# Peak Effect in the Critical Current of Type II Superconductors with Strong Magnetic Vortex Pinning

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We perform 2D Langevin simulations studying the peak effect (PE) of the critical current taking into account the temperature dependence of the competing forces. We observe and report that the PE results from the competition of vortex-vortex interactions and vortex-pin interactions which have different temperature dependencies. The simulations reveal that the PE can take place only for certain pinning strengths, densities of pinning centers, and driving forces, which is in good agreement with experiments. No apparent vortex order-disorder transition is observed across the PE regime. In addition, the PE is a dynamical phenomenon, and thermal fluctuations can speed up the process for the formation of the PE.

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One pronounced phenomenon in type II superconductors is the so-called peak effect (PE), which is the appearance of a peak in the critical current density $J_c$ before decreasing to 0 with increasing temperature or field [1]. The PE has widely been observed in a variety of low and high temperature superconductors by different experimental techniques [2–10]. It was proposed long ago that the PE originated from softening of the elastic moduli of a vortex lattice [1,11,12]. Considering the competition between the strength of the vortex lattice and the pinning force of the isolated pinning center, Labusch showed that $J_c > 0$ only if the pinning force dominates over the strength of the vortex lattice; otherwise, $J_c = 0$ [13]. According to the Labusch criterion, the weak pinning centers in superconductors cannot pin the vortex lattice. Larkin and Ovchinnikov showed that weak pinning centers can act collectively in the correlation volume $V_c$ ($=L_c R_c^2$, where $L_c$ and $R_c$ are the longitudinal and transverse correlation lengths, respectively), where the vortices are of long-range order, significantly reducing the Labusch criterion [14]. In the collective pinning theory, the PE was interpreted as resulting from the abrupt decrease of $V_c$ due to the reduction in elastic interaction [15]. The PE was recently shown to appear naturally at the crossover from weak collective to strong pinning [16]. Further calculations showed that the "Bragg glass" phases exist to be a quasi-long-range ordered vortex lattice on a length scale $r \gg R_c$ [2]. Furthermore, the occurrence of PE was explained as evidence for a Bragg glass disordering transition or an order-disorder (OD) transition [2,5,9,17]. The OD transition was suggested to be a thermodynamic phase transition induced by thermal fluctuations or pinning centers [2], which has been confirmed experimentally by the direct structural observation of the vortex lattice [9,18]. Also, the investigation of the reversibility of the OD transition provided strong support for its thermodynamical nature [19].

However, the exact nature of the PE phenomenon remains controversial [6,7,10,20–22]. Recently, across the PE regime no noticeable change in the order of the vortex lattice was observed by small-angle neutron scattering [20] and by Bitter decoration [21]. It was also reported that the PE is situated on a boundary separating the strong pinning regime from the thermal fluctuation dominated regime, while the weak-strong pinning crossover is located far from the PE regime [7]. On the other hand, for some superconductors with strong pinning centers, it has been found that the occurrence of the PE depends strongly on the strength and number of the pinning centers [3,6,23,24]. Especially, the PE can be adjusted by changing the pinning strength of the twin boundaries, which is difficult to explain through the OD mechanism [3].

In this work we investigated the PE of a vortex system with strong pinning by using Langevin dynamics simulations. Extensive molecular dynamics simulations of driven vortices have been carried out and studied [25]. However, the calculation of the peak effect of the critical current $J_c(T)$ taking into account the temperature dependencies of the vortex-vortex interactions and the vortex-pin interaction has—to the best of our knowledge—not been reported so far. The temperature dependence of pinning force $f_{pv}(T)$ is different from that for the elastic forces between vortices $f_{vv}(T)$. Thus, the competition between $f_{pv}(T)$ and $f_{vv}(T)$ takes place as temperature changes. The PE was clearly seen for certain pinning strengths and densities of pinning. No marked change in the order of the vortices was observed. In addition, the PE is a dynamical phenomenon, and thermal fluctuations are not crucial for the occurrence of the PE but can speed up its dynamical process.

