Relevance of Multiple Quasiparticle Tunneling between Edge States at $\nu = p/(2np + 1)$

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We present an explanation for the anomalous behavior in tunneling conductance and noise through a point contact between edge states in the Jain series $\nu = p/(2np + 1)$, for extremely weak backscattering and low temperatures [Y. C. Chung, M. Heiblum, and V. Umansky, Phys. Rev. Lett. **91**, 216804 (2003)]. We consider edge states with neutral modes propagating at finite velocity, and we show that the activation of their dynamics causes the unexpected change in the temperature power law of the conductance. Even more importantly, we demonstrate that multiple-quasiparticle tunneling at low energies becomes the most relevant process. This result will be used to explain the experimental data on current noise where tunneling particles have a charge that can reach p times the single-quasiparticle charge. In this Letter, we analyze the conductance and the shot noise to substantiate quantitatively the proposed scenario.

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Introduction.—Noise experiments in point contacts have been crucial to demonstrate the existence of fractionally charged quasiparticles (QPs) in fractional quantum Hall systems [1]. In particular, it was proved that, for filling factor $\nu = p/(2np + 1)$, with $n, p \in \mathbb{N}$, the QP charge is given by $e^* = e/(2np + 1)$ [2–4]. A suitable framework for the description of these phenomena is provided by the theory of edge states [5,6]. For the Laughlin series ($p = 1$), a chiral Luttinger liquid theory (χLL) with a single mode was proposed and shot-noise signatures of fractional charge were devised [7]. For the Jain series [8] ($p \geq 1$), extensions were introduced by considering $p - 1$ additional hierarchical fields, propagating with finite velocity [9], or two fields, one charged and one topological and neutral [10,11]. At intermediate temperatures, the experimental observations of tunneling through a point contact with $\nu = 1/3$ [12] are well described by the χLL theory [13], which interpolates between the strong and the weak backscattering limits. However, at low temperatures and weak backscattering, the current presents unexpected behaviors [12,14,15]. For instance, the backscattering conductance decreases for $T \to 0$ instead of increasing as the theories would require. Different mechanisms of renormalization of tunneling exponents were proposed to account for this discrepancy: coupling with additional phonon modes [16], interaction effects [17,18], or edge reconstruction [19]. For $p > 1$, there are other intriguing transport experiments on a point contact at low temperature and extremely weak backscattering [14,20] which are not yet completely understood. The main puzzling observations for $\nu = 2/5$ and $\nu = 3/7$ are (i) a change in the power-law scaling of the backscattering current with temperature and (ii) an effective tunneling charge, as measured with noise, that can reach the value pe^* for ultralow temperatures $T <$ 20 mK.

In this Letter, we propose a unified explanation of the above open points. We will describe infinite edges with two fields, one charged and one neutral, following the Lopez-Fradkin theory [10,11]. However, differently from that approach, where the neutral mode is nonpropagating and guarantees only the appropriate fractional statistics of QP excitations, we assume a *finite* velocity of propagation. We will show that the energy scaling of the single-QP tunneling is modified by the dynamics of neutral modes [21]. This will be sufficient to explain a change in slope of the linear conductance vs T. However, in order to find an "effective" tunneling charge larger than e^* at very low temperature, it is necessary to demonstrate that tunneling is dominated by an agglomerate of QPs. We will show that this is indeed the case.

Multiple-QP processes.—We start to describe tunneling through a point contact in a Hall bar with right or left edges ($j = R/L$) of infinite length [10,11]. Edge j consists of a charged mode ϕ_j^c and a neutral mode ϕ_j^n, mutually commuting [22]. The commutation relations are $[\phi_j^{c/n}(x), \phi_j^{c/n}(x')] = i\pi\eta^{c/n}\nu_{c/n}\mathrm{sgn}(x - x')$, with $\eta^{c/n} = +/-$, $\nu_c = \nu$, and $\nu_n = 1$. The electron number density is $\rho_j(x) = \partial_x\phi_j^c(x)/2\pi$. The real-time action $\mathcal{S}_j$ is ($\hbar = 1$)

$$\mathcal{S}_j = \frac{1}{4\pi\nu}\int dtdx\partial_x\phi_j^c(-\partial_t - v_c\partial_x)\phi_j^c + \frac{1}{4\pi}\int dtdx\partial_x\phi_j^n(+\partial_t - v_n\partial_x)\phi_j^n, \qquad (1)$$

where ϕ_j^n is counterpropagating with respect to ϕ_j^c and has velocity $v_n \ll v_c$. Consequently, the relation between the bandwidths $\omega_{c/n} = v_{c/n}/a$ will be $\omega_n \ll \omega_c$, where a^{-1} is the momentum cutoff.

