Quantum Hall Ferromagnetic States and Spin-Orbit Interactions in the Fractional Regime

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The competition between the Zeeman energy and the Rashba and Dresselhaus spin-orbit couplings is studied for fractional quantum Hall states by including correlation effects. A transition of the direction of the spin polarization is predicted at specific values of the Zeeman energy. We show that these values can be expressed in terms of the pair-correlation function, and thus provide information about the microscopic ground state. We examine the particular examples of the Laughlin wave functions and the 5/2-Pfaffian state. We also include effects of the nuclear bath.

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Two-dimensional electrons in strong magnetic fields have been a rich source of new physics, a prominent example being the discovery of fractional quantum Hall states [1,2]. At large cyclotron energy the ground state is well approximated assuming a small number of completely filled low Landau levels (LLs) while the large degeneracy of the partially filled highest LL is resolved by the electron interaction.

Additional spin degeneracy is obtained at vanishing Zeeman coupling, realized in GaAs/AlGaAs heterostructures by confinement [3], hydrostatic pressure [4], or gate modulation [5]. Under these conditions, the ground state can still be spin polarized due to the Coulomb interaction, but the polarization *direction* is determined by small spin anisotropies induced by the spin-orbit interaction. The effect of the spin-orbit coupling in the quantum Hall regime was studied in [6–12]. There, it was shown that below a critical value of the Zeeman energy the spin polarization deviates from the perpendicular direction and acquires an in-plane component. The previous treatment, however, was restricted to the case of integer filling factors while we examine here the fractional regime. This represents a nontrivial extension due to the highly correlated nature of the fractional wave functions, as opposed to the integer quantum Hall states. Furthermore, we obtain the effect of the simultaneous presence of Rashba and Dresselhaus spin-orbit couplings [13,14].

As a main result, we find that by including correlation effects the polarization transition explicitly depends on the quantum Hall ground state, and to leading order is determined by the pair-correlation function. This provides a new way to address many-body properties of the wave functions in the fractional regime. In fact, polarization measurements can be performed with established experimental techniques, as, in particular, photoluminescence [15] or NMR studies [16]. Furthermore, polarization properties are generally less affected by disorder [1] (in contrast to, e.g., gap measurements).

Our discussion is generally applicable to polarized quantum Hall states. We consider here the Laughlin wave functions and the Pfaffian state at $\nu = 5/2$ [17–20]. The latter has received special attention [21–25] since it might support excitations with non-Abelian statistics [26]. This proposal is consistent with the recent observation of $e/4$ charged quasiparticles [24,25].

Let us assume a high-field ground state with a partially occupied highest LL which is fully spin polarized along an arbitrary direction $\vec{n}$. Further, a certain number J of lower LLs are fully occupied for both spin orientations. The anisotropy in the polarization direction $\vec{n}$ is determined by the Zeeman energy and a general combination of Rashba (α) and Dresselhaus (β) spin-orbit interactions

$$\delta\hat{H} = \alpha(\hat{\pi}_x\hat{\sigma}_y - \hat{\pi}_y\hat{\sigma}_x) + \beta(\hat{\pi}_x\hat{\sigma}_x - \hat{\pi}_y\hat{\sigma}_y) - \frac{g\mu_B B}{2}\hat{\sigma}_z, \tag{1}$$

where $B > 0$ (an opposite polarization is obtained if the magnetic field $\vec{B}$ is along $+\hat{z}$), and $\hat{\boldsymbol{\pi}}/m$ is the standard kinematic velocity [1,2]. Second-order perturbation theory in the spin-orbit interaction gives us the angular-dependent energy correction, expressed in terms of the spherical coordinates (θ, φ) of $\vec{n}$

$$\frac{\delta E}{pN} = \left\{-\frac{g\mu_B B}{2} + [2J + 1 - \eta f_1(\eta,\nu)]m(\alpha^2 - \beta^2)\right\} \times \cos\theta - \frac{1}{2}\eta f_2(\eta,\nu)m(\alpha^2 + \beta^2 + 2\alpha\beta\sin 2\varphi)\sin^2\theta. \tag{2}$$

In Eq. (2), N is the total number of electrons, $p = (\nu - 2J)/\nu$ is the polarization without spin-orbit coupling, and we defined the interaction parameter [27] $\eta = (e^2/\epsilon\ell)/\hbar\omega_c$, where $\omega_c = eB/mc$, $\ell = \sqrt{\hbar c/eB}$ is the magnetic length, and ϵ the dielectric constant. The expressions for $f_{1,2}$ are provided in [28] and we discuss later their explicit form to leading order in η.

