Intruder Configurations in the $A = 33$ Isobars: ^{33}Mg and ^{33}Al

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(Received 2 July 2008; published 2 October 2008)

The β decay of ^{33}Mg ($N = 21$) presented in this Letter reveals intruder configurations in both the parent and the daughter nucleus. The lowest excited states in the $N = 20$ daughter nucleus, ^{33}Al, are found to have nearly $2p - 2h$ intruder configuration, thus extending the "island of inversion" beyond Mg. The allowed direct β-decay branch to the $5/2^+$ ground state of the daughter nucleus ^{33}Al implies positive parity for the ground state of the parent ^{33}Mg, contrary to an earlier suggestion of negative parity from a g-factor measurement. An admixture of $1p$-$1h$ and $3p$-$3h$ configurations is proposed for the ground state of ^{33}Mg to explain all of the experimental observables.

DOI: 10.1103/PhysRevLett.101.142504 PACS numbers: 23.20.Lv, 21.60.Cs, 23.40.−s, 27.30.+t

Investigations of exotic neutron-rich *sd* shell nuclei continue to yield surprising results [1–3]. They have highlighted how the filling of proton orbitals can change the position of neutron shells and also the effectiveness of neutron-neutron correlations in overcoming smaller shell gaps. Modern shell model calculations in a valence space including the *sd* shell and all *or* part of the *fp* shell [4–6] are coming close to accounting for the nuclear structure in this region. However, some important questions remain to be resolved, including difficulties in predicting theoretically the ground state configurations of many neutron-rich nuclei such as ^{31}Mg [7] and ^{33}Mg [8]. Satisfactory understanding of these cases will lead to reliable models with more confidence in extrapolating towards the drip line and also the investigation of $f_{7/2}$ intruder states in less exotic nuclei.

In this Letter, we look at the conflicting evidence for the ground state configuration of the odd mass isotope ^{33}Mg. An allowed β decay between the ground states of the $A = 33$ isobars is documented [9–11], namely, $^{33}\mathrm{Na} \rightarrow {}^{33}\mathrm{Mg} \rightarrow {}^{33}\mathrm{Al}(5/2^+) \rightarrow {}^{33}\mathrm{Si}(3/2^+)$. Although the argument gets weaker with increasing neutron excess, this would suggest positive parity for all of the ground states. For ^{33}Mg ($N = 21$), the valence neutron is in the $f_{7/2}$ orbital, and positive parity implies an even number of neutrons in the *fp* shell; i.e., 1 or 3 neutrons are excited from the lower *sd* shell. However, a recent g-factor measurement for ^{33}Mg has inferred a $J^\pi = 3/2^-$ ground state by comparing their measured negative g-factor value to theoretical predictions [8]. A $2p$-$2h$ configuration (3 neutrons in the $f_{7/2}$ orbital) is proposed to account for the negative magnetic moment. In the present work we studied the β decay of ^{33}Mg to ^{33}Al and confirmed the allowed ground state to ground state decay. We propose an alternative intruder configuration leading to $J^\pi = 3/2^+$ which will be consistent with all of the experimental observations.

The spin and parity of the ground state of ^{33}Mg are strongly dependent on the number of *sd* neutrons which move up to the *fp* shell. This in turn decides the states to which it decays in the daughter nucleus ^{33}Al, as the Gamow-Teller transformation would favor the $\nu(sd) \rightarrow \pi(sd)$ conversion. The promotion of neutrons to the upper shell represents a dynamic balance between the shell gap which varies with the proton number, due to the spin-isospin interaction [4], and the gain in correlation energy which can offset smaller gaps. Figures 1(a) and 1(b) display the decrease of the $N = 20$ shell gap with decreasing Z and the average number of neutrons occupying the *fp* shell in the ground states of the $N = 20$ nuclei, as predicted by Monte Carlo shell model (MCSM) calculations using the *sd* to *pf* mixed shell (SDPF-M) interaction [6,12]. A threshold behavior can be seen with a drop from an intruder to normal ground state configuration from ^{32}Mg [2] to ^{34}Si [13]. This makes the case that the transitional nucleus ^{33}Al should provide a sensitive test of the models.