The operator that annihilates an agglomerate of m QPs for the jth edge can be written in the bosonized form

$$\Psi_j^{(m)}(x) = \frac{\mathcal{F}^m}{\sqrt{2\pi a}}e^{i[\sqrt{\alpha_m}\phi_j^c(x)+\sqrt{\beta_m}\phi_j^n(x)]}. \qquad (2)$$

Here $\mathcal{F}^m$ corresponds to the ladder operator for changing the number of m QPs. It plays the role of a Klein factor and in lowest order in tunneling can be neglected. The coefficients are determined by requiring that $\Psi_j^{(m)}(x)$ satisfies the appropriate commutation relations with the electron density $[\rho_j(x), \Psi_j^{(m)}(x')] = -m(\nu/p)\delta(x-x')\Psi_j^{(m)}(x')$ and the statistical properties $\Psi_j^{(m)}(x)\Psi_j^{(m)}(x') = \Psi_j^{(m)}(x')\Psi_j^{(m)}(x)e^{-i\theta_m \mathrm{sgn}(x-x')}$. The statistical angle is [23]

$$\theta_m = \pi m^2\left(\frac{\nu}{p^2} - \frac{1}{p} - 1\right) + 2\pi k, \qquad (3)$$

where $k \in \mathbb{Z}$ takes into account the 2π periodicity. Thus, for every value of p, one has

$$\alpha_m = \frac{m^2}{p^2}; \qquad \beta_m = m^2\left(1 + \frac{1}{p}\right) - 2k. \qquad (4)$$

Equation (4) admits several solutions labeled by different $k \le k^{\max}$ with $k^{\max} = \mathrm{Int}[m^2(1+1/p)/2]$, where $\mathrm{Int}[x]$ is the integer part of x. For a given m, there is a family of $\Psi_j^{(m)}$ with the same fractional properties but different scaling behavior. The local scaling dimension Δ_m of the m-agglomerate operator is defined as half the power-law exponent at long times ($|\tau| \gg 1/\omega_c, 1/\omega_n$) [24] in the imaginary time Green function $\mathcal{G}_m(\tau) = \langle T_\tau[\Psi_j^{(m)}(0,\tau)\Psi_j^{(m)\dagger}(0,0)]\rangle \propto \tau^{-2\Delta_m}$. At $T = 0$,

$$\mathcal{G}_m(\tau) = \frac{1}{2\pi a}\left(\frac{1}{1+\omega_c|\tau|}\right)^{g_c\nu\alpha_m}\left(\frac{1}{1+\omega_n|\tau|}\right)^{g_n\beta_m}, \qquad (5)$$

where one can clearly recognize in the last term the dynamical contribution of the neutral modes. The scaling dimension is then $\Delta_m = (g_c\nu\alpha_m + g_n\beta_m)/2$. Note that, in order to take into account possible additional interaction effects, we considered in Eq. (5) renormalization parameters $g_{c,n} \ge 1$ [25]. They correspond to the renormalization of the dynamical exponents induced by a coupling of the fields with independent dissipative baths [16]. The microscopic models underlying these renormalizations were extensively treated in literature [16–19]. The *most relevant* operator in the m family will then have the minimal value $\Delta_m^{\min} = [g_c\nu\alpha_m + g_n\beta_m^{\min}]/2$ given by the minimal value of β_m in Eq. (4):

$$\beta_m^{\min} = m^2(1 + 1/p) - 2k^{\max}. \qquad (6)$$

Let us now identify the dominant process for specific cases. In the Laughlin series ($p = 1$), one finds $\beta_m^{\min} = 0$, and therefore the single-QP tunneling ($m = 1$) is always the dominant one since $\Delta_m^{\min} = m^2\Delta_1^{\min}$. A different scenario is present for $p \ge 2$. Here one has, for $m = 1$, $\beta_1^{\min} = 1 + 1/p$, while for the p agglomerate $\beta_p^{\min} = 0$. This allows us to conclude that agglomerates with $m > p$ are never dominant: $\Delta_{m>p}^{\min} > \Delta_{m=p}^{\min}$.

To find the most relevant operator, one has to choose within the class with $1 \le m \le p$. In the bare case $g_{n,c} = 1$, one can show that the p agglomerate is the most relevant for $p \le 6$. With renormalized exponents $g_{n,c} > 1$, the analysis is still possible but more cumbersome; we limit here the discussion to $p = 2, 3$, which are directly connected with the experiments at $\nu = 2/5, 3/7$ [14]. It is furthermore possible to show with the above relations that the p agglomerate is always dominant in the parameter region $g_n/g_c > \nu(1 - 1/p)$, while otherwise the single-QP tunneling prevails.