From Eq. (2) we immediately obtain that the polarization $\vec{n}$ satisfies $\sin 2\varphi_m = \text{sgn}(\alpha\beta)$ (we find $f_2 > 0$). The anisotropy in φ disappears for $\alpha = 0$ or $\beta = 0$ [6,7]. The polarization $\vec{n}$ can be tilted from the vertical direction, in which case θ is given by

$$\cos\theta_m = \frac{g\mu_B B - 2m\gamma_-^2[2J + 1 - \eta f_1(\eta, \nu)]}{2m\gamma_+^2 \eta f_2(\eta, \nu)}, \quad (3)$$

where $\gamma_\pm^2 = (|\alpha| + |\beta|)(|\alpha| \pm |\beta|)$. It is easiest to consider the case in which the g factor is changed at fixed external parameters [3–5]. The transition from $-\hat{z}$ to $+\hat{z}$ is illustrated in Fig. 1. It occurs around g_c, which corresponds to the condition of in-plane polarization $\vec{n}$

$$g_c = \frac{2m\gamma_-^2}{\mu_B B}[2J + 1 - \eta f_1(\eta, \nu)], \quad (4)$$

and the transition region has a width of $2\Delta g$, where

$$\Delta g = \frac{2m\gamma_+^2}{\mu_B B}\eta f_2(\eta, \nu). \quad (5)$$

At $\alpha = \beta = 0$ one has as usual $g_c = \Delta g = 0$, while for the noninteracting problem with spin-orbit interaction $g_c \neq 0$ but still $\Delta g = 0$. Thus, the Coulomb interaction results in a shift $\sim\eta f_1$ in g_c and opens a finite region $\sim\eta f_2$ in which the polarization $\vec{n}$ acquires an in-plane component. Note also that from Eqs. (4) and (5) we obtain $g_c = 0$, but still $\Delta g \neq 0$, in the special case $|\alpha| = |\beta|$.

The effect of an in-plane component of the external field is straightforward to include in Eq. (2), via a term $\frac{1}{2}g\mu_B B_\parallel \cos(\varphi - \varphi_\parallel)\sin\theta$. This results in a correction to the equilibrium values (θ_m, φ_m) of the polarization $\vec{n}$ which is anisotropic in the field angle $\varphi_\parallel$.

A calculation of $f_{1,2}$ is in general difficult, but the leading contribution in η can be obtained explicitly in terms of "generalized" pair-correlation functions $g_{i,j}^{h,k}(\mathbf{r}_1, \mathbf{r}_2)$ [28]. These are generated iteratively from the ordinary pair-correlation function, which we parametrize as in [29]

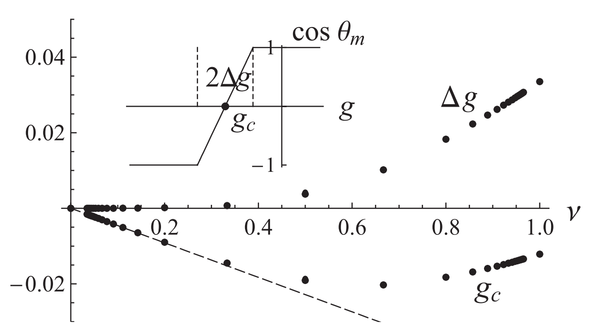

FIG. 1. Values of g_c (negative dots) and Δg (positive dots), as obtained from Eqs. (4) and (5) for states at $\nu = 1/M$ (and their particle-hole conjugates), and $\nu = 1/2$. The density is chosen such that $B = 5/\nu$ (in T), $\alpha = 0$, and other parameters are given in the main text. The dashed line is the noninteracting value of g_c ($\Delta g = 0$ for noninteracting electrons). Inset: general plot of the polarization angle θ_m [see Eq. (3)] as a function of the g factor.

$$g_{0,0}^{0,0} = 1 - e^{-|z|^2/2\ell^2} + \sum_m{}' \frac{2}{m!}\left(\frac{|z|^2}{4\ell^2}\right)^m c_m e^{-|z|^2/4\ell^2}, \quad (6)$$

where $z = z_1 - z_2$ ($z_\alpha = x_\alpha + iy_\alpha$) and the prime indicates a summation over positive odd values of m only. It is these $\{c_m\}$ that characterize the specific ground state and also parametrize the final results for $f_{1,2}$.