The transitional nature of ^{33}Al, however, is not reflected in its measured ground state properties, namely, the β-decay half-life [11] and g factor [14]. These can be largely explained without invoking excitations beyond the *sd* shell and thus are not sensitive to the occupancy of the $f_{7/2}$ orbital. The present detailed study of the excited-state structure of ^{33}Al following the β decay of ^{33}Mg, on the other hand, gives insight to the intruder dominance in this nuclide. The lowest excited states are attributed to neutrons occupying the lower *fp* shell and have no counterpart in a pure *sd* shell picture. This establishes the transitional nature of ^{33}Al and suggests similarities with ^{29}Na [15], both of which have $T_z = 7/2$.

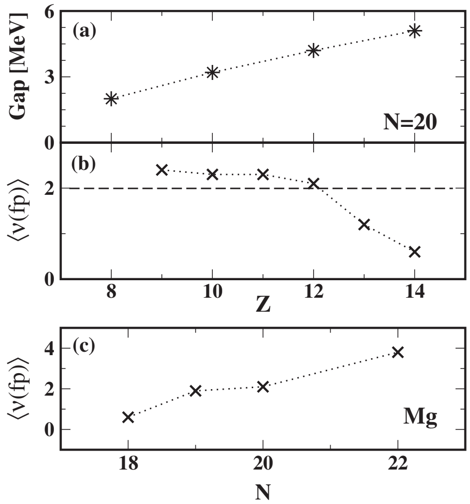

FIG. 1. MCSM calculation predictions for the (a) $N = 20$ shell gap and (b) the average number of neutrons promoted to the fp shell in the $N = 20$ isotopes of O to Si. (c) Average number of neutrons in the fp shell for the Mg isotopes extracted by comparing experimental information with MCSM calculations.

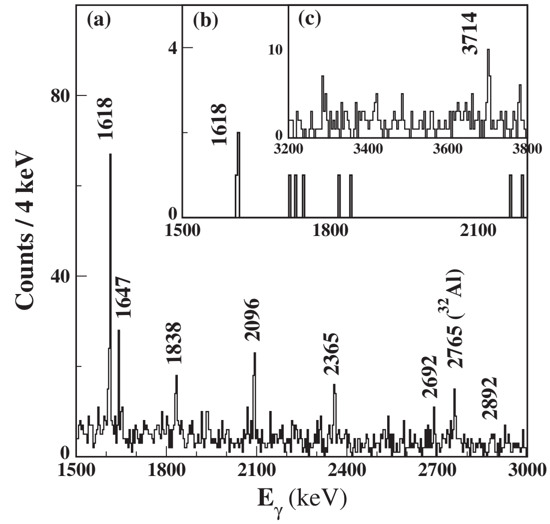

FIG. 2. (a) γ spectrum for events within the first 100 ms after a β^- correlated ^{33}Mg implantation (1.5–3.0 MeV range). The γ rays assigned to ^{33}Al are labeled by γ-ray energy. (b) Coincidence spectrum between the 1618 and 2096 keV (gate) transitions which produces the 3714 keV level. (c) High-energy part of the spectrum in (a) to show the 3714 keV transition which depopulates the state at the same energy.

The β^- decay of ^{33}Mg was studied at the National Superconducting Cyclotron Laboratory at Michigan State University. The exotic ^{33}Mg secondary beam was produced by the fragmentation of a 140 MeV/nucleon ^{48}Ca beam on a 752 mg/cm^2 Be target located at the object position of the A1900 fragment separator, separated using a 300 mg/cm^2 wedge-shaped Al achromatic wedge and identified at the focal plane of the A1900. With the magnetic rigidities of the A1900 magnets set to 4.7856 and 4.6558 Tm and a momentum acceptance of 2%, a cocktail beam of ^{30}Ne, ^{31,32}Na, and ^{33}Mg was transported to the beta counting system (BCS) [16]. The β decays of these exotic nuclei were studied by implanting in the 40×40 double-sided Si microstrip detector (DSSD), the key component of the BCS. The DSSD detected the high-energy fragments and the subsequent low-energy decay products. Each event was time stamped so that fragment-β events could be correlated in software based on the decay half-life. The electromagnetic transitions from the daughter and granddaughter nuclei were detected in 16 detectors of the Segmented Germanium Array [17] arranged in a compact geometry around the BCS. More details of the experimental setup and analysis are available in Refs. [15,18].