We conclude by emphasizing that, for a nonpropagating neutral mode with $v_n = \omega_n = 0$, the single-QP processes will *always* dominate because the neutral mode does not contribute to the scaling.

Transport.—In this part we restrict the analysis of tunneling through the point contact to $\nu = 2/5$ and $\nu = 3/7$. In these cases we consider the two most dominant processes only: the single QP and the agglomerate of p QPs. The tunneling at $x = 0$ is $H_T = t_1\Psi_R^{(1)\dagger}(0)\Psi_L^{(1)}(0) + t_p\Psi_R^{(p)\dagger}(0)\Psi_L^{(p)}(0) + \mathrm{H.c.}$, with t_1 and t_p the tunneling amplitudes. Here the operators $\Psi_j^{(m)}$ are the most relevant representatives in the m family. The tunneling rates at lowest order in t_m are ($m = 1, p$ and $k_B = 1$)

$$\Gamma_m(E) = \gamma_m\int_{-\infty}^{+\infty} dt e^{iEt}e^{-[\alpha_m W^c(t) + \beta_m^{\min}W^n(t)]}, \qquad (7)$$

with $\gamma_m = (|t_m|/2\pi a)^2$ and $W^{c/n}(t) = \sum_j\langle[\phi_j^{c/n}(0,0) - \phi_j^{c/n}(0,t)]\phi_j^{c/n}(0,0)\rangle$ the bosonic correlation functions. The explicit expression of the kernel is $W^r(t) = g_r\nu_r\ln[(1 + i\omega_r t)\mathbf{\Gamma}(\eta_r)^2/|\mathbf{\Gamma}(\eta_r + iTt)|^2]$, where $\eta_r = 1 + T/\omega_r$, with $r = c, n$, and $\mathbf{\Gamma}(x)$ is the Gamma function [26]. In the following, we assume that the neutral mode bandwidth ω_n can be comparable with T and with the external voltage energy e^*V, while the charge bandwidth ω_c is taken as the largest cutoff energy.

In lowest order, the total backscattering current through the point contact is given by the sum of the two independent processes contributions I_1^B and I_p^B:

$$I^B = \sum_{m=1,p} I_m^B = e^*\sum_{m=1,p} m(1 - e^{-E_m/T})\Gamma_m(E_m), \qquad (8)$$

with $E_m = me^*V$ the energy for m-QP tunneling in the presence of the bias V. The linear backscattering conductance is then $G^B(T) = \sum_{m=1,p}G_m^B(T)$, where $G_m^B(T) = (me^*)^2\Gamma_m(0)/T$. It will contribute to the total conductance via the relation $G(T) = \nu e^2/2\pi - G^B(T)$. Before analyzing it numerically, we discuss qualitatively the different scaling regimes. Let us start with $G_1^B(T)$: For $T \ll \omega_n$ the neutral modes participate in the temperature scaling giving $T^{2(g_c\nu\alpha_1 + g_n\beta_1^{\min} - 1)}$, while in the opposite limit $T \gg \omega_n$ the scaling is driven by the charged modes only with $T^{2[g_c\nu\alpha_1 - 1]}$. On the other hand, the p agglomerate follows the power law $G_p^B(T) \propto T^{2(g_c\nu\alpha_p - 1)}$ with a scaling driven always by the charged modes because $\beta_p^{\min} = 0$. The total backscattering conductance will depend on the relative weights between the single-QP and p-agglomerate contributions. We fix the ratio of the tunneling amplitudes t_1/t_p by introducing the temperature T^* at which $G_1^B(T^*) = G_p^B(T^*)$.

The experiments [14] suggest the relevance of the p agglomerate at extremely low temperature so $T^* < \omega_n$ and the renormalization coefficients satisfy $g_n/g_c > \nu(1 - 1/p)$. In this case the behavior of the backscattering conductance $G^B(T)$ presents three distinct power laws:

$$G^B(T) \approx \begin{cases} T^{2[\nu g_c - 1]} & T \ll T^*, \\ T^{2[\nu g_c/p^2 + g_n(1+1/p)-1]} & T^* \ll T \ll \omega_n, \\ T^{2[\nu g_c/p^2 - 1]} & \omega_n \ll T, \end{cases} \quad (9)$$