We find [28], at $0 < \nu < 1$,

$$f_1(0, \nu) = f_2(0, \nu) = \frac{\nu}{2}\sqrt{\frac{\pi}{2}} + \nu\sum_m{}' \frac{c_m\Gamma(m - 1/2)}{4m!}, \quad (7)$$

and at $2 < \nu < 3$,

$$f_1(0, \nu) = \frac{3\nu - 2}{8}\sqrt{\frac{\pi}{2}} + (\nu - 2)\sum_m{}' \frac{c_m\Gamma(m - 5/2)}{256m!} \times 3(8m - 15)(8m - 5), \quad (8)$$

$$f_2(0, \nu) = \frac{7(\nu - 2)}{8}\sqrt{\frac{\pi}{2}} + (\nu - 2)\sum_m{}' \frac{c_m\Gamma(m - 5/2)}{256m!} \times (105 - 112m + 64m^2), \quad (9)$$

where the summations are restricted to positive odd integers. The remarkable result $f_1 = f_2$ when $0 < \nu < 1$ is only established to lowest order in η.

As a first application, we consider now the case of the Laughlin trial wave functions, which are appropriate for $\nu = 1/M$ where M is an odd integer. For simplicity, we adopted the approximation used in [29]. This amounts to set $c_m = -1$ for $m < M$ and $c_m = 0$ for $m > M + 4$. The three remaining coefficients are determined by exact sum rules [29]. We obtain $f_{1,2} = 0.0710, 0.0301$ for $M = 3, 5$, respectively. These values show small deviations for more accurate parametrizations of the $\{c_m\}$ coefficients (e.g., using the $\{c_m\}$ of [30] gives $f_{1,2} = 0.0708, 0.0300$). The same approximation is used at $\nu = 2 + 1/M$ and, by making use of the particle-hole symmetry [28], the states at $\nu = 1 - 1/M$ and $\nu = 3 - 1/M$ can also be studied.

We turn now to the Pfaffian (Pf) trial state, which implies half filling of the highest LL ($\nu = 1/2, 5/2$). A closely related compressible state is the polarized composite Fermi sea (CFS). The Pfaffian state is produced by pairing of free composite Fermions (CFs) [31,32], due to their residual interaction. Therefore, the quantitative properties of these two trial states are very similar [33]. We list

TABLE I. Parameters of the pair-correlation function for the composite Fermi sea (CFS) and Pfaffian (Pf) wave functions.

	CFS	Pf		CFS	Pf
c_1	−0.5699	−0.4205	c_9	−0.3518	0.8761
c_3	0.4559	0.0333	c_{11}	0.9403	−1.406
c_5	−0.0261	0.3521	c_{13}	−0.7151	1.170
c_7	−0.1660	−0.4853	c_{15}	0.1825	−0.3703

in Table I their $\{c_m\}$ parametrizations, which we obtained by fitting the pair-correlation functions of [33]. At $\nu = 1/2$ we have $f_{1,2}^{\rm CFS} = 0.20$ and $f_{1,2}^{\rm Pf} = 0.22$, while at $\nu = 5/2$

$$f_1^{\rm CFS}(0, 5/2) = 1.16, \qquad f_1^{\rm Pf}(0, 5/2) = 1.01, \qquad (10)$$

$$f_2^{\rm CFS}(0, 5/2) = 0.49, \qquad f_2^{\rm Pf}(0, 5/2) = 0.45. \qquad (11)$$

Evidently, the values of $f_{1,2}$ reflect the different correlations of these two trial states. We obtain a $\sim$10% relative change in the values of $f_{1,2}$, which is significantly larger than the change in the total energy [33].