 

We use the overdamped Langevin equation of motion for a vortex at position $\mathbf{r}_i$, which is [26]

$$\mathbf{F}_i = \sum_{j\neq i}^{N_v} \mathbf{F}^{vv}(\mathbf{r}_i - \mathbf{r}_j) + \sum_{k}^{N_p} \mathbf{F}^{vp}(\mathbf{r}_i - \mathbf{r}_k^p) + \mathbf{F}^L + \mathbf{F}_i^T = \eta \frac{d\mathbf{r}_i}{dt},$$

where $\mathbf{F}_i$ is the total force acting on vortex $i$, $\mathbf{F}^{vv}$ and $\mathbf{F}^{vp}$ are the forces due to vortex-vortex and vortex-pin interactions, respectively, $\mathbf{F}^L$ is the driving force due to the current $\mathbf{J}$ ($\mathbf{F}^L \propto \mathbf{J} \times \hat{\mathbf{z}}$), and $\mathbf{F}^T$ is the thermal stochastic force; $\eta$ is the Bardeen-Stephen friction coefficient $\eta \propto \phi_0 B_{c2}/\rho_n$, $N_v$ the number of vortices, $N_p$ the number of pinning centers, and $\mathbf{r}_k^p$ the position of the $k$th pinning center. We choose $\mathbf{F}^{vv}(\mathbf{r}_i - \mathbf{r}_j) = (\phi_0^2 s)(2\pi\mu_0\lambda^2)^{-1}(\mathbf{r}_i - \mathbf{r}_j)(|\mathbf{r}_i - \mathbf{r}_j|)^{-2} = f_{vv}(\mathbf{r}_i - \mathbf{r}_j)(|\mathbf{r}_i - \mathbf{r}_j|)^{-2}$, where $\phi_0$ is the flux quantum and $s$ the length of the vortex. We employ periodic boundary conditions and cut off the logarithmic vortex-vortex repulsion potential smoothly [27]. Pinning centers exert an attractive force on the vortices: $\mathbf{F}^{vp}(\mathbf{r}_i - \mathbf{r}_k^p) = -f_{pv}(r_{ik}/r_p)\exp[-(r_{ik}/r_p)^2]\hat{\mathbf{r}}_{ik}$, where $f_{pv}$ tunes the strength of this force and $r_p$ determines its range [28]. We assume $r_p = 0.2\lambda$ and $f_{pv} \propto B_{c2}^2(1 - B/B_{c2})\xi^2/\kappa^2$ as core pinning is considered [29], where $\kappa = \lambda/\xi$, $B_{c2}$ depends on the temperature via $B_{c2}(T) = B_{c2}(0) \times [1 - (T/T_c)^2]$ [30]. $\mathbf{F}_i^T = f_{\rm th}\sum_j \delta(t - t_j)\Gamma(t_j)\Theta(p - q_j)$, where $f_{\rm th}$ represents the intensity of thermal force [31], $\Gamma(t_j)$ is a random number chosen from a Gaussian distribution of mean 0 and width 1, $p$ represents the frequency of the action to the vortex by thermal noise, and $q_j$ is a random number uniformly distributed between 0 and 1. To represent the temperature dependence of $\mathbf{F}^{vv}$, $\mathbf{F}^{vp}$, $\kappa$, and $\mathbf{F}^T$, we use $\lambda(T)/\lambda(0) = (1 - T/T_c)^{-1/2}$ and $\xi(T)/\xi(0) = (1 - T/T_c)^{-1/2}$ [30]. The motion equation is integrated by a Euler scheme with a normalized time step of $\Delta t = 0.005$ [32]. The average $x$ component of the velocities of the vortices is $\langle V_x\rangle = \frac{1}{N_v}\sum_i^{N_v} v_{xi}$, which is proportional to the resulting voltage. We normalize lengths by $\lambda_0 = \lambda(0)$, forces by $f_0 = (\phi_0^2 s)(2\pi\mu_0\lambda_0^3)^{-1}$, and time by $\tau_0 = \lambda_0\eta(0)/f_0$. All quantities shown in the following figures are expressed in simulation units.

For a fixed magnetic field, we perform simulations with a Lorentz driving force along the $x$ axis while cooling from $0.99T_c$ to $0.1T_c$. The total number of vortices $N_v = 676$ is used in the calculations presented here. For larger systems, similar results are observed. The size of the simulated system is $26\lambda_0 \times 26\lambda_0$ if we take $N_v = 676$. We employ $B = 0.015B_{c20}$, $\lambda_0 = 690$ Å, $s = 12$ Å, and $\eta_0 = 1.4 \times 10^{-17}$ kg/s, and, unless specified otherwise, the pinning strength at zero temperature is $f_{pv0} = 9f_0$, the rate of change of temperature is $dT/dt = -0.02T_c/t_0$, the driving force $F^L = 2f_0$, the number of pinning centers $N_p = 0.2N_v$, and $f_{\rm th} = 1$.

