Disorder-Induced Enhancement of Transport through Graphene *p-n* Junctions

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We investigate the electron transport through a graphene *p-n* junction under a perpendicular magnetic field. By using the Landauer-Büttiker formalism combined with the nonequilibrium Green function method, the conductance is studied for clean and disordered samples. For the clean *p-n* junction, the conductance is quite small. In the presence of disorders, it is strongly enhanced and exhibits a plateau structure at a suitable range of disorders. Our numerical results show that the lowest plateau can survive for a very broad range of disorder strength, but the existence of high plateaus depends on system parameters and sometimes cannot be formed at all. When the disorder is slightly outside of this disorder range, some conductance plateaus can still emerge with its value lower than the ideal value. These results are in excellent agreement with a recent experiment.

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Because of the recent success in the fabrication of graphene, a single-layer hexagonal lattice of carbon atoms, a great deal of attention has been focused on the research of graphene [1–3]. The unique band structure of graphene with a linear dispersion relation ($E = \pm\hbar v|\mathbf{k}|$) near the Dirac-points leads to many peculiar properties [4]. For instance, the quasiparticles obey the Dirac-like equation and have relativisticlike behavior with a zero rest mass. Its Hall plateaus assume the half-integer values $g(n + 1/2)e^2/h$ with the degeneracy $g = 4$ [1,2]. By varying the gate voltage, the charge carrier of graphene can be tuned from electronlike to holelike and vice versa. To examine the interplay between the electronlike and holelike quasiparticles, a graphene *p-n* junction would be a good candidate. Many exciting phenomena reflecting the massless Dirac character of carriers [5–7], such as relativistic Klein tunneling [5] and Veselago lensing [6], were predicted for the graphene *p-n* junction.

Very recently, the graphene junction has been realized experimentally [3]. As expected, it was found that in quantum Hall regime the two-terminal conductance exhibits quantized plateaus with half-integer values $4(n + 1/2)e^2/h$ for the *p-p* or *n-n* junctions. For the disordered *p-n* junction, new plateaus emerge at e^2/h and $(3/2)e^2/h$. At about the same time, a theoretical analysis qualitatively explained the appearance of these plateaus that is due to the mixture of the electron and hole Hall edge modes in the *p-n* boundary [8]. After that, subsequent works have also investigated the graphene *p-n* junction [9]. However, these theories cannot account for the experimentally observed plateau that appeared at about $1.4e^2/h$ which is lower than the expected value $(3/2)e^2/h$. In addition, the reason that the expected plateaus at $3e^2/h$ or higher values have not been observed remained mysterious. In view of this situation, a thorough and reliable analysis for the graphene *p-n* junction is urgently needed.

In this Letter, we theoretically study the electron transport through the *p-n* junction of disordered graphene under a perpendicular magnetic field B. By using the tight-binding model and the Landauer-Büttiker formalism combining with the nonequilibrium Green function method, the conductance is calculated for both clean and disordered samples. Numerical results show that the conductance is very weak in the clean *p-n* region at large B. Depending on its strength, the disorder can have two effects. At small disorder, it can mix the electron and hole edge states which in turn enhances the conductance. At large disorder, it will drive the system into the insulating regime. So the conductance is strongly enhanced at the small disorder but suppressed at the large disorder. At suitable disorders, new plateaus [with values e^2/h, $(3/2)e^2/h$, etc.] emerge. The range of the disorder strength W needed for the existence of the lowest plateau e^2/h is very broad, so this plateau can easily be observed. But for the plateaus with higher quantization values, the range of W can be very narrow. Sometimes these higher order plateaus cannot be formed. When the disorder is slightly off this disorder range, the conductance plateaus can still emerge, but its value is lower than the expected one. These results are in excellent agreement with experimental data.

In the tight-binding representation, the Hamiltonian of the graphene *p-n* junction [see Fig. 1(a)] is given by [10]: $H = \sum_i \epsilon_i a_i^\dagger a_i - \sum_{\langle ij\rangle} t e^{i\phi_{ij}} a_i^\dagger a_j$, where $a_i^\dagger$ and a_i are the creation and annihilation operators at the discrete site i, and ϵ_i is the on-site energy. In the left and right leads, $\epsilon_i = E_L$ or E_R, which can be controlled by the gate voltages. The disorder exists only in the center region. The potential drop from the right to the left leads is assumed to be linear, i.e., $\epsilon_i = k(E_R - E_L)/(2M + 2) + E_L + w_i$, where M is the length of the center region and $k = 0, 1, 2, \ldots, 2M + 1$ [see Fig. 1(a)]. The on-site disorder energy w_i is uniformly distributed in the range $[-W/2, W/2]$ with the disorder