A partial γ spectrum following the β^- decay of ^{33}Mg implantations is shown in Fig. 2, where the γ transitions proposed to be in ^{33}Al are marked. Some of these transitions have been reported in an earlier β-decay investigation [10]. The most intense 1618 [absolute intensity of 16 (2)%], 1838 [9(2)%], and 2365 keV [7(1)%] transitions are proposed to decay directly to the ground state. The 2096 keV [9(2)%] transition is found to be coincident with the 1618 keV one (Fig. 2), whereas the 2892 keV [3 (1)%] γ line represents the decay from the 4730 keV level to the 1838 keV second excited state. Direct decays from the 3714 and 4730 keV levels to the ground state have absolute intensities of 8(2)% and 10(3)%, respectively. The cascade of γ lines, 1466 [3(1)%], 1647 [7(1)%], and 1618 keV, further supports the 4730 keV level. The 596 [5(1)%] and 1046 keV [4(1)%] transitions are proposed to depopulate a level at 4310 keV. The two strongest transitions in ^{32}Al produced in β delayed neutron emission, 2765 and 735 keV [19], are also observed with absolute intensities of 11(2)% and 8(1)%, respectively.

Displayed in Fig. 3 is the proposed level scheme for ^{33}Al where the absolute β-decay branches to the excited states were obtained using the measured γ-ray efficiencies and the total β-decaying implants. The β correlated decay curve of the ^{33}Mg implantations was fitted to extract the total implants [$19.45(215) \times 10^3$] and yielded a half-life of 89(1) ms in agreement with the previously reported value [11]. For determining the ground state branch, the neutron emission probability P_n of 14(2)% was taken from the previous β-decay investigation of ^{33}Mg [10], which measured the β delayed neutrons. The absolute intensities along with measured half-life and the Q_{β^-} value [20] were used to obtain the apparent logft values for the ground as well as the excited states.

The large apparent β-decay branch of 37(8)% to the ground state of ^{33}Al (Fig. 3) implies an allowed decay from the ^{33}Mg ground state. The ground state spin and parity of ^{33}Al are known to be $5/2^+$ [21]; hence, the ground state of ^{33}Mg based on the above information should have positive parity. However, negative parity is inferred for the ^{33}Mg ground state ($J^\pi = 3/2^-$) based on the measurement of a negative g factor in Ref. [8]. An assignment of negative

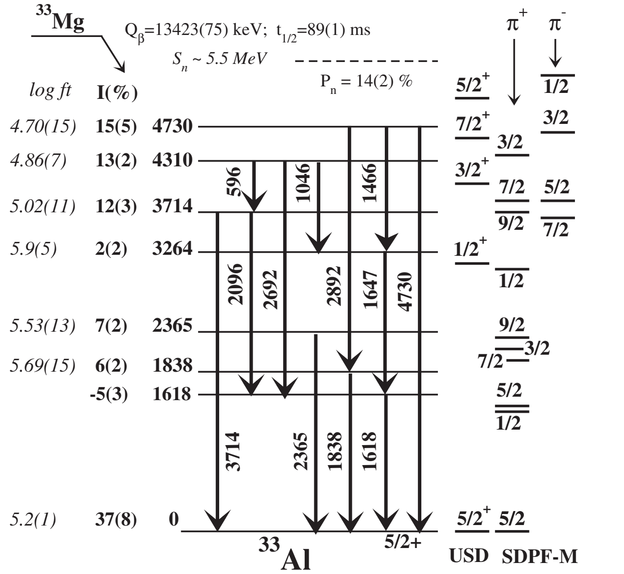

FIG. 3. Level scheme for ^{33}Al from the present work along with predictions of the MCSM calculation with SDPF-M interaction [6] (both positive and negative parity states). Also shown are the results using the universal "sd" (USD) interaction [26].

parity is inconsistent with our findings and also contradicts the β-decay results reported for ^{33}Na [9], where an allowed β decay to the ground state of ^{33}Mg was observed from the ^{33}Na ground state which has a proposed spin and parity of $3/2^+$.

The results of the previous β-decay experiment on ^{33}Mg [10] were interpreted in Ref. [8] as being consistent with their assignment of negative parity for the ground state of ^{33}Mg (though inconsistent with the β decay of ^{33}Na). However, from Fig. 2 of Ref. [10] it can be seen that the ground state branch is 18(9)% (the ground state and other observed branches, bound and unbound, add up to 100%), which yields a $\log ft$ value in the allowed range. In fact, the 2761 keV transition proposed in Ref. [10] as a direct decay to the ^{33}Al ground state is likely the 2765 keV γ ray in ^{32}Al [19]. Removing this transition from the level scheme of Ref. [10], and with the 2096 keV γ ray now shown to feed the 1618 keV level instead of the ground state, makes the apparent ground state branch closer to our results.