Figure 1(a) shows a typical plot of the average velocity of the vortices in the $x$ direction $V_x$ against the reduced temperature $T/T_c$ (solid triangles). The figure also shows the number of pinned vortices $N_v^p$ normalized by $N_v$ as a function of reduced temperature $T/T_c$ (open circles). A dip of $V_x$ can be seen in the temperature regime from $T_{ps}$ to $T_{pe}$. Since the average vortex velocity is proportional to the resistance, the dip in the $V_x$ implies a peak in $J_c$ identified as the PE [4]. Considering $J_c \sim M \sim 1/(1 - |\chi'|) \sim 1/|\chi''|$ [33], $M$ is magnetization, and $\chi'$ and $\chi''$ are the real and imaginary parts of the susceptibilities, respectively. We see that a dip in $V_x(T)$ corresponds to a peak in $M(T)$, a dip in $\chi'(T)$, and a dip in $\chi''(T)$ in experiments, respectively. The simulated results show three distinct

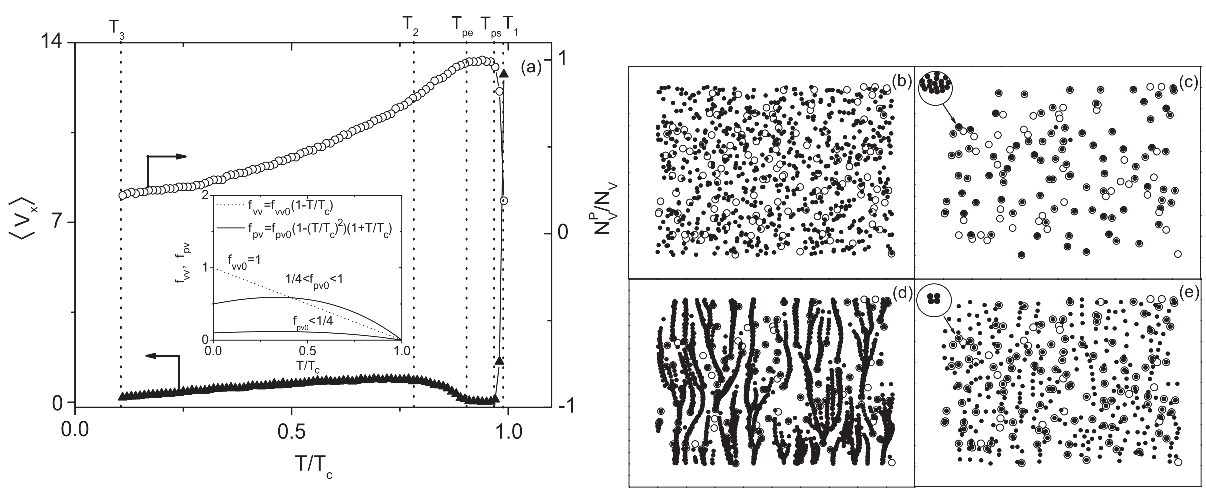


FIG. 1. (a) The dip effect in the velocity $V_x$ (solid triangles) and the number of pinned vortices $N_v^p/N_v$ (open circles) as a function of temperature. Inset: Schematic diagram of elastic force $f_{vv}(T)$ and pinning force $f_{pv}(T)$ as a function of temperature. (b)–(e) Vortex (solid circles) distribution at temperatures marked in (a). (b) $T = T_1$, where almost all vortices are unpinned. (c) $T = T_{ps}$, where almost all of the vortices are pinned, and each pin (open circles) can trap several vortices, shown in the enlarged graph. (d) $T_{pe} > T \geq T_2$, where part of the pinned vortices are unpinned and the vortex channels are formed. (e) $T = T_3$, where most of vortices are unpinned and only a smaller number of vortices are trapped, shown in the enlarged graph.

regimes. We discuss the features going from high to low temperature.

At higher temperatures ($T_{ps} < T < T_c$), the number of pinned vortices increases with decreasing temperature, which results in the average velocity of vortices decreasing accordingly. Close to $T_c$, the pinning force is small and thermal fluctuations are strong, so pinning centers cannot pin vortices. This is supported by most of the vortices being unpinned and randomly distributed in the simulation cell at $T_1$, as shown in Fig. 1(b). When decreasing the temperature, the pinning force becomes more important than both the thermal fluctuations and the elastic force [$f_{pv} > f_{vv}$; see also the inset in Fig. 1(a)]. Thus, more and more vortices are trapped by the random pinning centers. Close to $T = T_{ps}$, almost all of the vortices are pinned.