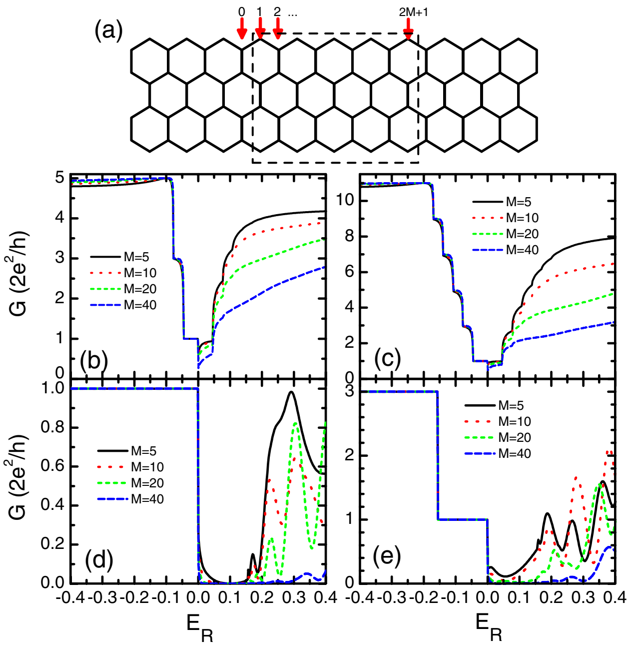

FIG. 1 (color online). (a) The schematic diagram for a zigzag edge graphene p-n junction. (b)–(e) the conductance G vs E_R for different center lengths M at $W = 0$. The parameters $E_L = -0.1$ for (b) and (d), $E_L = -0.2$ for (c) and (e), $\phi = 0$ for (b) and (c), and $\phi = 0.007$ for (d) and (e).

strength W. The size of the center region is described by the width N and length M, and it has $2N(2M + 1)$ carbon atoms. In Fig. 1(a), it shows a system with $N = 2$ and $M = 4$. The second term in Hamiltonian describes the nearest-neighbor hopping. Because of the existence of the perpendicular magnetic field B, a phase ϕ_{ij} is added in the hopping element, and $\phi_{ij} = \int_i^j \vec{A} \cdot d\vec{l}/\phi_0$ with the vector potential $\vec{A} = (-By, 0, 0)$ and $\phi_0 = \hbar/e$.

The current flowing through the graphene p-n junction is calculated from the Landauer-Büttiker formula [11]: $I = (2e/h) \int d\epsilon T_{LR}(\epsilon)[f_L(\epsilon) - f_R(\epsilon)]$, where $f_\alpha(\epsilon) = 1/\{\exp[(\epsilon - eV_\alpha)/k_B T] + 1\}$ ($\alpha = L, R$) is the Fermi distribution function in the left and right graphene leads. Here, $T_{LR}(\epsilon) = \mathrm{Tr}[\mathbf{\Gamma}_L \mathbf{G}^r \mathbf{\Gamma}_R \mathbf{G}^a]$ is the transmission coefficient with the linewidth functions $\mathbf{\Gamma}_\alpha(\epsilon) = i[\mathbf{\Sigma}_\alpha^r(\epsilon) - \mathbf{\Sigma}_\alpha^a(\epsilon)]$, the Green functions $\mathbf{G}^r(\epsilon) = [\mathbf{G}^a(\epsilon)]^\dagger = 1/[\epsilon - \mathbf{H}_{\mathrm{cen}} - \mathbf{\Sigma}_L^r - \mathbf{\Sigma}_R^r]$, and the Hamiltonian in the center region $\mathbf{H}_{\mathrm{cen}}$. The retarded self-energy $\mathbf{\Sigma}_\alpha^r(\epsilon)$ due to the coupling to the lead-α that can be calculated numerically [12]. After obtaining the current I, the linear conductance is given by $G = \lim_{V\to 0} dI/dV$.

In the following numerical calculations, we use the hopping energy $t \approx 2.75$ eV as the energy unit. Since the hopping energy t corresponds to 10^4 K, we can safely set the temperature to zero in our calculation. The width N is chosen as $N = 50$ in all calculations. Since the nearest-neighbor carbon-carbon distance is $a = 0.142$ nm, the width is $(3N - 1)a \approx 21.2$ nm for $N = 50$. The magnetic field is expressed in terms of ϕ with $\phi \equiv (3\sqrt{3}/4)a^2 B/\phi_0$ and $(3\sqrt{3}/2)a^2 B$ is the magnetic flux in the honeycomb lattice. In the presence of disorder, the conductance is averaged over up to 2000 random configurations except for Fig. 2(b) where only 400 random configurations were used for each data. In the experiment, the typical concentration of electrons or holes is around $10^{13}/\mathrm{cm}^2$ that corresponds to the on-site energies $E_L, E_R \leq 0.1t$. So we will mainly focused on the region of E_L and E_R within $0.3t$. In this range of energy, the dispersion relation is linear and exhibits Dirac behaviors.