where we explicitly used Eqs. (4) and (6). A sketch of these behaviors is shown in Fig. 1(a). The solid line is the backscattering conductance, and the dashed lines are the three different asymptotic power laws in Eq. (9). At very low temperatures (region I) the p agglomerate dominates, while at higher temperatures (regions II and III) the single QP is dominant. Note that the intermediate temperatures regime (II), where the neutral modes are effective, will be accessible only if $T^* \lll \omega_n$. Otherwise, we expect a mixing of regions II and I. Figure 1(b) shows the backscattering current I^B in Eq. (8) for $\nu = 2/5$ (solid gray line) evaluated numerically. The parameters were adjusted in order to fit the experimental data (black squares) taken from Fig. 2(a) of Ref. [14]. With respect to the sketch in Fig. 1(a), the best fit of the experimental data is mainly given by region II, where the p agglomerate is not fully effective. We warn,

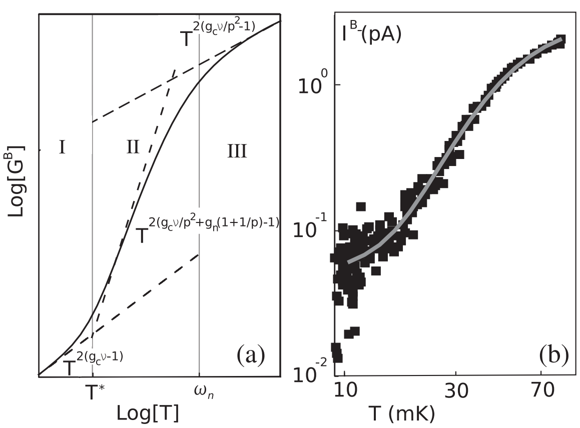

FIG. 1. (a) Sketch of the backscattering conductance G^B vs temperature in a log-log plot. The dashed lines are the asymptotic power laws, and the solid line is the conductance in different temperature regimes: I, low; II, intermediate; and III, high T. In this scheme the parameters are chosen with $T^* \ll \omega_n$, $\omega_n \ll \omega_c$, and $g_c\nu/p^2 > 1$. (b) Comparison between the theoretical backscattering current I^B (solid gray line) and the experimental data (black squares) at $\nu = 2/5$ ($p = 2$) from Ref. [14] with courtesy of M. Heiblum. Plotting parameters: $g_c = 3$, $g_n = 4$, $\omega_n = 50$ mK, $\omega_n/\omega_c = 10^{-2}$, $T^* = 20$ mK, $e^*V = 1.16$ mK, $\gamma_1/\gamma_2 = 1.66$, and $\gamma_1/\omega_c^2 = 4 \times 10^{-2}$.

however, that due to the restricted experimental range of temperatures (roughly one decade) it is not possible to extract meaningful values for power-law exponents. Anyway, an estimate of the neutral modes bandwidth of $\omega_n \sim 50$ mK appears reasonable. This fact could explain why in experiments at higher temperatures the effects of the neutral modes are not easily detectable.

Shot noise.—Direct information concerning the effective charge transferred through the point contact can be obtained via the current noise spectrum S at zero frequency. In the following, we analyze the shot-noise regime with $T \ll e^*V$. Since in the experiments [14] the edges are extremely weakly coupled, we will restrict our analysis to lowest order in t_m. In this limit the transport through the point contact has a Poissonian nature, and a Schottky formula $S_m = 2me^*I_m^B$ for the noise S_m in each channel can be safely applied [7,13,27], with $m = 1, p$ being the most relevant tunneling processes. In the same limit, the different tunneling processes are independent, and the total noise is the sum of the two individual contributions $S = S_1 + S_p = 2e^*(I_1^B + pI_p^B)$. Then the effective charge $q_{\rm eff}$ of the tunneling process will be evaluated from the behavior of the Fano factor $F = S/2eI^B$, via the relation $q_{\rm eff} = eF$. F is, in general, a measure of a weighted average charge transmitted via the two processes. Only when one of the two processes becomes strongly dominant does the Fano reproduce the corresponding charge, viz. e^* for single-QP tunneling and pe^* for the p agglomerate.