In the PE region ($T_{pe} \leq T \leq T_{ps}$), the vortices are pinned and disordered. It is clear that the high $J_c$ is due to $f_{pv}(T)$ dominating over either $f_{vv}(T)$ or thermal fluctuations. Besides, because the vortex pinning is strong enough, several vortices are trapped by one pinning site; see, for example, the inset in Fig. 1(c). This is consistent with the recent experimental and numerical observations [34].

At low temperatures ($T < T_{pe}$), $f_{vv}(T)$ becomes more important and thus more and more vortices are depinned; see, for example, the inset in Fig. 1(e). The vortex paths form channel flows as shown in Fig. 1(d). These channels will be destroyed by the quickly enhancing elastic force at very low temperatures, shown in Fig. 1(e). Note that no ordered vortex structure is observed even in low temperature $T \sim 0$.

Our calculations clearly show that there exists the PE for a 2D strongly disordered vortex system. Furthermore, the vortices are always in disordered states across the PE regime. In other words, no apparent change in the vortex order can be observed when the PE takes place. The PE can be simply explained by the competition between pinning and elastic forces of the vortices, which results from their different temperature dependencies. It is evident that our interpretation on the PE is different from that based on the OD transition or on the abrupt reduction in the correlation volume. Therefore, the simulations show that at the peak position no marked change in the order of the vortices and nothing like a Bragg glass-vortex glass transition, indicating a possible new mechanism based on dynamics rather than thermodynamics for the PE. This is the central result of this work.

Figure 2 demonstrates the $V_x - T/T_c$ curves for different pinning strengths $f_{pv0}$ and densities $n_p(= N_p/N_v)$. It can be seen that the PE appears only for certain $f_{pv0}$ at a given density $n_p$ and Lorentz force. For small $f_{pv0}$, the vortices cannot be effectively pinned, resulting in a very low $J_c$ and no observable PE, while, for very large $f_{pv0}$, the vortices are always pinned until very low temperatures; see the data for $f_{pv0} = 90f_0$ shown in Fig. 2. Therefore, the PE can be numerically realized by changing pinning strength,

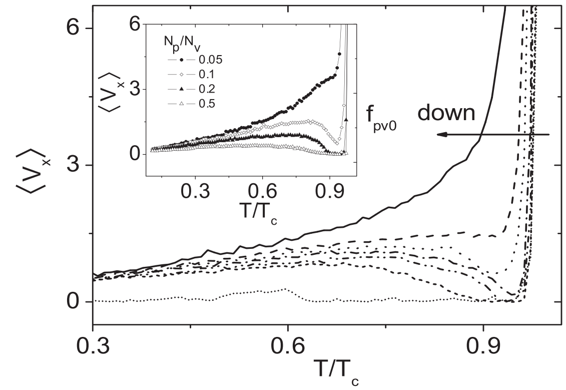


FIG. 2. Effect of pinning strength $f_0$ on the PE at a fixed density of pinning centers. The Lorentz force $f_{pv0}$ varies from 4.5 to $90f_0$ (4.5, 6, 6.6, 7.5, 9, 10.5, 90)$f_0$. Inset: Effect of density of the pinning center on the PE at a fixed pinning strength and Lorentz force.

which is in good agreement with transport experiment by Kwok and co-workers, who successfully observed the PE by adjusted pinning strength of two twin boundaries in yttrium barium copper oxide [3].

The inset in Fig. 2 shows $V_x - T/T_c$ curves at several $n_p$ at given $f_{pv0}$. The PE is hardly seen if $n_p$ is either too large or too small. That is, the situation for low density is similar to that for low pinning strength. For the presence of the PE, therefore, these results imply that the vortex-pin interactions should be comparable with the vortex-vortex interactions. Our simulated results are supported by the experimental investigation about the dependence of the PE on the pin density, which was adjusted by radiation or annealing treatments [6,35].

We now study the effects of the experimental speed on the PE. Figure 3 shows the $V_x - T/T_c$ curves for several temperature ramping rates $dT/dt$ at fixed pinning and driving forces. For high $dT/dt$, the moving vortices cannot