A negative parity ground state of ^{33}Mg also contradicts the apparent feeding to states below 2.5 MeV in ^{33}Al seen in the present work and in Ref. [10]. If the ^{33}Mg ground state is $3/2^-$, as proposed in Ref. [8], these states should have $J^\pi = 1/2^-$–$5/2^-$ based on the allowed nature of the transitions. The MCSM calculations using the SDPF-M interaction predict no negative parity states below 3.6 MeV (Fig. 3). Although this argument appears model-dependent, one can use experimental systematics to estimate the excited states in ^{33}Al as a test for the reliability of the MCSM calculations. The lowest excitations in ^{33}Al can be considered as $d_{5/2}$ proton hole states relative to $N = 20$ ^{34}Si. Coupling of the 2^+ state in ^{34}Si [13] to $\pi d_{5/2}$ gives states from $1/2^+$ to $9/2^+$, the same range of spins as predicted by the MCSM calculations shown in Fig. 3. A magnetic substate weighted average of these predicted states gives an "effective" 2^+ energy of 1.9 MeV for ^{33}Al, which lies between the 885 keV of ^{32}Mg and 3.3 MeV of ^{34}Si. Thus, the average position of the lowest positive parity MCSM states is consistent with the nearby experimental systematics. Similarly, the lowest negative parity state predicted by the MCSM calculations lies almost midway between the experimentally observed lowest π^- states in ^{32}Mg (2.9 MeV) [22] and ^{34}Si (4.3 MeV) [13].

All of the known experimental signatures from the β-decay investigation of ^{33}Na and ^{33}Mg refute a negative parity for the ground state of ^{33}Mg in favor of positive parity [9,23]. To reiterate, the negative parity assignment to the ^{33}Mg ($N = 21$) ground state in Ref. [8] is made considering that only a $2p$-$2h$ configuration (Fig. 4) could theoretically reproduce the measured spin of $3/2$ and g factor of $-0.4971(4)$. Contrary to the argument in Ref. [8], promoting an odd number of neutrons (1 or 3) for this $N = 21$ isotope to the fp shell yielding positive parity can also give a negative g factor if the fp neutrons couple to nonzero spin. This will be similar to the $2p$-$2h$ configuration proposed for the ^{31}Mg ($N = 19$; $J^\pi = 1/2^+$) ground state (Fig. 4) which has a magnetic moment of $-0.883\,55(15)\mu_N$ [7]. Here the spin and magnetic moment can be reproduced only by coupling the $f_{7/2}$ neutrons to spin 2.

The ground state g factor of ^{31}Mg and ^{33}Mg, considering coupling of valence neutrons in the $d_{3/2}$ and $f_{7/2}$ orbitals, n and m, respectively, to $I_{3/2}$ and $I_{7/2}$ and total spin I, i.e., a configuration $[(j^n_{3/2})_{I_{3/2}}(j^m_{7/2})_{I_{7/2}}]_I$, is given by [24]

$$g = \frac{1}{2}[g(j_{3/2}) + g(j_{7/2})] + \frac{1}{2}[g(j_{3/2}) - g(j_{7/2})] \times \frac{I_{3/2}(I_{3/2}+1) - I_{7/2}(I_{7/2}+1)}{I(I+1)}.$$

Thus for ^{31}Mg ($2p$-$2h$ configuration), with $I_{3/2}$, $I_{7/2}$, and I equal to $3/2$, 2, and $1/2$, respectively, we obtain $\mu = -0.796\,21\mu_N$ [$g(j_{3/2}) = 0.681\,12$ from ^{39}Ca and $g(j_{7/2}) = -0.455\,65$ from ^{41}Ca [25]]. Similarly, a $1p$-$1h$ or $3p$-$3h$ ground state neutron configuration for ^{33}Mg ($N = 21$) will give positive parity for the ground state and a negative g factor with $I_{3/2}$, $I_{7/2}$, and I equal to $3/2$, 2, and $3/2$, respectively (Fig. 4). That is, a pair of the $f_{7/2}$ neutrons is coupled to $I = 2$ rather than $I = 0$. While this is not the lowest energy coupling for spherical nuclei, the "island of inversion" nuclei are not spherical, and the $I = 2$ coupling has already been seen to form the lowest energy configuration in ^{31}Mg [7].