We first study the clean graphene junction. Figures 1(b) and 1(c) show the conductance G versus the Fermi level of right lead E_R when magnetic field $B = 0$. In the n-n region with $E_L, E_R < 0$, G is approximatively quantized and exhibits a series of equidistant plateaus at the half-integers (in the unit of $4e^2/h$) due to the transverse sub-bands of the lead with finite width. Because of the linear dispersion relation, the transverse sub-bands E_n of the confined graphene are in equidistant instead of $E_n \sim n^2$ of the usual two-dimensional electron gas, while for $E_R < E_L$, due to the fixed sub-band numbers in the left region, no more higher plateaus appear. On the other hand, in the p-n region with $E_L < 0$ and $E_R > 0$, there are no plateaus. The conductance G in the p-n region ($E_R > 0$) is always less than the corresponding plateau value in the n-n region ($E_R < 0$). Because of the occurrence of the Klein tunneling processes [5], the conductance is quite large, e.g., $G > e^2/h$ for almost all positive E_R at $M = 5$. With the increase of M, the Klein tunneling processes are slightly weakened and so is the conductance.

Next, we examine the effect of the magnetic field B in the clean sample. With the increase of B, the equidistant sub-bands gradually evolve into the Landau levels which scales as $E_n \propto \sqrt{n}$ for the Dirac particle. The conductance plateaus in the n-n region evolve into the Hall plateau, and the conductance G in the p-n region is strongly suppressed at small E_R. Figures 1(d), 1(e), and 2(a) show G at a high magnetic field with the magnetic flux (or phase) $\phi = 0.007$. We see perfect Hall plateau in the n-n junction with equidistant in the scale of E_R^2 [see Fig. 2(a)]. The plateau values are given by $\min(|\nu_L|, |\nu_R|)e^2/h$ where ν_α

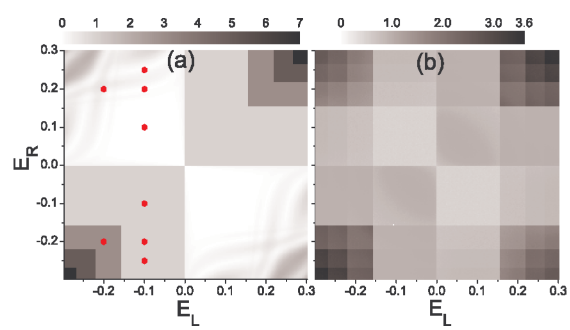

FIG. 2 (color online). The conductance G (in the unit of $2e^2/h$) vs E_L and E_R with $M = 20$, $\phi = 0.007$. (a) $W = 0$ and (b) $W = 2$.

is the filling factors in the lead-α. However, in the p-n region ($E_R > 0$ and $E_L < 0$), no plateaus exist. G is small and strongly depends on the junction length M. For small filling factors ν_L and ν_R or the large junction length M, G is almost zero. This is because for the clean p-n junction, the Hall edge states for electrons and holes are well separated in space and cannot form mixture states leading to a very small conductance G.

In the following, we shall focus on how the conductance G is affected by disorders. Figure 2 plots G versus the energy E_L and E_R with disorder strength $W = 0$ and $W = 2$. In the presence of disorders, G in the p-n and n-p region are strongly enhanced due to the mixture of the electron and hole Hall edge states, while G in the n-n and p-p regions are slightly weakened. At fixed filling factors ν_L and ν_R, G is approximatively a constant. As ν_L or ν_R varies, a jump occurs in G with the borders between ν_L and ν_R regions clearly seen in Fig. 2(b).