For simplicity, we consider the limit $T = 0$. The current (8) can be evaluated without any further assumption:

$$I_m^B = m \frac{4e^*\pi\gamma_m}{\omega_c^{a_m}\omega_n^{b_m}} \frac{e^{-E_m/\omega_c}}{\Gamma(a_m + b_m)} E_m^{a_m+b_m-1} \times {}_1F_1\left(b_m, a_m + b_m, \frac{E_m}{\omega_c} - \frac{E_m}{\omega_n}\right), \quad (10)$$

with ${}_1F_1(a, b, z)$ the Kummer confluent hypergeometric function, $a_m = 2g_c\nu\alpha_m$, and $b_m = 2g_n\beta_m$. Similarly to the conductance, the current exhibits different regimes. For $E_1 \gg \omega_n$ the single-QP contribution scales as $I_1^B \propto E_1^{2g_c\nu/p^2-1}$, while for $E_1 \ll \omega_n$ it receives additional contributions from the neutral modes $I_1^B \propto E_1^{2[g_c\nu/p^2+g_n(1+1/p)]-1}$. This twofold power law is present only for the single-QP tunneling since the p-agglomerate current depends only on the charged mode $I_p^B \propto E_p^{2\nu g_c-1}$. We define V^* as the voltage at which $I_1^B(V^*) = I_p^B(V^*)$. From the previous scaling argument, we conclude that for $V \ll V^*$ the p agglomerate dominates, while for $V \gg V^*$ single-QP tunneling is more relevant.

In Fig. 2, the Fano factor is shown as a function of the V for $\nu = 2/5$ (solid line) and $\nu = 3/7$ (dashed line). One can easily recognize two regimes with distinct effective charges: For $V \gg V^*$ the noise is dominated by the single-QP processes and $q_{\rm eff} = e^*$, while for $V \ll V^*$ the p agglomerate will prevail with $q_{\rm eff} = pe^* = \nu e$. Note that

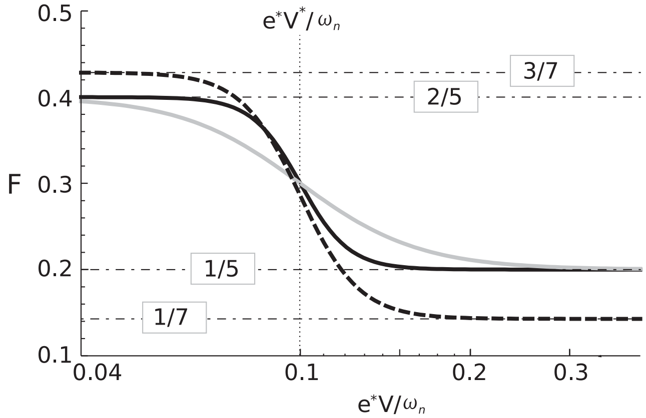

FIG. 2. Zero temperature Fano factor F vs source-drain potential e^*V/ω_n in log scale. The solid lines represent $\nu = 2/5$ for $g_c = 3$, $g_n = 2$, and $\kappa = 2.75$ (gray) and $g_c = 3$, $g_n = 4$, and $\kappa = 5.25$ (black). The dashed line is $\nu = 3/7$ with $g_c = 3$, $g_n = 4$, and $\kappa = 4.26$. Other parameters are $\omega_n/\omega_c = 10^{-2}$ and $e^*V^*/\omega_n = 0.1$.

the width of the transition region is determined by the difference between the power-law exponents of I_1 and I_p. Indeed, defining the ratio $\kappa = \Delta_1^{\min}/\Delta_p^{\min} > 1$, one has a sharper transition for larger κ values (see Fig. 2 for $\nu = 2/5$). The smoothness of the Fano factor could be then relevant to determine the renormalized parameters and the voltage at which the p-agglomerate tunneling is clearly visible.

We observe that the above results on the possibility to detect an effective tunneling charge $q_{\mathrm{eff}} = pe^*$ will remain valid also at finite temperatures as long as $T \ll e^*V^*$. At higher temperature, the dominance of the p agglomerate is progressively compromised.

The above facts could explain why in the experiment for $\nu = 3/7$ the limiting value $F = 3/7$ is not fully reached, while for $\nu = 2/5$ the limiting value is observed.

Conclusions.—We have shown that p-QP agglomerates can be the most dominant tunneling process through a point contact at extremely low temperatures in the weak backscattering regime. Direct signatures of this relevance are shown in the behavior of the shot noise. The main point underlying this result is the assumption of neutral modes propagating at finite velocity. Their dynamical activation affects the single-QP tunneling scaling and makes it less relevant than multiple-QP tunneling. In addition, we explain the double power law observed in the temperature scaling of the backscattering current.

Though in this work we mainly investigated the experimental observations of Ref. [14], we expect that our results hold for more general experimental situations. For instance, a super-Poissonian noise has been found in the complementary regime of tunneling in the strong backscattering limit [28], and indeed an analogous application of our theory to that limit demonstrates that electron agglomerates can be the dominant tunneling events at low energy for filling factors in Jain's series. A new generation of experimental studies of shot noise in point contacts at low temperatures is thus desirable in order to shed light on the physics of tunneling of agglomerates.

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