The values of $f_{1,2}$ can be accessed through the measurement of g_c and Δg [see Eqs. (4) and (5)], which makes them an experimentally relevant characterization of the quantum Hall state. As it is clear from Eqs. (7)–(9), $f_{1,2}$ provide information about the c_m coefficients, and therefore on the pair-correlation function. In fact, by truncating the series (7)–(9) to the two lowest terms, an estimate of c_1 and c_3 from the measured values of f_1 and f_2 is obtained. This procedure is justified since the c_m prefactors decrease like $m^{-3/2}$.

For example, using our "exact" values of $f_{1,2}$ in (10) and (11) one obtains for the Pfaffian $c_{1,3} \simeq -0.43$, 0.07, in reasonable agreement with Table I and clearly distinct from the CF sea values. Furthermore, to obtain approximate values of $c_{1,3}$ at $\nu = 1/3$ and $1/5$ would test the distinct short-range behavior of the pair-correlation functions ($\sim r^6$ and r^{10}, respectively) of the Laughlin wave functions. Also in the controversial case $\nu = 7/3$ (see, e.g., [34]), to measure c_1 would provide a test of the Laughlin model at this filling factor.

Let us now estimate the effects for typical GaAs parameters, and thereby demonstrate that our predictions are within experimental reach. We evaluate Eqs. (4) and (5) using $m = 0.067m_0$, $\epsilon = 12.4$, and for a symmetric well with thickness $L = 6$ nm, close to the value at which the g factor is zero [3,4]. We obtain for the Dresselhaus coupling $\hbar\beta = \lambda(\pi/L)^2 \simeq 27$ meV Å, where $\lambda \simeq 10$ eV Å^3 [35], and $\alpha = 0$. The results for g_c and Δg are plotted in Figs. 1 and 2 in the range $0 < \nu < 1$ and $2 < \nu < 3$, respectively. We assumed a constant density $\rho = 1.21 \times 10^{11}$ cm^{-2} in the first case and $\rho = 3.02 \times 10^{11}$ cm^{-2} for the second one [17]. As seen, the values of g_c, Δg are in the range already realized in practice [3–5].

Figures 1 and 2 show that the effect of the interaction on the values of g_c can be rather large. This is clearly identified as the deviation of the g_c values (points) from the dashed line, which refers to the noninteracting result. Furthermore, the transition region Δg can be sizable, as opposed to the noninteracting case $\Delta g = 0$. When $\nu \to 0$ (see Fig. 1), the values of g_c approach the noninteracting linear dependence of Eq. (4) (with $J = 0$, $\eta = 0$, and $\gamma_-^2 = -\beta^2$). This limit allows one to extract β, independently from other methods known in the literature.

We note that the difference between specific realizations at $\nu = 5/2$ (CFS and Pf) is small compared with the total effect. Nevertheless, we suggest that the relative change could be detected from temperature-dependent measurements. When the temperature exceeds the pairing energy (but remains still smaller than the CFs kinetic energy), the existence of a CF sea was experimentally demonstrated in [32]. We expect that the CFS values of Fig. 2 would be observed in this high-temperature regime. With decreasing temperature, g_c and Δg evolve into the Pfaffian values, due to the formation of the incompressible state of paired CFs [31,32].

Higher order corrections in η affect the precise values of g_c and Δg. Since η is often not particularly small under the typical conditions at which the $\nu = 5/2$ state is observed ($\eta \simeq 0.74$ at the highest field 12.6 T in [36]), measurements at larger values of B would be desirable. We show in Fig. 3 the high-field dependence of g_c, Δg from Eqs. (4) and (5). The f_1 coefficient determines the $\propto 1/\sqrt{B^3}$ correction to the noninteracting background, which is linear in $1/B$. The f_2 coefficient gives the leading $\propto 1/\sqrt{B^3}$ contribution to Δg. Alternatively, higher orders in $1/\sqrt{B}$ have to be explicitly computed.

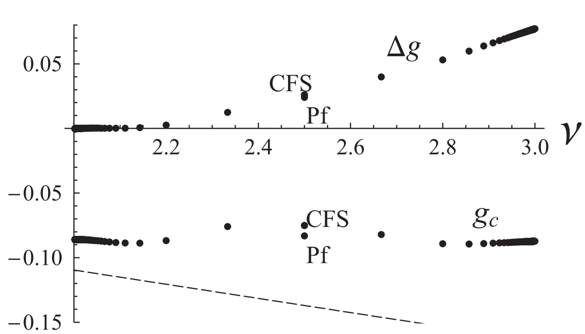

FIG. 2. Same as Fig. 1, in the range $2 < \nu < 3$. At $\nu = 5/2$ both composite Fermi sea (CFS) and Pfaffian (Pf) results are displayed. The density is such that $B = 12.5/\nu$ (in T) and $\alpha = 0$. The noninteracting g_c is also plotted (dashed line).