Now we investigate the effect of disorders on the conductance in more detail. Figure 3 depicts the conductance vs E_R at fixed $E_L = -0.1$ ($\nu_L = -2$) and -0.2 ($\nu_L = -6$). When $W = 0$, G is small in the p-n region and G exhibits Hall plateaus in the n-n region. With the increase of W from 0, the conductance G in the p-n region is strongly enhanced even for very small W. For example, for $W = 0.02$ or $W = 0.05$, G is greater than $0.2e^2/h$, which is much larger than that ($G < 0.001e^2/h$) at $W = 0$ [see Figs. 3(a) and 3(c)]. When $W = 0.1$, the lowest conductance plateau with $\nu_L = -2$ and $\nu_R = 2$ is well established with its plateau value at e^2/h. In particular, this plateau remains for a broad range of disorder strength W (from 0.1 to 3). For higher filling factors, the conductance is also enhanced by the disorder, but it requires much larger disorder to reach its ideal plateau value at $[|\nu_L||\nu_R|/(|\nu_L| + |\nu_R|)]e^2/h$. For example, for $\nu_L = -2$ and $\nu_R = 6$ or $\nu_L = -6$ and $\nu_R = 2$, the conductance reaches the plateau of $(3/2)e^2/h$ when $W = 2$ [see Figs. 3(b) and 3(d)]. In the n-n region, the Hall plateau is not affected by small disorders and kept their values at $\min(|\nu_L|, |\nu_R|)e^2/h$. If the disorder strength W is increased further, the conductance G starts to drop in both n-n and p-n regions. For very large W (e.g., $W = 6$ or larger), the system enters the insulating regime and G is very small for all E_L and E_R. Here, we wish to emphasize two points: (i) We have seen that the new plateau survives only within certain range of $W = [W_{\min}, W_{\max}]$. When the disorder is slightly below $W_{\min}$ or above $W_{\max}$, G still exhibits a plateau, but its value is less than the value of ideal plateau. For example, the plateau of $\nu_L = -2$ and $\nu_R = 6$ is less than $(3/2)e^2/h$ when $W = 1$ and $W = 3$ [see Figs. 3(c) and 3(d)]. (ii) For some high filling factor region (e.g., $\nu_L = -6$ and $\nu_R = 6$), the conductance plateau does not emerge at all for any W. Because it is much more difficult to completely mix all states for the case of high filling factor, so the system goes to the insulating regime before the occurrence of the complete state mixing. These numerical results are in excellent agreement with the experiment [3].

We now focus on the conductance vs disorder strength for energies E_L and E_R shown in Fig. 2(a) (solid dots). In the n-n region, the Hall edge states are very robust against disorders so the conductance remains quantized at small W [see Fig. 4(b)]. At large disorders, the edge states are destroyed, and the Hall conductance decreases monotonically with increasing of W. In the p-n region [Figs. 4(a), 4(c), and 4(d)], the enhancement of conductance due to the states mixing at moderate disorders is clearly seen. For the lowest filling factors with $\nu_L = -2$ ($E_L = -0.1$) and $\nu_R = 2$ ($E_R = 0.1$), G reached its ideal plateau value e^2/h at $W = 0.09$ and stayed there until $W = 3$ [see Fig. 4(a)]. We emphasize that this range of disorder W (from 0.09 to 3) is very broad, extending in almost 2

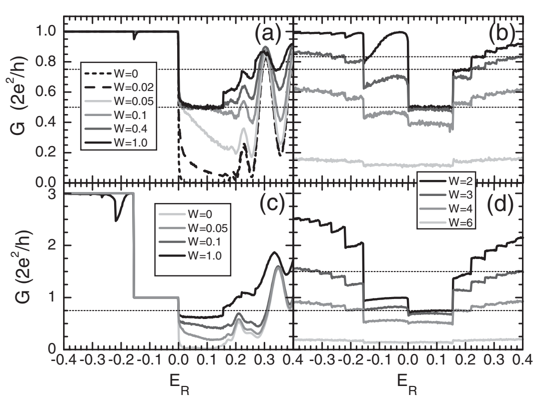

FIG. 3. The conductance G vs E_R for the different disorder strengths W, with the parameters $M = 20$, $\phi = 0.007$. (a), (b) $E_L = -0.1$. (c), (d) $E_L = -0.2$. The panels (b) and (d) share the same legend.

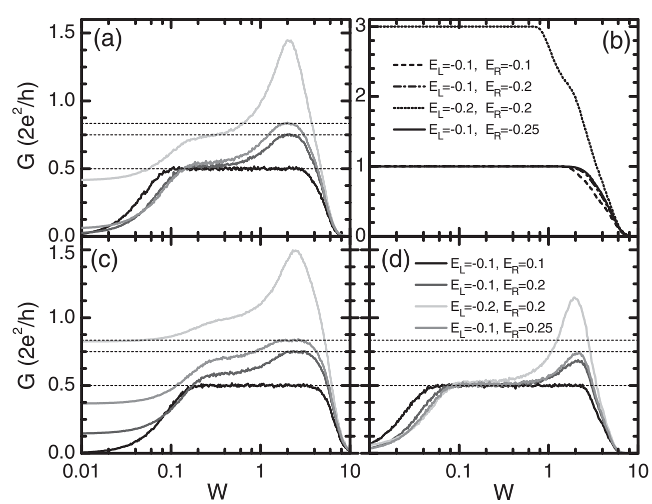

FIG. 4. The conductance G vs disorder strength for E_L and E_R fixed at different points shown in Fig. 2(a) with $\phi = 0.007$. The system size, $M = 20$ for (a) and (b), $M = 10$ for (c), and $M = 40$ for (d). E_L and E_R in (a) and (c) are same as in (d).

