark also receives a small contribution to its mass from the y_2 term and nonrenormalizable terms, which can account for the small mixings with the light quarks.

It is also interesting to note that if Φ_j , $\bar{\Phi}_j$ fields have additional interactions, for instance, in the case that μ_2 , μ_3 may come from the VEV of some other field, the Higgs quartic coupling can receive a large tree-level correction, proportional to \hat{g}_5^2 , compared with the MSSM value after integrating out the heavy states.

Since we unify the right-handed top quark with the Higgs field in the same $SU(5)$ multiplet, one may ask whether R parity is conserved in this model. We have already seen that the scalar pieces of Φ_2 and $\bar{\Phi}_2$ that have VEVs must have even R parity. We have implicitly assumed above that the color triplet fermions in Φ_j , $\bar{\Phi}_j$, and \bar{H} have even R parity and therefore that the scalar partners have odd R parity. It is easy to check that a modified R parity with a twist $P = (-1, -1, -1, 1, 1)$ in the $SU(5)$ sector will satisfy the above constraints. This twisted R parity plays the same role as the usual R parity in

MSSM and it is conserved in all the interactions. More explicitly, the bosonic and fermionic components of \bar{H} are

$$\bar{H} \supset (\bar{T}, H_1) + \theta(\bar{T}, \bar{H}_1). \quad (14)$$

Similarly, if we use a subscript s and a to denote the $SU(3)_C$ singlet and adjoint pieces of Φ_3 , and s and a for the $SU(2)_L$ singlet and adjoint pieces of Φ_2 , we have

$$\Phi_j \supset (\Phi_{js}, \Phi_{ja}, \bar{\Phi}_{jt}) + \theta(\bar{\Phi}_{js}, \bar{\Phi}_{ja}, \Phi_{jt}). \quad (15)$$

Similarly, there is a new baryon number symmetry which is a linear combination of the original baryon number (with Φ_3 , $\bar{\Phi}_3$ carrying original baryon number $-1/3$, $+1/3$ respectively) and a gauge transformation $2(V + aT_{24})$. It is left unbroken by the VEVs, so there is no dangerous proton decay or neutron-antineutron oscillation induced.

The couplings of W' , B' , and B'' to the light SM fermions are suppressed by \hat{g}_2/\hat{g}_5 , $\hat{g}_{1H}/\sqrt{15}\hat{g}_5$ and $\hat{g}_{1H}/\hat{g}_{1V}$ relative to the SM W and B gauge bosons, respectively. The experimental constraints on Z' put a lower bound on $m_{W'}$ at about 800 GeV. The more stringent constraints come from the mixing of W , B and W' , B' , B'' due to the Higgs VEVs because the two Higgs doublets and the light fermions transform under different gauge groups. In terms of the usual S , T parameters [4], the strongest constraint comes from T for $\hat{g}_{1V} \geq 2-3$ and it depends only on $f_2^2 + \bar{f}_2^2$. For the light Higgs field (~ 115 GeV) $f_2^2 + \bar{f}_2^2$ is required to be $\geq (3 \text{ TeV})^2$. For the heavier Higgs field the bound is weaker and $f_2^2 + \bar{f}_2^2$ might be as low as $\sim (2 \text{ TeV})^2$. Another constraint comes from the corrections to the $Z \rightarrow b\bar{b}$ vertex. The b_L quark in this model is a linear combination of several fields and is mostly the gaugino of $SU(5)$. It couples to the heavy gauge bosons differently from d_L , s_L . It induces a correction to $Z b_L \bar{b}_L$ coupling after Higgs VEVs mix Z with W'_3 , B' , and B'' . Requiring that $\delta g_{Lb}/g_{Lb} < 0.6\%$ puts a lower bound of ~ 1.6 TeV for $m_{W'}$. Note, however, that these are indirect constraints. It is possible that these contributions are cancelled by some additional contributions. It is still quite possible that \bar{Q} has a mass below 2 TeV.

The model predicts the existence of new gauge bosons G' , W' , B' , B'' and several new fermions t' , b' around the same scale as the spin-1 top superpartner \bar{Q} . The masses of the gauge bosons are closely related through Eqs. (7) and (9). For heavy t' and b' quarks, there are three doublets and one singlet. Their masses depend on the model parameters. For the set of the parameters considered earlier above Eq. (11) and (12) and $\hat{g}_3 = 2.0$, $\hat{g}_2 = 0.75$, $\hat{g}_{1H} = 0.36$, $\hat{g}_{1V} = 3.5$ at ~ 2 TeV (to produce the measured values of SM gauge couplings), the sample spectrum for the heavy gauge bosons and fermions is listed below.