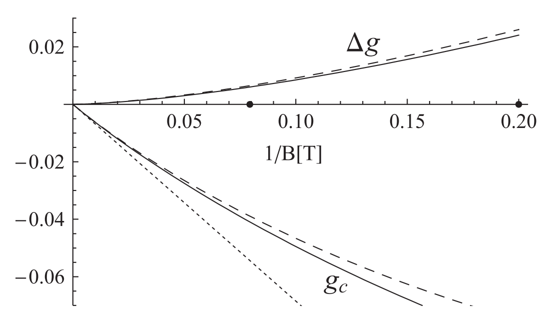

FIG. 3. Plot of the values of g_c (negative) and Δg (positive) from Eqs. (4) and (5) at $\nu = 5/2$, $B > 5$ T and other parameters as in the text. The solid and dashed lines correspond to the Pfaffian state and CF sea, respectively [$f_{1,2}$ as in Eqs. (10) and (11)]. The dotted line is the linear noninteracting contribution to g_c ($f_1 = 0$). The dots mark the B-field values of [17,36].

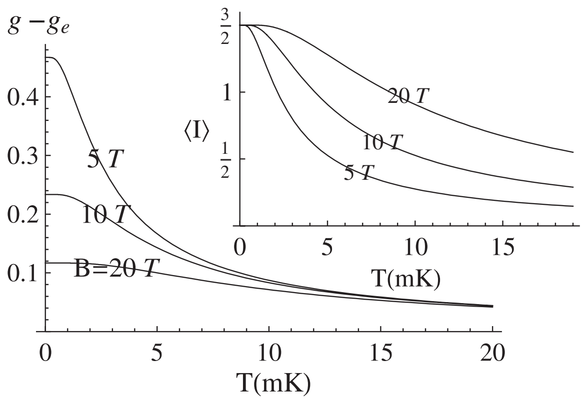

FIG. 4. Nuclear shift of the hyperfined modified electron g factor (see text) as a function of temperature T at different values of B. In the inset, the average nuclear polarization $\langle I\rangle = \sum_i x_i \langle \hat{I}_z\rangle_i$ is also shown.

The assumption of full polarization of the highest LL is justified at several values of ν (e.g., $\nu = 1/M$ and $\nu = 5/2$). At other fractional values the ground state can be unpolarized (e.g., $\nu = 1/2$, $2/3$) or partially polarized (e.g., $\nu = 3/5$, $3/7$). For the latter case, a similar effect is expected, driven to leading order by the noninteracting contribution ($f_{1,2} = 0$), but our calculation of $f_{1,2}$ does not apply. It is of conceptual interest to consider a large value of β, such that full polarization is obtained around g_c in the whole intervals $0 < \nu < 1$ and $2 < \nu < 3$. It would then be possible to observe nonanalytic features at the incompressible values (see [28]), similarly to the predicted cusps in the total energy [37].

Finally, we note that at ultralow temperatures at which the $\nu = 5/2$ state is observed, there is a significant effect from the nuclear spin bath. This contribution can be easily included by interpreting g in Eq. (1) as $g = g_e - \sum_i x_i A_i \langle \hat{I}_z\rangle_i / \mu_B B$, where g_e is the "bare" electron g factor of the heterostructure and the second term is the Overhauser shift produced by the hyperfine interaction. Here, x_i are the fractions relative to the different nuclear species (equal to 0.5, 0.3, 0.2 for ^{75}As, ^{69}Ga, ^{71}Ga, respectively) and A_i are the corresponding hyperfine couplings (with estimated values [38] 94, 77, 99 μeV). In Fig. 4 we plot the shift g-g_e as function of T for different values of B. The high-T limit gives $g - g_e \simeq 0.9/T$ (T in mK), independent of B. We see that a change of temperature might provide a practical way of tuning the small Zeeman energies involved.

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