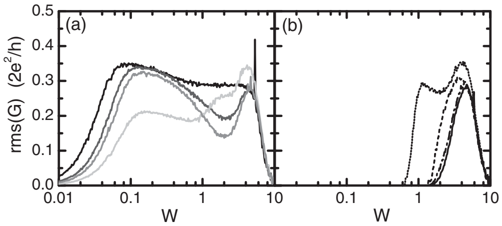

FIG. 5. rms(G) vs E_R. The parameters in (a) and (b) are the same as in Figs. 4(a) and 4(b), respectively.

orders of magnitude. So the lowest plateau can easily be observed experimentally. For $\nu_L = -2$ and $\nu_R = 6$ ($E_R = 0.2$) (or $\nu_R = 10$ with $E_R = 0.25$), the left side of the sample has an electron Hall edge state and the right side has three (or five) holes Hall edge states. As a result of the mixing of the left-side electron state and one of right-side hole states, the conductance G develops a step around e^2/h. Upon further increasing W, the complete mixing of the left-side electron state and all right-side hole states occurs at $W = 1.6$ (or 1.7), and G reaches the ideal plateau value $(3/2)e^2/h$ [or $(5/3)e^2/h$]. Note that this ideal plateau exists only within the disorder window $1.7 < W < 2.6$ (or $1.6 < W < 2.4$) that is much narrower than that of the lowest plateau. Our results also show that for the case of higher filling factors (e.g., $\nu_L = -6$ and $\nu_R = 6$), the ideal plateau $3e^2/h$ cannot be reached for any disorders. When the center region becomes longer or shorter, the conductance G shows similar results [see Figs. 4(c) and 4(d)]. For a short center region (e.g., $M = 10$), G reaches the ideal lowest plateau at a larger W with a smaller plateau width. The high conductance plateau at $\nu_L = -6$ and $\nu_R = 6$ also appears. On the other hand, for a longer center region (e.g., $M = 40$), G reaches the ideal lowest plateau at a smaller W with a wider plateau. Except for the lowest plateau, all other plateaus do not appear for $M = 40$. When the width N of graphene ribbon becomes wider or narrower, similar results are obtained. In addition, for the wider ribbon, more conductance plateaus emerge due to the fact that it has longer distance for the mode mixing.

Up to now, we only consider the zigzag edge graphene. In fact, for the armchair edge as well as edges with other chirality, they all have very similar behaviors: the conductance G exhibits plateau structure at suitable range of W. The lowest plateau is the easiest to emerge and can survive for a broad range of W, but the existence of high plateaus depends on system parameters. In addition, we also find that these effects are the strongest for the zigzag edge graphene, and the weakest for the armchair edge graphene. For graphene edges with other chirality, the effects are in between.

Finally, we study the conductance fluctuation $\mathrm{rms}(G) \equiv \sqrt{\langle G - \langle G \rangle \rangle^2}$, where $\langle \ldots \rangle$ is the average over the disorder configurations with the same disorder strength W. Figure 5 shows rms(G) versus W with the same set of parameters as in Figs. 4(a) and 4(b). In the n-n region [see Fig. 5(b)], there is no fluctuation of the Hall edge states at small W. When disorder increases, the conductance fluctuates when the edge states are partially destroyed. At large disorders, rms(G) eventually goes to zero, and the system enters the insulating regime. On the other hand, in the p-n region [see Fig. 5(a)], the fluctuation rms(G) is small for both small and large W. But rms(G) is large for intermediate W and usually exhibits a double-peak structure. In particular, rms(G) does not have the plateau although the conductance has a very long plateau especially at $\nu_L = -2$ and $\nu_R = 2$.

In summary, the electron transport through graphene p-n junctions under the perpendicular magnetic field is numerically studied. We find the conductance is quite small for the clean p-n junction. But the disorder can drastically enhance the conductance leading to the conductance plateaus. The lowest conductance plateaus can sustain for a very broad range of disorder strength (about 2 orders of magnitude), but the higher plateaus are difficult to form. When the disorder is slightly outside of this disorder range, some conductance plateaus in G vs E_R curve can also emerge with plateau value smaller than the ideal value.

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