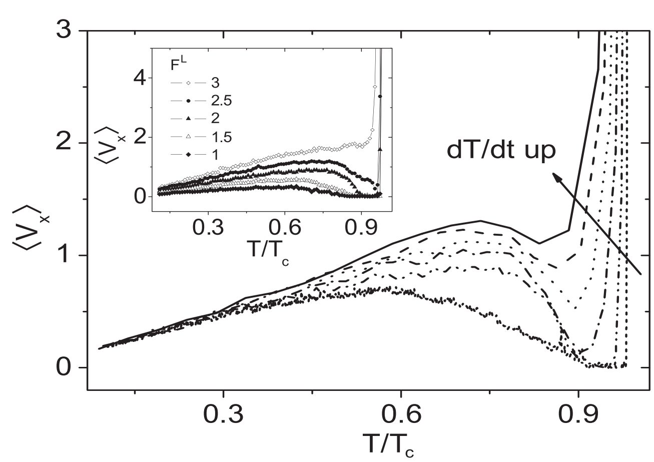


FIG. 3. Effect of temperature ramping rates $dT_c/dt_0$ on the PE, $dT_c/dt_0$ ranging from $-0.002$ to $0.1T_c/t_0$ ($-0.002$, $-0.02$, $-0.04$, $-0.06$, $-0.08$, $-0.1$)$T_c/t_0$. Inset: Effect of the driving force on the PE.

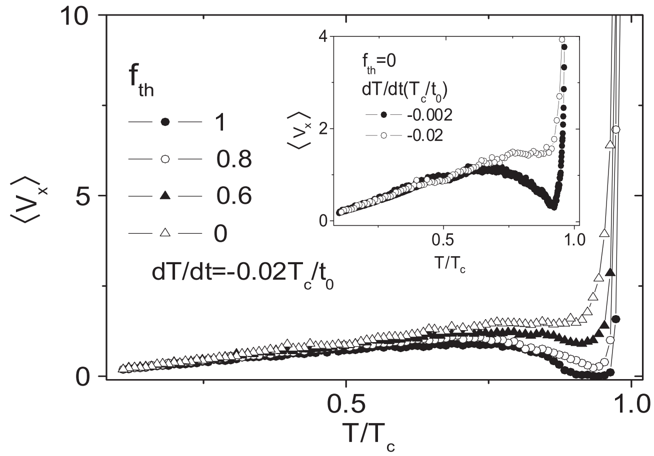


FIG. 4. $V_x - T/T_c$ characteristics for various thermal fluctuation forces. Inset: $V_x - T/T_c$ characteristics without the thermal fluctuations for different temperature ramping rates.

be pinned due to the (relatively slow) vortex relaxation. Thus the PE will progressively disappear with increasing $dT/dt$. At low temperature ramping rates, the PE is progressively visible because the vortices have enough time to move into the pinning wells. These simulated results indicate that the PE is a metastable phenomenon.

It is expected that an increment of pinning force at a fixed driving current is equivalent to a decrement of applied current at a fixed pinning force. We now study the effect of external current on the PE. In the inset in Fig. 3, we show the $V_x - T/T_c$ curves for various driving forces at a fixed pinning force. For large $F^L$ $(=3f_0)$, no PE could be observed, just as the case for a small pinning force. Decreasing the driving force $F^L$ $(=2.5f_0)$, a dip in the $V_x - T/T_c$ curve is progressively more pronounced, just as seen here; see Fig. 3 in Ref. [36]. As the driving force is further decreased $(F^L = 2f_0)$, the pinning force becomes dominant, leading to a broadening of the PE [4].

Finally, in order to clarify the role of thermal fluctuations [7], we studied the effects of thermal fluctuations on the PE. Shown in Fig. 4 are $V_x - T/T_c$ characteristics for various $f_{\rm th}$. It can be seen that the PE is growing with enhanced thermal fluctuations. The reason is that the ''shaking'' effect of thermal fluctuation can effectively reduce the relaxation time within which the vortices diffuse into the potential wells. To further confirm this, we study the effects of the $dT/dt$ on the PE under $f_{\rm th} = 0$, which is shown in the inset in Fig. 4. It is found that the PE occurs for low $dT/dt$ $(=-\,0.002T_c/t_0)$ but disappears for large $dT/dt$ $(=-\,0.02T_c/t_0)$. This is strong evidence that thermal fluctuations are not crucial for the occurrence of the PE in our 2D vortex system, contrasting with the reported result [7].

In summary, 2D Langevin simulations reveal that the PE is ubiquitously observed for certain strengths and densities of pinning sites and the Lorentz force, which is in good agreement with the experiments. No vortex order-disorder transition or collective-individual vortex pinning cross is observed. The PE can be simply explained by the competition between pinning and elastic forces due to their different temperature dependencies. The relaxation relating to the PE is observed by changing the temperature ramping rate or thermal fluctuation force, demonstrating its dynamical characteristics. In addition, thermal fluctuations are not crucial to the PE in the strongly pinned system, but they can speed up the process for the formation of the PE.

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