	G'	W'	B'	B''	\bar{Q}	Q'	Q''	Q'''	\bar{T}'
M/TeV	1.7	3.2	0.83	2.6	2.0	0.65	3.0	5.8	0.95

(16)

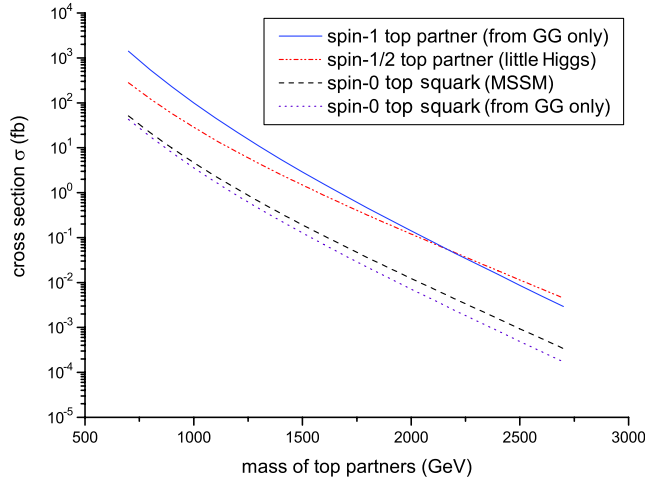


FIG. 1 (color online). Total cross sections vs mass of top partners.

The spectrum of superpartners depends on the soft SUSY-breaking terms. The superpartners of the light fermions can have masses in the multi-TeV range without affecting the naturalness of electroweak symmetry breaking because of the small Yukawa couplings. The SUSY-breaking masses of $\Phi_{2,3}$, $\tilde{\Phi}_{2,3}$ should also be in the TeV range in order to induce symmetry breaking VEVs of the TeV scale. We assume that all soft SUSY-breaking scalar masses except those of H and \tilde{H} are large (\gtrsim a few TeV). In this case, most of the scalar superpartners are beyond the reach of the LHC. On the other hand, the soft masses of H and \tilde{H} are relevant for stabilizing the electroweak scale as they contain the Higgs fields and the right-handed top quark. They need to be at ~ 1 TeV or below. Electroweak gauginos also give significant contributions to the Higgs field masses. We assume that the SUSY-breaking gaugino masses are of the order a few hundred GeV, smaller than (most of) the soft scalar masses. This can be naturally achieved if there is an approximate R symmetry in the SUSY-breaking sector.

With the spectrum assumed above, the superpartners of the SM particles that can be seen at the LHC are the spin-1 partner of the left-handed top-bottom doublet, the scalar partner of the right-handed top, gauginos of the SM gauge group, and Higgsinos. In addition, we expect to see new gauge bosons, t' , b' , and some of their superpartners. We will focus on the spin-1 partner of the top and bottom doublet. At the LHC, the main production mechanism is $GG \rightarrow \tilde{Q}\tilde{Q}^*$. The processes with $q\bar{q}$ initial states are suppressed by destructive interference between G and G' exchanges because q , \bar{q} and \tilde{Q} transform under different UV gauge groups. The dominant production cross sections of the spin-1 and spin-0 top partners of this model from gluons are shown in Fig. 1 as functions of their masses. In comparison, the cross sections of the (right-handed) top

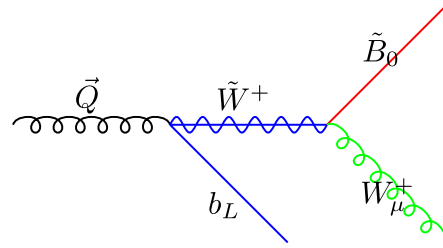


FIG. 2 (color online). A decay chain for the spin-1 top partner.

squark in the MSSM and fermionic top partner in little Higgs models are also shown.

The total cross sections of the top partners have been used as a way to distinguish little Higgs models from SUSY models as the total cross section of the fermionic top partner is much larger than that of a scalar partner for the same mass [5]. However, we see that the spin-1 top partner can have an even larger total cross section for the same mass.

With a similar spectrum the \tilde{Q} decay is similar to the usual top squark decay. If the gauginos are light, it will decay to a gaugino and a top or bottom quark, and eventually end up with a lightest R -parity odd particle through cascade decays. One such possible decay channel is shown in Fig. 2. It may have a different angular distribution for its decay products compared with the usual top squark, but it would be very challenging to tell at the LHC.

In conclusion, we have shown that a spin-1 top partner requires an extended gauge symmetry which can explain why one quark is much heavier than the others, and provide a larger Higgs quartic coupling than in the MSSM. The spin-1 top partner thus provides an interesting (and experimentally challenging) new scenario for LHC physics.

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