Positive parity of the ^{33}Mg ground state implies that the lowest excited states in ^{33}Al are positive parity having no counterparts in pure sd shell excitations and hence are intruders (Fig. 3). The MCSM calculations predict an average occupation of two neutrons in the fp shell for

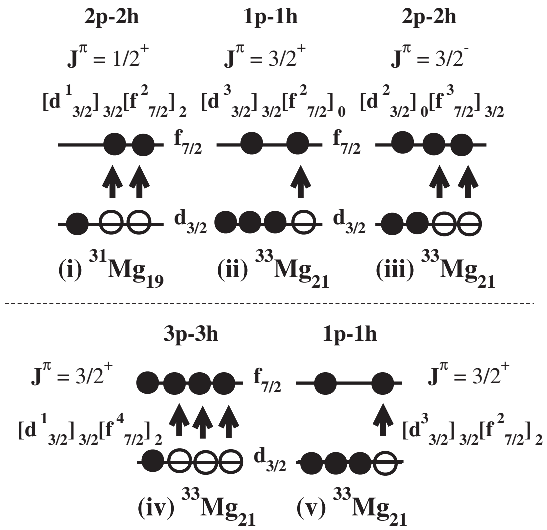

FIG. 4. (i) Neutron configuration proposed for ^{31}Mg [7]; (ii), (iii) configuration considered for ^{33}Mg in Ref. [8], only (iii) yields a negative magnetic moment; (iv),(v) configurations for ^{33}Mg proposed in the present work which yield positive parity and a negative magnetic moment. For clarity, we are counting particles and holes relative to the simplest configuration implied by filling the normal (spherical) ordering of the shells.

these states (except the $5/2^+$ states), similar to other $N = 20$ isobars in the island of inversion. Thus excited states in ^{33}Al are likely to have a predominant $2p$-$2h$ configuration, which suggests a $1p$-$1h$ configuration for the parent nucleus ^{33}Mg. A $1p$-$1h$ configuration would favor a $\nu(d_{3/2}) \rightarrow \pi(d_{5/2})$ Gamow-Teller transition in its β decay, creating $2p$-$2h$ excited states in the daughter nucleus. The large feeding to the ground state indicates a significant $2p$-$2h$ configuration in the ground state of ^{33}Al also, based on the allowed nature of the direct ground state to ground state transition. This conclusion differs from the earlier suggestions that ^{33}Al can be considered outside of the island of inversion, based on the good agreement of its β-decay half-life and only $\sim$4% departure of its g factor from the USD predictions [14]. The significant intruder configuration admixtures for the ground state and excited states of ^{33}Al seen in the β decay of ^{33}Mg thus expand the island of inversion region beyond the $Z = 10$–12 range. ^{33}Al with $Z = 13$ seems to start the transition as a function of the proton number, as ^{34}Si displays "normal" behavior with its small binding energy and high-lying 2_1^+ state at 3.3 MeV [13]. ^{32}Mg, on the other hand, has a low-lying 2_1^+ state at only 0.885 MeV [2] and a large deformation and clearly classifies as a member of the island of inversion.

While the β-decay arguments stated above favor a $1p$-$1h$ configuration for the ^{33}Mg ground state, the increasing occupancy of the fp shell with neutron excess, as shown in Fig. 1(c), suggests excitation of more neutrons. A $3p$-$3h$ configuration (4 neutrons in fp) thus is also possible for the positive parity ground state in accordance with the large deformation. Taken together, the most likely configuration for the ^{33}Mg ground state seems to be a combination of $1p$-$1h$ and $3p$-$3h$ (Fig. 4). This mixed configuration is consistent with all of the observed properties, including positive parity, g factor, and β decay, if the $f_{7/2}$ neutrons generate a total spin of 2, as is the case for ^{31}Mg.

In summary, the β-delayed γ spectroscopy of ^{33}Mg has provided strong evidence for a positive parity ground state in ^{33}Mg. A mixture of $1p$-$1h$ and $3p$-$3h$ configurations is proposed which is consistent with all of the experimental evidence, including the negative magnetic moment. The experimental evidence for excited states populated in the β decay also helps confirm the theoretical prediction of a mixed $0p$-$0h$ and $2p$-$2h$ ground state configuration in ^{33}Al with mostly $2p$-$2h$ low-lying excited states. This places ^{33}Al within the island of inversion, which was proposed to end with the Mg isotopes.

The authors appreciate the NSCL operations staff for their help. This work was supported by the NSF Grants No. PHY-04-56463 and No. PHY-06-06007 and in part by a Grant-in-Aid for Specially Promoted Research (13002001) from the MEXT of Japan.

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